Chapter 12 Notes

These notes correspond to Cournot and Bertrand competition in chapter 12 of Mas-Colell, Whinston, and Green as well as the Stackelburg model. The text goes into much more detail about different methods of tweaking the basic models than the notes do.

1 Introduction

It is now time to apply the tools of game theory to the problem at hand – how do we find equilibrium in markets where firms recognize that they have some market power but are not a monopolist. Recall that in the competitive market it was fairly straightforward – the market price was given and each firm found the quantity where price equaled marginal cost and then produced that quantity to earn zero economic profit. The monopolist's problem was also fairly straightforward – the monopolist found the quantity where marginal revenue equaled marginal cost and produced that quantity. The price was determined by finding the price the market was willing to pay for the quantity produced. The price was greater than the monopolist's marginal cost and the monopolist earned positive economic profit. Alternatively, the monopolist could have maximized profit by choosing the price and letting the market determine the quantity at that price – either way, the resulting quantity in the market, price in the market, and profit for the monopolist would be identical. In the case of multiple firms with market power whether the firms compete using quantity or price will have a significant effect on the outcome in the market (at least using the simplest models).

2 Bertrand competition (choosing prices)

When firms compete by choosing prices this is called Bertrand competition after the person who initially developed it. The structure of the market is as follows. Consider 2 firms that simultaneously choose prices, $p_1 \in \mathbb{R}_+$ and $p_2 \in \mathbb{R}_+$. There is a demand function for the good given by $x(p)$, where $x(\cdot)$ is continuous and strictly decreasing at all p where $x(p) > 0$. There exists $\overline{p} < \infty$ such that $x(p) = 0$ for all $p \geq \overline{p}$ (if price is too high then there is no demand). Assume that the 2 firms are identical and face constant marginal cost of c. There is a socially optimal level of production $x(c) \in (0, \infty)$. Sales for firm j are given by:

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{cases}$$

Thus, firm j is the only seller in the market if its price is less than its competitor's, firm j and firm k split the market evenly if they choose equal prices, and firm j sells nothing if its price is greater than its competitors. This is a produce to order market, so costs are only incurred on when units are actually sold. For a given p_j and p_k, firm j's profit is

$$(p_j - c)\, x_j(p_j, p_k).$$

Consider a one-shot game of the Bertrand model. There is a unique NE to this game, where $p_j^* = p_k^* = c$. Note that profits are equal to zero under this proposed strategy because price equals marginal cost. First ask whether any firm would wish to deviate unilaterally from this proposed strategy. If firm j chooses a price less than c while firm k chooses c, then firm j captures the entire market, which is good, but is now charging a price less than marginal cost, which is bad because its profit is now negative. Thus, if one firm chooses a price equal to c, the other has no incentive to decrease price. Firm j also has no incentive to charge a higher price than c if firm k is charging c, because firm j would still earn zero profit, only now firm

j would earn zero profit because it sells nothing. Thus, there is no incentive for either firm to deviate from this proposed strategy so it is a NE.

As for uniqueness, we know that neither firm will choose a price below c. This leaves 3 cases:

Case 1: Both firms choose the same price that is greater than c, or $p_j = p_k = \widetilde{p} > c$. This is not a NE. Each firm receives half of the market at price $\widetilde{p}$, but either firm could do better by charging a slightly lower price, $\widetilde{p} - \varepsilon$, and capturing the whole market.

Case 2: Both firms choose a price strictly greater than marginal cost, but one firm chooses a price strictly (but only slightly) greater than the other firm, or $p_j > p_j - \varepsilon = p_k > c$. This is not a NE. Firm j would wish to change its price, as it could capture the entire market by choosing $p_j - \varepsilon - \varepsilon$. It is also possible to argue that firm k would wish to change its price UPWARD if there is some price $p_j - \phi$ such that $p_j > p_j - \phi > p_j - \varepsilon$.

Case 3: One firm chooses a price strictly greater than marginal cost while the other firm chooses a price equal to marginal cost, or $p_j > p_k = c$. We have already seen that firm j can do no better by choosing a different price. However, given that the action space is continuous (prices are chosen from the positive real numbers), there must exist a price $p_j - \varepsilon$ for $\varepsilon \in \mathbb{R}_{++}$ such that $p_j > p_j - \varepsilon > p_k = c$ for some arbitrarily small ε. Thus, firm k would wish to change its price UPWARD to shift from earning zero profits to earning positive profits.

As you can see, with only 2 firms competing in Bertrand competition (at least this version) the competitive outcome is achieved and firms are earning zero economic profit. Intuitively this does not seem logical as we might think that if there are only 2 firms in a market they should earn some positive economic profit. It is possible to modify the Bertrand model in ways that removes this problem – we are assuming here that products are perfect substitutes and that firms can serve the entire market at any price level. If either is removed (firms produce differentiated products or are capacity constrained) then the competitive market outcome disappears.

3 Cournot competition (choosing quantities)

Now, consider the case of 2 firms that compete by simultaneously choosing quantity levels. This is also a one-shot game. There are two symmetric firms with constant marginal cost of c. There is an inverse market demand function, $p(Q)$, where $Q = q_1 + q_2$. The function $p(\cdot)$ is differentiable, with $p'(q) < 0$ at all $q \geq 0$. We also have $p(0) > c$ (so that a market exists) and a unique output level $q^0 \in (0, \infty)$ such that $p\left(q^0\right) = c$. Firm j's problem is to maximize profit conditional on the output of the other firm.

$$\max_{q_j \geq 0} \; p\left(q_j + \overline{q_k}\right) q_j - c q_j$$

This maximization problem has the following first-order condition:

$$p'\left(q_j + \overline{q_k}\right) q_j + p\left(q_j + \overline{q_k}\right) \leq c, \text{ with equality if } q_j > 0$$

For each $\overline{q_k}$, let $b_j\left(\overline{q_k}\right)$ denote firm j's choice of quantity. Thus $b_j(\cdot)$ is firm j's best response correspondence. To find b_j simply solve the above equation for q_j. There is one slight modification – it is possible that firm j's best response to firm k is to choose a quantity less than 0. If that is the case, then firm j should choose 0. A pair of quantity choices (q_1^*, q_2^*) is a NE if and only if $q_j^* \in b_j\left(q_k^*\right)$ for $k \neq j$ and $j = 1, 2$. Thus, the following need to hold for firm 1 and firm 2:

$$\begin{aligned} p'\left(q_1^* + q_2^*\right) q_1^* + p\left(q_1^* + q_2^*\right) &\leq c \\ p'\left(q_1^* + q_2^*\right) q_2^* + p\left(q_1^* + q_2^*\right) &\leq c \end{aligned}$$

We will argue from intuition that $q_1^* > 0$ and $q_2^* > 0$ so that these equations hold with equality. If $q_1^* = 0$, then q_2^* should be the monopoly quantity. But if q_2^* is the monopoly quantity, then firm 1 can produce some small amount and make positive profit. Thus, $q_1^* > 0$ and $q_2^* > 0$, and these equations hold with equality. You should verify the intuitive argument mathematically for practice.

We can show that price is greater than marginal cost by adding the two equations above to get:

$$p'\left(q_1^* + q_2^*\right)\left(\frac{q_1^* + q_2^*}{2}\right) + p\left(q_1^* + q_2^*\right) = c$$

Since $p'\left(q_1^* + q_2^*\right)\left(\frac{q_1^*+q_2^*}{2}\right) < 0$, we must have $p\left(q_1^* + q_2^*\right) > c$.

3.1 Linear inverse demand

Now, suppose that $p(Q) = a - bQ$, where $Q = q_j + q_k$. Firms still have constant marginal cost of c, with $a > c \geq 0$ and $b > 0$. We can find firm j's best response function either by solving the maximization problem directly or by using the previous results. We know that firm j's best response function can be found by solving the following equation for q_j:

$$p'\left(q_j + \overline{q_k}\right) q_j + p\left(q_j + \overline{q_k}\right) = c$$

Substituting in for $p'\left(q_j + \overline{q_k}\right)$ and $p\left(q_j + \overline{q_k}\right)$ we have:

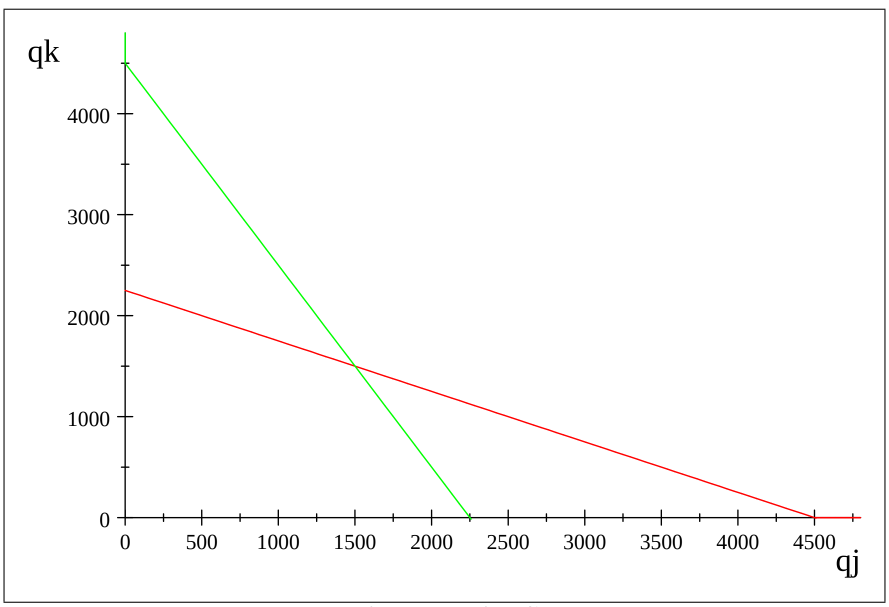
$$-bq_j + a - b\left(q_j + \overline{q_k}\right) = c$$

Solving for q_j we have:

$$q_j = \frac{a - b\overline{q_k} - c}{2b} \text{ or } q_j = \frac{a-c}{2b} - \frac{1}{2}\overline{q_k}$$

Note that if $\overline{q_k} > \frac{a-c}{b}$ then $q_j < 0$. This makes sense on an intuitive level when you realize that $\frac{a-c}{b}$ is the socially optimal quantity where $p(Q) = c$. Thus, if firm k produces more than the socially optimal quantity, firm j would want to produce a negative quantity to "remove" units from the market and bring the price back up to c. But since firm j cannot produce less than 0 units, then firm j will choose to produce 0 units if $\overline{q_k} > \frac{a-c}{b}$. Thus, firm j's best response function is $b_j(q_k) = Max\left[0, \frac{a-bq_k-c}{2b}\right]$. Firm k has a similar best response function, $b_k(q_j) = Max\left[0, \frac{a-bq_j-c}{2b}\right]$. One other useful piece of information is that in this setup the monopoly quantity is $\frac{a-c}{2b}$, so that if $q_k = 0$ then $q_j = \frac{a-c}{2b}$. Again, this conforms with intuition because if one firm chooses not to produce any units the other should choose to produce the amount of units that it would produce if it was a monopolist.

Since these best responses are functions we can depict them graphically.



Best response functions for Cournot game.

The green line shows the best response function for firm j while the red line shows the best response function for firm k. The intersection point of the two best response functions is the NE for this game. To find this point, simply solve the system of equations for the two best response functions. We ignore the zero portion of $b_j(q_k) = Max\left[0, \frac{a-bq_k-c}{2b}\right]$ and $b_k(q_j) = Max\left[0, \frac{a-bq_j-c}{2b}\right]$ because neither firm will choose quantity greater than $\frac{a-c}{b}$.

$$\begin{aligned} q_j^* &= \frac{a-bq_k^*-c}{2b} \\ q_k^* &= \frac{a-bq_j^*-c}{2b} \end{aligned}$$

Or:

$$\begin{aligned} 2bq_j^* &= a-b\left(\frac{a-bq_j^*-c}{2b}\right)-c \\ 4bq_j^* &= 2a-a+bq_j^*+c-2c \\ 3bq_j^* &= a-c \\ q_j^* &= \frac{a-c}{3b} \end{aligned}$$

To find q_k^*:

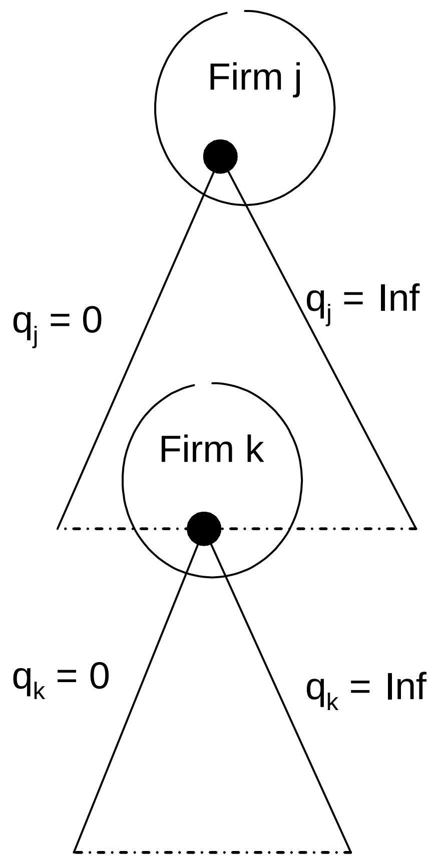
$$\begin{aligned} q_k^* &= \frac{a-b\left(\frac{a-c}{3b}\right)-c}{2b} \\ 2bq_k^* &= a-\left(\frac{a-c}{3}\right)-c \\ 6bq_k^* &= 3a-a+c-3c \\ 6bq_k^* &= 2a-2c \\ q_k^* &= \frac{a-c}{3b} \end{aligned}$$

Thus, the NE for this game is a pair of quantities $\left(q_j^*, q_k^*\right) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)$. Note that the total quantity in the market, Q, is equal to $\frac{2}{3}\frac{a-c}{b}$, which is greater than the monopoly quantity, $\frac{1}{2}\frac{a-c}{b}$, but less than the quantity from the purely competitive outcome, $\frac{a-c}{b}$. Thus, the firms in Cournot competition produce between the monopoly and the competitive level, which seems like a more intuitive result than the one we found in Bertrand competition.

The price in the market when this NE is played is $\frac{a+2c}{3}$. Profit to each firm is $\frac{(a-c)^2}{9b}$.

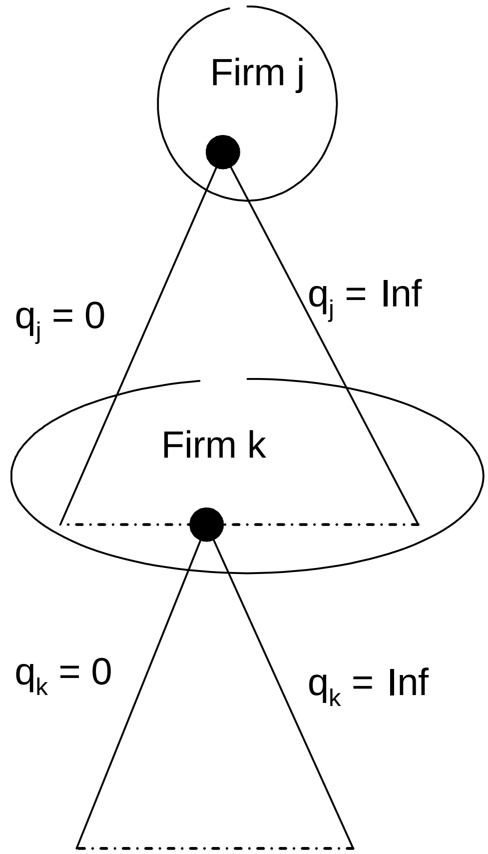
4 Stackelburg competition (sequential quantity choice)

Now, consider the case of 2 firms that compete by choosing quantity levels, but one firm makes an observable quantity choice before the other. This is also a one-shot game. There are two symmetric firms with constant marginal cost of c. There is an inverse market demand function, $p(Q)$, where $Q = q_1 + q_2$. The function $p(\cdot)$ is differentiable, with $p'(q) < 0$ at all $q \geq 0$. We also have $p(0) > c$ (so that a market exists) and a unique output level $q^0 \in (0, \infty)$ such that $p\left(q^0\right) = c$. Let firm j be the first-mover and firm k be the second-mover. The extensive form version of the game will look like:



Stackelburg game

Note that this is slightly different than the extensive form games we have seen. The dotted line between the branches labeled $q_j = 0$ and $q_j = Inf$ represents a continuum of strategies since it is impossible to write out every possible strategy for firm j (recall that $q_j \in \mathbb{R}_+$). There is a similar dotted line for firm k. However, note that the circle representing the information set for firm k does NOT encompass all the possible actions by firm j. This means that firm k observes firm j's quantity choice and then makes a decision. If the game were simultaneous, as the typical Cournot game is, then the figure would look like:



Extensive form Cournot game

Note the subtle difference. Also note that we do not list payoffs, since it would be impossible to write all of those down.

Now, as for solving the game it is fairly similar. The first question to ask is what constitutes a strategy (in the Stackelburg game) for each player. The second-mover will have to specify the quantity he will produce given any quantity choice by the first-mover. Thus, the second-mover will need to have a best response function just like in the Cournot model. The first-mover will not need to have a best response function. The first-mover makes one decision – what quantity level do I choose? Thus, while the second-mover's strategy is a best response function, a first-mover's strategy is simply a quantity choice.

Solving the game we work backwards. Consider the second-mover's decision. The second-mover needs to specify a quantity choice for any decision made by the first-mover. This is the same problem as the Cournot problem – hold the first-mover's quantity choice constant and then maximize profit based on that quantity choice.

$$\max_{q_k \geq 0} \; p\left(\overline{q_j} + q_k\right) q_k - c q_k$$

This yields the first-order condition:

$$p'\left(\overline{q_j} + q_k\right) q_k + p\left(\overline{q_j} + q_k\right) \leq c, \text{ with equality if } q_k > 0.$$

Note that this is the same first-order condition as we had in the Cournot problem. By a similar argument to the one made in the Cournot case we can assume that $q_k > 0$, so the first-order condition holds with equality. We can then specify a general best response function $b_k\left(\overline{q_j}\right)$ which represents the quantity that firm k will produce given that firm j produces $\overline{q_j}$.

Now, firm j need only make a single quantity decision. Firm j will take into consideration firm k's best response function when making its decision, so that firm j's profit maximization problem is:

$$\max_{q_j \geq 0} \; p\left(q_j + b_k\left(q_j\right)\right) q_j - c q_j.$$

Thus, firm j now has the first-order condition:

$$p'(q_j + b_k(q_j))\, b'_k(q_j)\, q_j + p(q_j + b_k(q_j)) \leq c, \text{ with equality if } q_j > 0.$$

Note that this first-order condition is different than the one in the Cournot model because firm j is now explicitly incorporating firm k's best response function into its profit function.

4.1 Linear inverse demand

Now, suppose that $p(Q) = a - bQ$, where $Q = q_j + q_k$. Firms still have constant marginal cost of c, with $a > c \geq 0$ and $b > 0$. Firm k's best response function in the Stackelburg game is identical to its best response function in the Cournot game, so:

$$b_k(\overline{q_j}) = Max\left[0, \frac{a - b\overline{q_j} - c}{2b}\right].$$

Again, recall that if Firm j will then explicitly incorporate this best response function into its maximization problem. We focus on the part of the best response function where $\frac{a-bq_j-c}{2b} > 0$. The reason for this is that firm k will only choose from the 0 portion of its best response function if firm j chooses $q_j > \frac{a-c}{b}$. Firm j will not choose $q_j > \frac{a-c}{b}$ because this will lead to a negative profit. Firm j's maximization problem is then:

$$\max_{q_j \geq 0} \left(a - b\left(q_j + \frac{a - bq_j - c}{2b}\right)\right) q_j - cq_j.$$

This yields the first-order condition:

$$a - 2bq_j - \frac{a}{2} + bq_j + \frac{c}{2} - c \leq 0, \text{ with equality if } q_j > 0.$$

We can easily check if $q_j > 0$ by imposing equality and determining whether or not profit is greater than or equal to zero. If profit is greater than or equal to zero when $q_j > 0$, then this is as last as good for the firm as when $q_j = 0$. Solving the first order condition gives:

$$\frac{1}{2}\frac{a - c}{b} = q_j.$$

Note that this is the monopoly quantity when the inverse demand function is linear. So an SPNE to this game is:

$$\left(q_j^*, b_k^*(\overline{q_j})\right) = \left(\frac{1}{2}\frac{a - c}{b}, Max\left[0, \frac{a - b\overline{q_j} - c}{2b}\right]\right).$$

The outcome from this SPNE is that firm j produces $q_j = \frac{1}{2}\frac{a-c}{b}$ and firm k produces $q_k = \frac{1}{4}\frac{a-c}{b}$, with $p(Q) = \frac{a+3c}{4}$ and $\pi_j = \frac{a+3c}{4}\frac{a-c}{2b} - c\frac{a-c}{2b} = \frac{(a-c)^2}{8b}$ and $\pi_k = \frac{a+3c}{4}\frac{a-c}{4b} - c\frac{a-c}{4b} = \frac{(a-c)^2}{16b}$. Note that firm j makes twice as much profit as firm k since firm j produces twice as much as firm k. In comparison with the Cournot outcome, note that market quantity in the Cournot model is $\frac{2}{3}\frac{a-c}{b}$ while market quantity in the Stackelburg model is $\frac{3}{4}\frac{a-c}{b}$. Thus, consumers are better off in the Stackelburg model than in the Cournot model because prices are lower ($\frac{a+3c}{4}$ versus $\frac{a+2c}{3}$).