Chapter 10 Notes

These notes correspond to chapter 10 of Mas-Colell, Whinston, and Green.

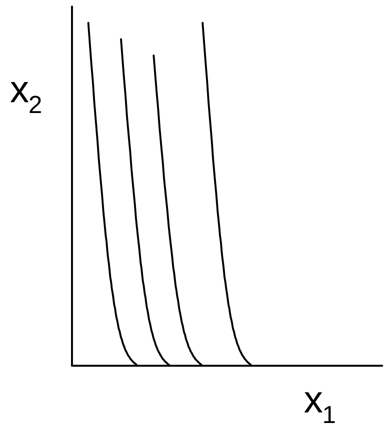
1 Introduction

This section is the first introduction to an economy. Up until now we have studied consumer behavior and producer behavior in isolation, deriving results for both. In this section we will combine the two and discuss the concepts of optimal allocations and competitive equilibrium. Our discussion will focus on partial equilibrium (PE) analysis as opposed to general equilibrium (GE) analysis (next semester you get GE). A partial equilibrium analysis assumes that there are no feedback effects to other markets when changes occur in the market under study. So, if we are studying the Beanie Babies market, it is fairly likely (as of the current day) for a vast amount of consumers that changes in the Beanie Babies market do not affect behavior in other markets, or if they do then that effect is minimal. Partial equilibrium allows us to determine prices, profits, production, and other variables of interest without concern for these feedback effects. Now consider the market for gasoline. Given all the uproar when the price of gasoline increases 3 cents, it is unlikely that a partial equilibrium analysis would suffice if the market for gasoline were the one to be studied. Also, when the topic of interest requires that the economy as a whole be studied, such as when one wants to model economic growth of a country, a general equilibrium analysis is more appropriate because increasing production in one sector may very well impact the production in other sectors. Finally, in PE analysis individual's wealth is exogenous, whereas it is endogenous in GE analysis.

Consider a market for a single good for which each consumer's expenditure constitutes a small portion of his overall budget. Here are the basic assumptions of PE laid out in bullet points.

- Changes in the market for this good will leave the prices of all other commodities unaffected
- Negligible wealth effects in the market under study – thus, when wealth changes there is essentially no change in demand for the product in question. Consider the market for toothpicks – when an individual's wealth increases, it is unlikely (unless the individual is *extremely* poor and does not purchase any toothpicks) that the individual purchases more toothpicks because wealth increases. Also, if the individual's wealth decreases it is unlikely that the individual decreases consumption of toothpicks since the amount spent on toothpicks is likely negligible, maybe a few dollars a year.

Our focus is on a two-good economy. One good will be the good in question (call this x_2). A second good will be a composite of all other goods in the economy (call this the numeraire good x_1). We also assume that consumer utility is quasilinear in the numeraire good. By quasilinear this means "nearly" linear, so that consumer indifference curves look like (the indifference curves should be parallel):



Quasilinear preferences

Why quasilinear preferences? With quasilinear preferences, it can be shown that demand functions of non-numeraire goods are independent of wealth. It can also be shown that $v(p, w) = w + \phi(p)$ for some function ϕ. Note that this is consistent with our assumption that wealth effects for the good in question are negligible.

There are many interesting aspects of a market to study besides the properties of optimality and equilibrium, which will be the focus of our study. One is the actual market process itself – how are the goods sold? Is it a bilateral bargaining institution (used-car market)? Is it a posted-offer market (retail outlets)? Is it a double auction (NYSE)? Is it a one-sided auction (Ebay)? Another may be how the rules and norms in place in the market may affect behavior. But optimality (or efficiency) and equilibrium provide a starting point that is used in nearly every discussion of markets. Economists want to know if (1) a particular allocation of items is leaving people as "well off as possible" (we'll qualify this statement shortly) and (2) whether or not the market is balanced or at rest (equilibrium). The question then becomes what happens when change occurs in the market. This may be something completely out of the control of humans, like a natural disaster, or something completely within the control of humans, like a policy change in the market (tax or the imposition of a price control). We can then examine the impact of the change in the market by examining whether or not the new allocation is optimal (or efficient) and how different market participants are affected in the new equilibrium. These are important concepts even when the discussion centers around a particular market process, like an auction. The primary question (from a societal view) is does the mechanism allocate the goods to the individual who values them the most (does the mechanism provide an efficient allocation). A secondary question is how the equilibrium is affected by a particular mechanism (if you are a seller of a good in an auction you would like to choose an auction design that yields more revenue rather than less).

2 Pareto Optimality and Competitive Equilibria

There are two key concepts that we will discuss. The first is what it means for the allocation in the economy to be "optimal". The second is what it means for an economy to be in "equilibrium". But first we need to define an economy:

1. There are I consumers, indexed $i = 1, ..., I$
2. There are J firms, indexed $j = 1, ..., J$
3. There are L goods, indexed $\ell = 1, ..., L$

Each consumer i has preferences over consumption bundles $x_i = (x_{1i}, ..., x_{Li})$, where x_i is a vector of L goods for consumer i available in consumer $i's$ consumption set $X_i \subset \mathbb{R}^L$ and represented by the utility function $u_i(\cdot)$.

The total amount of each good initially available in the economy (the endowment) is denoted by $w_\ell \geq 0$ for $\ell = 1, ..., L$.

Firms use a production technology to transform some of the initial endowment into additional amounts of other goods. Each firm has production possibilities given by $Y_j \subset \mathbb{R}^L$. Each element of Y_j is a production vector $y_j = (y_{1j}, ..., y_{Lj}) \in \mathbb{R}^L$, and $(y_1, ..., y_J) \in \mathbb{R}^{LJ}$ are the production vectors for the J firms. The total net amount of good ℓ available in the economy is given by

$$w_\ell + \sum_{j=1}^{J} y_{\ell j}$$

Definition 1 *An economic allocation* $(x_1, ..., x_I, y_1, ..., y_J)$ *is a specification of a consumption vector* $x_i \in X_i$ *for each consumer* $i = 1, ..., I$ *and a production vector* $y_j \in Y_j$ *for each firm* $j = 1, ..., J$. *The allocation is* feasible *if*

$$\sum_{i=1}^{I} x_{\ell i} \leq w_\ell + \sum_{j=1}^{J} y_{\ell j}$$

Thus, the allocation is feasible if the amount consumed is less than or equal to the amount available (endowment $\pm$ production) in the economy. That seems fairly reasonable.

Definition 2 *A feasible allocation* $(x_1, ..., x_I, y_1, ..., y_J)$ *is* Pareto optimal *(or Pareto efficient) if there is no other feasible allocation* $\left(x'_1, ..., x'_I, y'_1, ..., y'_J\right)$ *such that* $u_i(x'_i) \geq u_i(x_i)$ *for all* $i = 1, ..., I$ *and* $u_i(x'_i) > u_i(x_i)$ *for at least one consumer* i.

Again, this seems fairly reasonable. An allocation is optimal if there is no other allocation that can keep all consumers at the same level of utility while raising the level of at least one other consumer. Thus, the minimal criteria of optimality is that all the goods are consumed and resources used by someone. There is nothing in this definition that discusses the equity of the final distribution. It could be that $I-1$ individuals are at subsistence and that $I = 1$ individual is living like Louis XIV. However, this allocation is optimal in our sense because we cannot make any individuals better off without making someone else worse off. Returning to the auction example, does optimality imply that the highest valued user of the item receive the item, or just that some user receive the item? Well, if the second highest valued user has a "value" of \$20 (denote value in \$) and the highest valued user has a value of \$50, and the second highest pays \$15 for the item, this is not a Pareto optimal allocation. The second highest valued individual has a surplus of \$5 in value, while the highest valued individual has a surplus of zero (because he has not consumed the item). The highest valued individual could pay the second highest valued individual \$26 – then the second highest value individual would have a surplus of \$6, the highest valued individual would have a surplus of \$24, and all other participants (including the original auctioneer) would be left just as well off as before. So the allocation with the second highest valued individual receiving the good cannot be optimal in our sense even though someone has received the resource. Thus, there is slightly more to Pareto optimality then the resources/goods are being used – in fact, if Louis XIV could find some trade with a subsistence individual that would make either himself or the farmer better off without hurting the other, then there would be a Pareto improvement that could be made.

The focus is on competitive market economies, where society's initial endowments and production technologies (firms) are owned by the consumers. While we will not discuss GE in detail in this course, the real beauty of the theory is that it shows how "the competitive market" can achieve efficient allocations without some central agency planning production and setting prices. Many people forget the time period during which the theory was developed – it was developed during the height of the Cold War, when the US was more of a market-based economy than the USSR, which was more of a command economy.[1] The goal was to show how competitive markets could achieve efficient allocations, and some minimal assumptions necessary for that achievement. 50 years ago this was not strongly believed (and there are still many doubters out there). In fact, I believe in early editions of Samuelson's text there are pictures showing the growth rates for the US and the USSR economies, and that the USSR economy was supposed to have outgrown the US in the late 1980s. And this is Samuelson, the youngest ever Nobel Laureate in Economics. Enough for the digression. By assumption, in a competitive market economy:

[1] It is naive to state that the US was purely capitalist and that the USSR was purely command. The US was simply further towards the capitalism end of the spectrum than the USSR.

1. Consumer i initially owns $w_{\ell i}$ of good ℓ and $\sum_i w_{\ell i} = w_\ell$
2. Consumer i owns a share of firm j, denoted θ_{ij}, where $\sum_i \theta_{ij} = 1$. This share entitles consumer i to θ_{ij} of firm $j's$ profits
3. There is a market for each of the L goods. All consumers and producers are small relative to the market and thus act as price-takers.

Definition 3 *The allocation* $\left(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*\right)$ *and price vector* $p^* \in \mathbb{R}^L$ *constitute a competitive (Walrasian) equilibrium if the following conditions are met:*

1. *Profit Maximization: For each firm* j*,* y_j^* *solves*

$$\max_{y_j \in Y_j} \; p^* y_j$$

2. *Utility Maximization: For each consumer* i*,* x_i^* *solves*

$$\begin{aligned} &\max_{x_i \in X_i} \; u_i(x_i) \\ s.t. \; p^* x_i \;\; &\leq \;\; p^* w_i + \sum_{j=1}^{J} \theta_{ij} \left(p^* y_j^*\right) \end{aligned}$$

3. *Market clearing: For each good* ℓ*:*

$$\sum_{i=1}^{I} x_{\ell i}^* = w_\ell + \sum_{j=1}^{J} y_{\ell j}^*$$

Thus there are 3 conditions that must be met to find a competitive equilibrium: firms maximize profits, consumers maximize utility, and the market for each good clears. If a market does not clear (there is excess demand or excess supply) then a consumer or producer can bargain for a lower or higher price. This seems slightly inconsistent with the notion of a price-taker, but economic agents are price-takers when the price vector is the equilibrium price vector. The process by which the equilibrium prices are found is traditionally called *tatonnement*, which means that prices are set and consumers state how much of each good they are willing to buy at those prices while firms state how much they are willing to sell. If all markets clear at the price vector then trades are made; if there is excess supply or demand in a market, then NO TRANSACTIONS take place and the price in the disequilibrium markets are changed accordingly (raised if excess demand or lowered if excess supply). This raising and lowering of prices occurs until all markets clear, at which point the competitive equilibrium has been reached.

A useful result is that if the price vector $p^* >> 0$ is an equilibrium price vector resulting in the competitive equilibrium allocation $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$ then the same equilibrium allocation also occurs when the price vector is $\alpha p^* >> 0$ for any scalar $\alpha > 0$. This result allows us to normalize prices, meaning we can fix the price of some good at a particular level (usually we will set one price in the vector equal to 1).

Another useful result is something else that I have heard called "Walras Law". It states that if consumer budget constraints hold with equality and there are L goods that if $L-1$ markets clear then the L^{th} market clears.[2] This is useful because it is only necessary to check market clearing for $L-1$ markets, and when the economy is only a two-good economy then it is only necessary to check market clearing for one good. This concept is formalized below.

Lemma 4 *If the allocation* $(x_1, ..., x_I, y_1, ..., y_J)$ *and price vector* $p >> 0$ *satisfy the market clearing condition for all goods* $\ell \neq k$ *and if every consumer's budget constraint is satisfied with equality, so that* $px_i = pw_i + \sum_i \theta_{ij} py_j$ *for all* i*, then the market for good* k *clears.*

[2] Actually, if you look this up in Wikipedia as of 10/30/2006 Walras Law is defined as "if $N-1$ markets clear then the N^{th} market clears", and not as the consumer's budget constraint holds with equality as we have earlier defined Walras Law.

3 Partial Equilibrium Competitive Analysis

Now it is time to discuss "how" to find a competitive equilibrium mathematically − we already know that a competitive equilibrium is found through tatonnement. Essentially, there are $I+J$ maximization problems that must be solved (for L goods) and $L-1$ market clearing conditions that must be checked. Assume that there is a two-good economy, where good m_i is a composite numeraire and good ℓ is the market under study represented by x_i. Consumers have quasilinear preferences such that $u_i(m_i, x_i) = m_i + \phi(x_i)$. The consumption set is assumed to be $\mathbb{R}x\mathbb{R}_+$, so that the numeraire good may be consumed in negative quantities. This avoids boundary problems. We assume that $\phi(\cdot)$ is bounded above and twice differentiable, with $\phi_i'(x_i) > 0$, $\phi_i''(x_i) < 0$, and $\phi(0) = 0$ as a normalization (essentially this function begins at the origin and is a concave function that approaches some upper bound). The price of good m is set equal to 1 (price normalization) and p is the price of good ℓ. All firms can produce good ℓ from good m, and each firm has a cost function $c_j(q_j)$ to produce $q_j \geq 0$ units of good ℓ. Recall that the price of good m, which is the input in this model, has been fixed at 1, so that we write the cost function as only a function of the quantity. The cost function is twice differentiable such that $c_j'(q_j) > 0$ and $c_j''(q_j) > 0$ (the cost function is convex). There is no initial endowment of good ℓ. Each consumer i has $w_{mi} > 0$ of the numeraire and $w_m = \sum_i w_{mi}$.

Consider the profit maximization and utility maximization problems. For a price p^* for good ℓ, q_j^* must solve:

$$\max_{q_j \geq 0} \; p^* q_j - c_j(q_j)$$

The first order conditions that result are:

$$p^* \leq c_j'\left(q_j^*\right) \text{ for all } j = 1, ..., J.$$

These first order conditions hold with equality if $q_j^* > 0$ for all $j = 1, ..., J$. We also know that in the consumer's utility maximization problem that m_i^* and x_i^* must solve:

$$\begin{aligned} &\max_{m_i \in \mathbb{R}, x_i \in \mathbb{R}_+} \; m_i + \phi_i(x_i) \\ \text{s.t. } m_i + p^* x_i &\leq w_{mi} + \textstyle\sum_j \theta_{ij}\left(p^* q_j^* - c_j\left(q_j^*\right)\right). \end{aligned}$$

Alternatively, because the budget constraint holds with equality for all consumers (our first "Walras Law") we can solve for m_i and replace m_i in the consumer's utility function so that the consumer's problem is:

$$\max_{x_i \geq 0} \; \phi_i(x_i) - p^* x_i + w_{mi} + \textstyle\sum_j \theta_{ij}\left(p^* q_j^* - c_j\left(q_j^*\right)\right).$$

This yields the first order conditions:

$$\phi_i'(x_i^*) \leq p^* \text{ for all } i = 1, ..., I.$$

These first order conditions hold with equality if $x_i^* > 0$ for all $i = 1, ..., I$. We will define an equilibrium allocation as $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ with the understanding that $m_i^* = w_{mi} + \sum_j \theta_{ij}\left(p^* q_j^* - c_j\left(q_j^*\right)\right) - p^* x_i^*$ and $z_j^* = c_j\left(q_j^*\right)$.

Now check to see if the good ℓ market clears, so that $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ and $p^* >> 0$ constitute a competitive equilibrium if:

$$\begin{aligned} p^* &\leq c_j'\left(q_j^*\right) \text{ for all } j = 1, ..., J \\ \phi_i'(x_i^*) &\leq p^* \text{ for all } i = 1, ..., I \\ \textstyle\sum_i x_i^* &= \textstyle\sum_j q_j^*. \end{aligned}$$

If the solution is interior then the first order conditions are equalities and marginal revenue equals marginal cost from the firm's problem and marginal utility equals marginal cost from the consumer's problem. Thus, these $I+J+1$ conditions characterize the equilibrium. If $Max_i \; \phi_i'(0) > Min_j \; c_j'(0)$, then aggregate consumption and production of good ℓ is positive. This condition means that the largest marginal utility of some initial consumption of good ℓ is greater than the smallest marginal cost of producing some initial amount of good ℓ. In other (slightly less formal and technically incorrect) words, the consumer with the

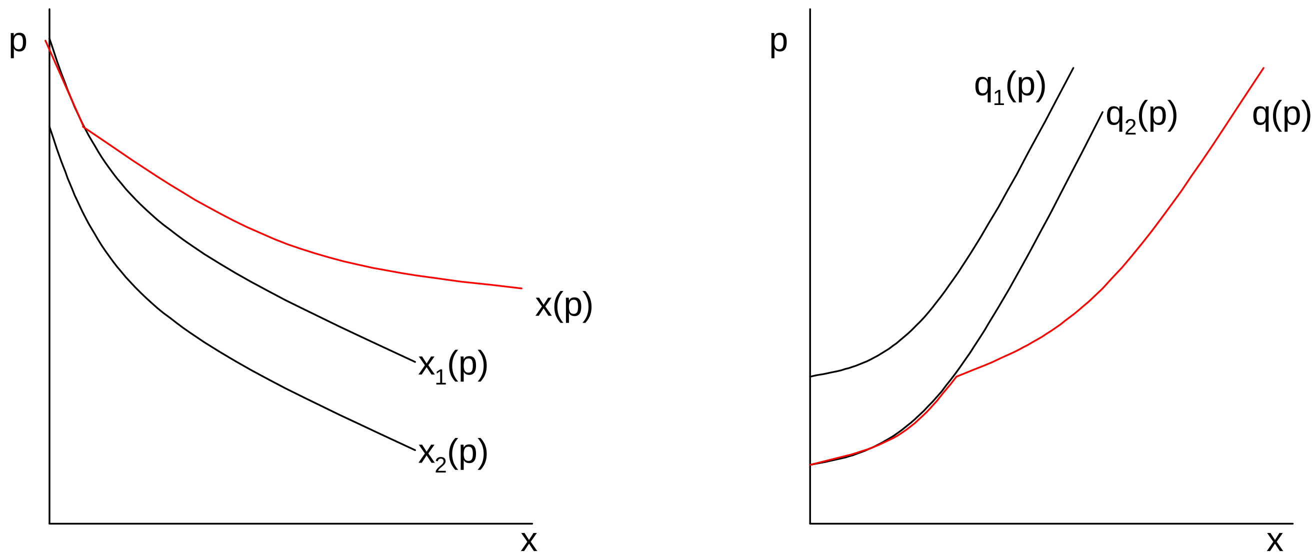


Figure 1: Individual demand and aggregate demand (left). Individual supply and aggregate supply (right).

highest value for the good has a value higher than the cost of the lowest cost producer. This condition is assumed to be met – otherwise, there would be no market for the good.

The 3 conditions that characterize the equilibrium do not depend on the endowment of m_i or the ownership shares θ_{ij}. This result is due to the quasilinear preference structure. We can derive aggregate demand and aggregate supply for good ℓ. Since $\phi_i''(\cdot) < 0$ and $\phi_i(\cdot)$ is bounded, $\phi_i'(\cdot)$ is a strictly decreasing function over $\left[0, \phi_i'(0)\right]$. For each level of $p > 0$ it is possible to solve for the unique level of x_i, $x_i(p)$, that satisfies $\phi_i'(x_i^*) = p^*$. If $p \geq \phi_i'(0)$, then $x_i(p) = 0$ (if price is greater than the marginal utility at zero units of good ℓ, then the consumer demands zero of the good at that price). A demand curve for each individual can be constructed. Note that this demand curve depends only on price, and not on wealth. Also, we can construct the aggregate demand function for good ℓ by summing the individual consumer demands at each price level p. Thus, let $x(p) = \sum_i x_i(p)$. This is a nonincreasing function at all $p > 0$. The aggregate demand function $x(p) = 0$ when $p \geq Max_i\ \phi_i'(0)$. Figure 1 shows an individual demand curve as well as the construction of an aggregate demand curve for a two consumer economy.

In a similar manner we can derive aggregate supply from $p^* = c_j'\left(q_j^*\right)$. We have earlier assumed that $c_j(\cdot)$ is convex for all j, and $c_j(\cdot)$ is such that $c_j'(q_j) \to \infty$ as $q_j \to \infty$. In other words, it is really, really costly to produce an infinite amount of q_j. For any $p > 0$, $q_j(p)$ denotes the unique level of q_j that satisfies $p = c_j'(q_j)$. If $p \leq c_j'(0)$, then $q_j(p) = 0$ (if price is below marginal cost then the firm does not supply any q_j to the market). Each firm's supply level of good ℓ is then given by $q_j(p)$. This function is continuous and nondecreasing at all $p > 0$, and is strictly increasing at any $p > c_j'(0)$. The aggregate supply of good ℓ is given by $q(p) = \sum_j q_j(p)$, which is continuous and nondecreasing in all $p > 0$ and strictly increasing at any $p > Min_j\ c_j'(0)$. Figure 1 shows an individual supply curve as well as the construction of an aggregate supply curve for a two firm economy.

Now we have a simple task – find the p^* that equates $x(p^*)$ with $q(p^*)$. We can then find the equilibrium allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ by finding $x_i^*(p^*)$ for all $i = 1, .., I$ and $q_j^*(p^*)$ for all $j = 1, .., J$. In our specification with quasilinear preferences and convex costs there is a unique equilibrium allocation.

When each firm has constant marginal costs $c_j(q_j) = cq_j$, there is still an equilibrium result for the economy (as long as supply and demand intersect the quantity traded in the market will be at their intersection and the price of good ℓ will be equal to c). However, the equilibrium allocation in this setting will not be unique if $J > 1$, as there are many ways to divide firm production among multiple firms.

Once again, finding equilibrium is a good starting point, and is typically used as the benchmark model, but economists are interested in how changes impact the equilibrium allocation and price vector. It is possible to consider the consumer's utility function as affected by exogenous variables $\alpha \in \mathbb{R}^M$ and the producer's cost function affected by exogenous variables $\beta \in \mathbb{R}^S$, so that $\phi(\cdot)$ is now a function of both x_i and α and $c_j(\cdot)$ is now a function of both q_j and β. Essentially, changes in the exogenous factors will increase or decrease the demand and/or supply for good ℓ, causing equilibrium allocation and price vector to change.

4 Fundamental Welfare Theorems: Partial Equilibrium Style

We began with a discussion of Pareto optimality. In our economy with quasi-linear preferences we want to find allocations that are Pareto optimal and compare them with the set of competitive equilibria that we found. It would be very nice if the Pareto optimal allocations coincided with the competitive equilibria, and indeed that is what we will see. Taking that result as given for now, we now have that the "market" can reach a competitive equilibrium with no excess supply or demand and that the resulting competitive equilibrium satisfies our definition of optimality. This is a tremendously important result – it is essentially the result that every pro-market, no government intervention individual relies upon when making his or her argument.[3] The skeptic will say (rightly so) that the assumptions of the model are not likely to be met in the "real world". However, the competitive equilibrium can (1) be used as a benchmark to determine the extent to which relaxing the assumption damages the economy and (2) can be reattained by breaking the assumptions that yield the competitive equilibrium result and finding new (and hopefully more realistic) assumptions that return the economy back to competitive equilibrium. This is similar to an econometrician's goal when analyzing data. There is a very nice theorem (Gauss-Markov theorem) in econometrics that provides efficiency and unbiasedness results about the linear regression model, provided that some initial assumptions about (primarily) the error structure are met. The problem is those assumptions are unlikely to be met in the "real world", and so much time is spent (as well as many trees and disk space) showing how the assumptions can be obtained in the face of potential problems (such as heteroscedasticity and serial correlation). It is a similar process with competitive equilibria – once the assumptions are broken, what fixes can we make that lead us back to the powerful theorem that has been established.

We have consumers with quasi-linear preferences, which means that the boundary of the economy's utility possibility set is linear. The economy's utility possibility set is the set of attainable utility levels for the individuals. The boundary of that set is the set of Pareto optimal bundles, because we cannot make one individual better off while keeping the others at the same utility level. Technically, we can fix x and q at $\overline{x}_i$ and $\overline{q}_j$, with the total numeraire available as $w_m - \sum_j c_j(\overline{q}_j)$. The set of utilities attainable is:

$$\left\{(u_1 ..., u_I) : \textstyle\sum_i u_i \leq \sum_i \phi_i(\overline{x}_i) + w_m - \sum_j c_j(\overline{q}_j)\right\}.$$

From the view of Pareto optimality in the economy, the optimal consumption and production vectors will satisfy:

$$\begin{aligned} & \max_{\substack{(x_i,...,x_I)\geq 0 \\ (q_1,...,q_J)\geq 0}} \textstyle\sum_i \phi_i(\overline{x}_i) - \sum_j c_j(\overline{q}_j) \quad + w_m \\ \text{s.t. } \textstyle\sum_i x_i - \sum_j q_j &= 0. \end{aligned}$$

Solving this problem we find:

$$\begin{aligned} \lambda &\leq c_j'(q_j^*) \text{ for all } j = 1, ..., J \\ \phi_i'(x_i^*) &\leq \lambda \text{ for all } i = 1, ..., I \\ \textstyle\sum_i x_i &= \textstyle\sum_j q_j. \end{aligned}$$

These are the $I + J + 1$ conditions that characterize a Pareto optimal equilibrium. Note that the term $\sum_i \phi_i(\overline{x}_i) - \sum_j c_j(\overline{q}_j)$ is called the Marshallian aggregate surplus. In principles we would call it "gains from trade". When $\lambda = p^*$, we have the competitive equilibrium that we have previously derived. Thus, any competitive equilibrium, given our structure, is a Pareto optimal allocation.

[3] We will get to the government intervention individuals in a moment – it is possible to be an economist and allow for some government intervention.

Proposition 5 *(The First Fundamental Theorem of Welfare Economics) If the price vector p^* and allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ constitute a competitive equilibrium, then this allocation is Pareto optimal.*

This is essentially a proof of the "Invisible Hand Theorem", which says that everyone in the economy maximizes their own self-interest (the firm and consumer maximization problems), and this will maximize the welfare in the economy. However, if markets are not complete or economic agents are not price-takers, then this result may not hold.

Now, back to those government intervention types. I claimed it was possible to be pro-redistribution policy and an economist. This is due to the Second Fundamental Theorem of Welfare Economics. We have seen that p^* and the competitive equilibrium allocation are unaffected by wealth levels. A transfer of one unit of the numeraire commodity from one consumer to the other will only cause the consumers' consumption levels to change by that amount and there will be no other changes. Thus, with an appropriate transfer of endowments, the market can reach any point on the boundary of the utility possibility set.

Proposition 6 *(Second Fundamental Theorem of Welfare Economics) For any Pareto optimal levels of utility $(u_1^*, ..., u_I^*)$ there are transfers of the numeraire commodity $(T_1, ..., T_I)$ satisfying $\sum_i T_i = 0$ such that a competitive equilibrium reached from the endowments $(w_{m1} + T_1, ..., w_{mI} + T_I)$ yields precisely the utilities $(u_1^*, ..., u_I^*)$.*

Thus, any Pareto optimal point can be reached by "simply" redistributing the endowments correctly and letting the market work. This argument can be used by government intervention types, and the justification will be that there are different points along the boundary of the utility possibilities set that yield different SOCIETAL utility. Thus, a point with one individual having everything above a subsistence level, which is Pareto optimal, may not be viewed as having as high a societal utility as a point where everyone has the same amount of all goods, which may also be Pareto optimal. Thus, a normative judgement is made that ranks the different Pareto optimal equilibria, and the idea is to find the one where societal utility is the largest. Be aware that this result also relies on the same assumptions of the model that says that every competitive equilibrium is Pareto optimal – one cannot argue that the assumptions that yield the First Fundamental Theorem of Welfare Economics are not met in the "real world" and so this requires that the First Fundamental Theorem of Welfare Economics be dismissed while simultaneously arguing for redistribution based upon the results of the Second Fundamental Theorem of Welfare Economics. There may be good reasons for redistribution policy, but those reasons will have to be established on their own merits if the assumptions of the model are not met.

5 Welfare Analysis in PE

It is possible to measure the change in social welfare that would occur if there is some change in the market, which could represent either improvements or restrictions. To do this we need to assume that there is a social welfare function, $W(u_1, ..., u_I)$. The social welfare function assigns a social welfare value to each utility vector $(u_1, ..., u_I)$. The goal of the "benevolent social planner" is to pick the feasible utility vector that generates the highest value of $W(\cdot)$. It should be noted that we are now assuming that utility levels are cardinal, rather than ordinal, as making a positive monotonic transformation of the utility function may change the value of $W(\cdot)$. In our current analysis there is no need to specify a social utility function as we are focusing on the single market for good ℓ and have consumers with quasilinear preferences – any social welfare function will be maximized at the same quantity of good ℓ consumed in our economy. Chapter 22 of Mas-Colell, Whinston, and Green provides some examples of social welfare functions – there is a utilitarian social welfare function, in which either (1) each consumer's utility receives the same weight and the utilities are summed and the sum of utilities is the value of the social welfare function or (2) each consumer's utility receives a different weight and the utilities are summed and the sum of utilities is the value of the social welfare function. There is also the "Rawlsian Social Welfare Function". In this social welfare function, $W(u_1, ..., u_I) = \min(u_1, ..., u_I)$. Thus, the person with the lowest level of utility determines the social welfare level of society.

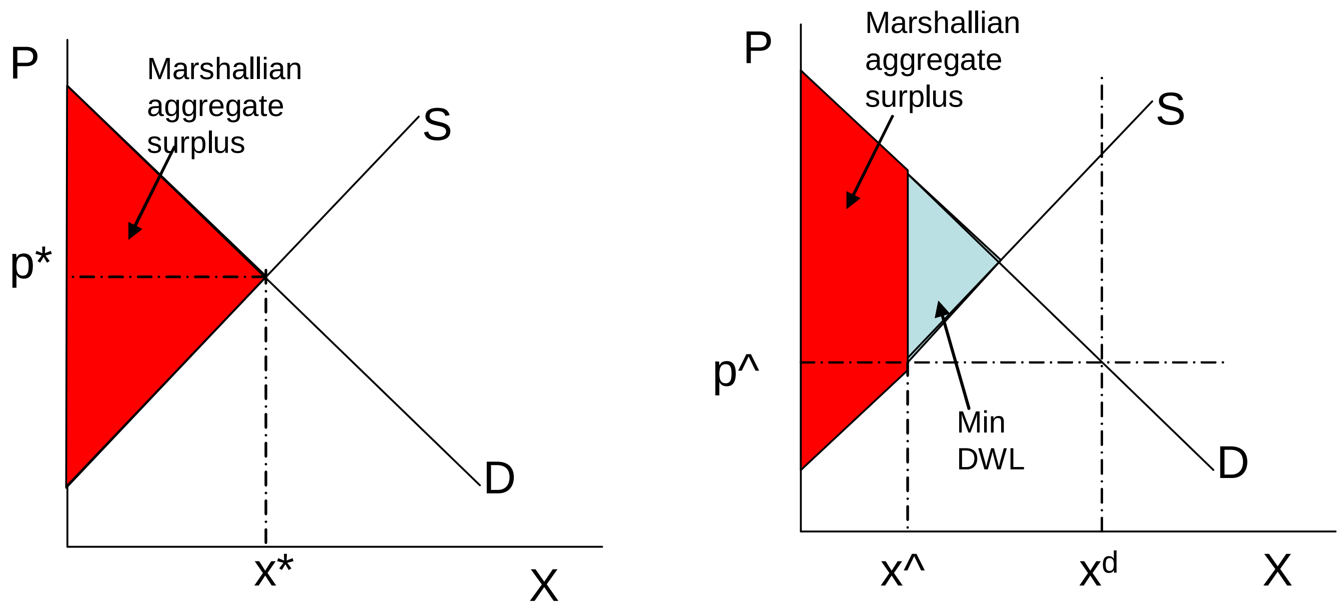


Figure 2: Marshallian aggregate surplus and minimum deadweight loss from a price control.

In the current set-up, changes in social welfare can be measured through the Marshallian aggregate surplus (gains from trade). The utility possibilities set for the economy is:

$$\left\{(u_1, ..., u_I) : \sum_i u_i \leq w_m + \sum_i \phi_i(x_i) - \sum_j c_j(q_j)\right\}.$$

The social welfare function must increase the larger the set is (the further out the boundary of the set – think of the consumer's budget constraint in the consumer problem). This increase can happen in two ways. One is for the endowment w_m to increase. But that is uninteresting. The other is for the Marshallian aggregate surplus, $\sum_i \phi_i(x_i) - \sum_j c_j(q_j)$, to increase. The Marshallian aggregate surplus can be easily represented in our graphical analysis of aggregate demand and aggregate supply of good ℓ. For any amount x of good ℓ, the Marshallian aggregate surplus is the area between the demand and supply curves. So we can take the difference of the integrals up to any point x and find the Marshallian aggregate surplus. Let $P(\cdot)$ represent the "inverse demand curve" (all the inverse demand curve does is let price be a function of quantity as opposed to quantity being a function of price) and $C'(\cdot)$ be the supply curve, so that the Marshallian aggregate surplus at any point x is:

$$S(x) = \int_0^x [P(s) - C'(s)]\, ds.$$

It is easy to see that the maximized value of $S(x)$ occurs at the intersection of supply and demand, which is the competitive equilibrium outcome. At quantities less than the competitive equilibrium outcome, there are additional quantities of good ℓ that can be consumed where the marginal value of those quantities is larger than the marginal cost. At quantities greater than the competitive equilibrium outcome, the additional quantities have a higher marginal cost than their marginal value.

There are various ways in which the Marshallian aggregate surplus can be increased or decreased. One is if the supply or demand for good ℓ changes. If demand for good ℓ decreases, perhaps due to changing tastes of the consumers, then the Marshallian aggregate surplus will decrease (assuming supply remains the same). If supply of the good increases, perhaps due to a change in technology that lowers cost, then Marshallian aggregate surplus will increase (assuming demand remains the same). It is also possible for policies, such as taxes or price controls, to impact the Marshallian aggregate surplus. The "simplest" case is for a price control to be implemented in the market for good ℓ. Suppose that x^* is the competitive equilibrium quantity and p^* is the competitive equilibrium price. We fix $\widehat{p} < p^*$ and state that price cannot rise above $\widehat{p}$. Thus, there is a price ceiling, such as a rent control policy. Now, the amount willing to be supplied at that price is $\widehat{x} < x^*$, since $\widehat{p} < p^*$. Since there are no trades beyond $\widehat{x}$, Marshallian aggregate surplus is $S(\widehat{x}) = \int_0^{\widehat{x}} [P(s) - C'(s)]\, ds$. But this is less than the competitive equilibrium Marshallian aggregate surplus, $S(x^*) = \int_0^{x^*} [P(s) - C'(s)]\, ds$ because $\widehat{x} < x^*$. The amount of Marshallian aggregate surplus lost in a market is known as the deadweight loss. Under the assumption that the highest valued consumers receive the good, the deadweight loss can be calculated as the difference between $S(x^*)$ and $S(\widehat{x})$. Or, representing deadweight loss as $DWL(x^*, \widehat{x})$:

$$DWL(x^*, \widehat{x}) = \int_{\widehat{x}}^{x^*} [P(s) - C'(s)]\, ds.$$

It should be noted that in this example of the price control there is not a unique equilibrium. There is a unique aggregate quantity traded ($\widehat{x}$) and price ($\widehat{p}$), but the individual x_i's are indeterminate because there is excess demand in the system. The deadweight loss function as defined above places a minimum on the deadweight loss in the system – if low valued consumers take units away from high valued consumers, then the deadweight loss will be larger. Thus, by Figure 2, if any consumer who has a value on the demand curve corresponding to the range from the price that corresponds to $\widehat{x}$ to the one that corresponds to x^d actually receives the item instead of a higher valued consumer, then deadweight loss will be greater.

6 Monopoly

This is from chapter 12 of the text. We will begin analyzing market power with the study of a single seller, the monopolist. Eventually we will move to multiple sellers with market power, but a discussion of game theory is needed first. The monopoly results fit in well with the analysis we have been doing in chapter 10.

Consider the market for good ℓ, only now there is a single seller. The seller's goal is to maximize profit by choosing a price and then selling all it can at that price. Thus, the problem is:

$$\max_p px(p) - c((x(p))),$$

where p is the price, $x(p)$ is the quantity demanded at that price, and $c(x(p))$ is the cost of producing the quantity $x(p)$. Alternatively, we can represent this problem as a quantity choice problem, so that the monopolist chooses quantity and lets the market determine the price at which the monopolist sells good ℓ. Note that in either formulation of the problem the monopolist sells good ℓ at the same price to all consumers. The quantity and price setting problems are analogous in the case of monopoly, but not necessarily so when we move to the case with multiple firms and market power. The monopolist's problem in the quantity choice setting is:

$$\max_q p(q)\, q - c(q),$$

where $p(q)$ is the inverse demand function, q is the quantity, and $c(q)$ is the monopolist's cost. Assume that $p(q)$ and $c(q)$ are twice differentiable and continuous at all $q \geq 0$, $p(0) > c'(0)$, and there is a unique output level $q^0 \in (0, \infty)$ such that $p(q^0) = c'(q^0)$. Thus, q^0 is the socially optimal level of output. This is where society's marginal benefit, represented by $p(q^0)$ is equal to society's marginal cost, represented by the monopolist's marginal cost at q^0.

To find the monopolist's choice of quantity, q^m, we simply solve the monopolist's maximization problem. Differentiating with respect to q and evaluating at q^m we find:

$$p'(q^m)\, q^m + p(q^m) - c'(q^m) \leq 0.$$

Or:

$$p'(q^m)\, q^m + p(q^m) \leq c'(q^m)$$

This inequality holds with equality if $q^m > 0$. Thus, the price in the market will be greater than marginal cost as long as $p'(q^m) < 0$. Essentially, if the demand curve slopes downward the price will be greater than the monopolist's marginal cost. The monopolist can gain revenue (relative to the competitive equilibrium

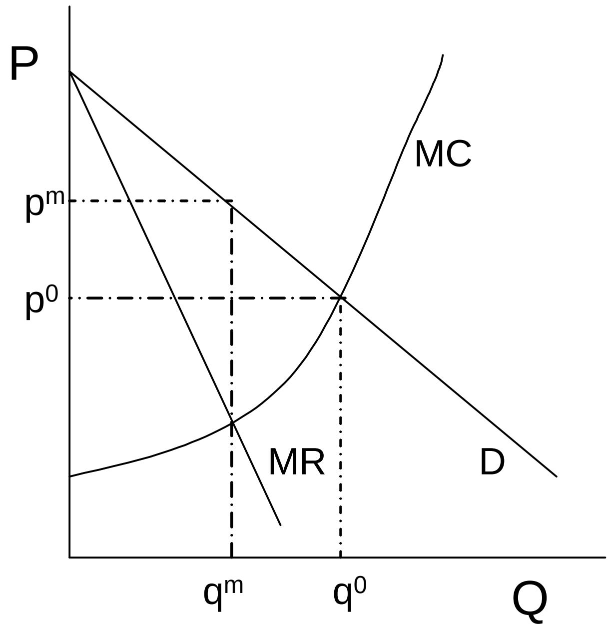


Figure 3: Monopolist's decision with increasing marginal cost.

optimal level of output) if it sells less units and allowing the price to rise on all the remaining units. Since $q^m < q^0$, there is deadweight loss in society due to monopoly. In this case, the deadweight loss function, $DWL(q^m, q^0)$, is the actual deadweight loss in the economy as there is no question as to which consumers buy (only those with a marginal value for the good greater than p^m) and there is no question which producer sells the good (it is a monopoly after all). The reason for the deadweight loss is that the monopolist's marginal revenue curve does not coincide with society's marginal benefit (demand) curve. Figure 3 shows the monopolist's decision graphically.

Now, there are plenty of criticisms of this simple model of monopoly. One is that the monopolist may be able to price discriminate (charge different prices to different consumers). If this is the case, then deadweight loss may be reduced. To see this, consider the monopolist charging each consumer exactly his or her value (or just a shade under his or her value) for every quantity up to the competitive equilibrium of q^0. Then the monopolist extracts all (or nearly all) of the Marshallian aggregate surplus, the consumers extract none (or very little) but the competitive equilibrium outcome is reached. This is known as first-degree price discrimination. Whether or not a particular consumer prefers this outcome to the one where the monopolist charges a single price depends on the value that the consumer has for good ℓ.