

Preference and Choice Lecture notes*

1 Introduction

Preference and choice are the fundamental building blocks of economic theory. Nearly every theory in economics either assumes some sort of preferences over objects or is based on actual choices made over sets of objects. Recall the definition of economics from a principles textbook:

Definition 1 *Economics: the study of the choices people make as the result of scarcity*

Thus, our discussion begins with preference and choice, remembering all the while that economics is an observational science. That is, even though we will use mathematical methods to model economic behavior, it is always beneficial to bear in mind the assumptions that you are imposing on the agents in your economy. It may not seem like it throughout the course, but we use mathematics to simplify what we see, and to give our thoughts and ideas some structure. By the way, I hope you prefer Greek letters and mathematical symbols, as that might be the last definition without one or the other.

We will focus on 2 approaches, although throughout the course we will likely focus more heavily on the first than the second. The first is the preference relation approach. Begin with a set of possible alternatives, X . Let's say that X is {go to class, go to the movies, go to the ballet, go home, stay where you are, go to sleep on N. Tryon and West WT Harris Blvd.}.¹ As a decision maker, you have certain tastes for each of the objects in the set X . Tastes are the primitive characteristic of the decision maker, and they will be summarized in the preference relation. We will impose rationality axioms on decision-maker preferences and analyze the consequences of these preferences for the individual's choice behavior.

The 2nd approach treats the individual's choice behavior as the primitive feature and proceeds by making assumptions directly concerning this behavior. The central assumption is the Weak Axiom of Revealed Preference (WARP), which imposes an element of consistency on choice behavior. Essentially, the choice approach looks at the decisions made by the decision-maker (i.e., which of those elements of X did you choose when you made your decision).

To summarize, the primitive characteristic of the preference relation approach is tastes, and we impose rationality assumptions. The primitive characteristic of the choice approach is actual choice behavior, and we impose WARP. Eventually, we will tie the two concepts together.

2 Preference Relations

We begin with the symbol \succsim , which represents our preference relation. It is a binary relation on the set of alternatives X , allowing the comparison of pairs of alternatives $x, y \in X$.

We read $x \succsim y$ as “ x is at least as good as y ”. There are two other binary relations that we can derive from this:

1. The strict preference relation \succ . If $x \succ y$, we have $x \succsim y$, but not $y \succsim x$. We read $x \succ y$ as “ x is preferred to y ”.

*Based on Chapter 1 of Mas-Colell, Whinston, and Green (1995).

¹Be careful when reading the notes and the text. There is a difference between X and x . Typically, a capital letter will refer to a set of alternatives, while a lowercase letter refers to one element of that set.

2. The indifference relation \sim . If $x \sim y$, we have $x \succsim y$ and $y \succsim x$. We read $x \sim y$ as “ x is indifferent to y ”.

We assume that preference relations are rational.

Definition 2 *The preference relation \succsim is rational if:*

- (1) *Completeness: $\forall x, y \in X$, we have that $x \succsim y$ or $y \succsim x$ or both.*
- (2) *Transitivity: $\forall x, y, z \in X$, if $x \succsim y$, and $y \succsim z$, then $x \succsim z$.*

Completeness says that the decision-maker can rank ALL alternatives of X . Note that this does NOT mean that one alternative must be preferred to the other, as we allow for indifference. However, this is still a rather strong assumption – think about all the possible combinations of goods and services that you could consume, and now consider ranking them.

If a decision-maker faces a sequence of pairwise choices, transitivity will not allow for these preferences to cycle. That is, if $a \succsim b$, and $b \succsim c$, we CANNOT have $c \succ a$.

Proposition 3 *If the preference relation \succsim is rational, then*

- (1) *\succ is both irreflexive ($x \succ x$ never holds) and transitive (if $x \succ y$ and $y \succ z$, then $x \succ z$)*
- (2) *\sim is reflexive ($x \sim x \forall x$), transitive (if $x \sim y$ and $y \sim z$, then $x \sim z$), and symmetric (if $x \sim y$ then $y \sim x$)*
- (3) *if $x \succ y \succsim z$, then $x \succ z$*

Proof. Assume \succsim is rational. If $x \succ y \succsim z$, then $x \succ z$. ■

We will only show part 3. We first need to show that $x \succ z$.

Statement	Reason
1. $x \succ y$	1. Definition of \succ
2. $x \succ z$	2. \succsim is rational
3. Suppose that $z \succsim x$	3. Contradictory statement
4. Implies that $y \succ z$	4. Transitivity of \succsim since $y \succ x$ and $x \succ z$

But step 4 contradicts the assumption that $x \succ y$, so we cannot have $z \succsim x$. Since we have $x \succ z$ (step 2) but not $z \succsim x$ (step 4), then we have shown that $x \succ z$ because the definition of \succ is that $x \succ z$ but not $z \succsim x$.

3 Utility Functions

We will use utility functions to describe preference relations. Our utility function, $u(x)$, assigns a numerical value to each element in X , ranking the elements of X in accordance with the individual’s preferences.

Definition 4 *A function $u : X \rightarrow \mathbb{R}$ is a utility function representing preference relation \succsim if, $\forall x, y \in X$*
 $x \succsim y \iff u(x) \geq u(y)$

Note that $X \rightarrow \mathbb{R}$ is read as “ X maps into \mathbb{R} ” or that “a function $u : X \rightarrow \mathbb{R}$ ” is read as a function u that maps X into \mathbb{R} .

Utility functions are NOT unique. For any strictly increasing function,² $f : \mathbb{R} \rightarrow \mathbb{R}, v(x) = f(u(x))$.

Proposition 5 *If there is a utility function that represents preferences \succsim , then \succsim must be complete and transitive.*

Proof. If there is a utility function that represents preferences \succsim , then \succsim must be complete and transitive. ■

Proof for completeness.

²A strictly increasing function always increases, or alternatively has no decreasing or “flat” spots.

Statement	Reason
1. For any $x, y \in X$, either $u(x) \geq u(y)$ or $u(y) \geq u(x)$	1. $u(\cdot)$ is a real-valued function (one that maps from some domain D into \mathbb{R}) on X
2. Either $x \succsim y$ or $y \succsim x$	2. $u(\cdot)$ is a real-valued function on X , def. 4
3. \succsim must be complete	3. Follows from step 2 and def. of complete

Proof for transitivity. Begin with the assumption that $x \succsim y$ and $y \succsim z$.

Statement	Reason
1. $u(x) \geq u(y)$ and $u(y) \geq u(z)$	1. $u(\cdot)$ represents \succsim
2. $u(x) \geq u(z)$	2. transitivity of \geq
3. $x \succsim z$	3. $u(\cdot)$ represents \succsim and step 2
4. \succsim must be transitive	4. Follows from step 3 and def. of transitive

The importance of this proposition is that if we are going to assume that utility functions represent preferences (which we are), then these preferences must be complete and transitive.

4 Revealed Preference Theory

Choice behavior is the primitive object of the theory. Formally, a choice structure, $(\beta, C(\cdot))$, consists of

1. β is a family (set) of nonempty subsets of X ; every element of β is a set $B \subset X$. We call the elements $B \in \beta$ "budget sets". The budget sets should be thought of as an exhaustive list of ALL the choice experiments that the physically, institutionally, or otherwise restricted social situation can conceivably pose to the decision-maker. It need not include all subsets of X .
2. $C(\cdot)$ is a choice rule that assigns a nonempty set of chosen elements $C(B) \subset B$ for every budget set $B \subset \beta$. When $C(B)$ contains a single element, that element is the individual's choice among the alternatives in B . The set $C(B)$ may contain more than one element.

What does all this mean? Let $X = \{chicken, beef, fish\}$. Let $\beta = \{\{chicken, beef\}, \{chicken, beef, fish\}\}$. A choice structure $(\beta, C_1(\cdot))$ could be $C_1(\{chicken, beef\}) = \{beef\}$ and $C_1(\{chicken, beef, fish\}) = \{chicken\}$. It could also be $(\beta, C_2(\cdot))$ where $C_2(\{chicken, beef\}) = \{chicken, beef\}$ and $C_2(\{chicken, beef, fish\}) = \{chicken\}$. You should ask yourself if these examples seem logical – in other words, do the choice rules seem consistent?

Following Samuelson (1947), we impose some "reasonable" restrictions regarding an individual's choice behavior.

Weak Axiom of Revealed Preference (WARP)

Definition 6 *The choice structure $(\beta, C(\cdot))$ satisfies WARP if*

for some $B \in \beta$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \beta$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

What this says is that if we have two elements (x, y) common to two different budget sets (B, B') , if x is chosen when the budget set is B , then y CANNOT be chosen when the budget set is B' . Note that WARP rules out the C_1 and C_2 choice rules from the chicken, beef, fish example.

Definition 7 *Given a choice structure $(\beta, C(\cdot))$, the revealed preference relation \succsim^* is defined by:*

$$x \succsim^* y \iff \text{there is some } B \in \beta \text{ such that } x, y \in B \text{ and } x \in C(B)$$

The equation " $x \succsim^* y$ " is read as " x is revealed at least as good as y ". The definition says that x is revealed at least as good as y if and only if x and y are both in the same budget set and x is chosen from that budget set.

The revealed preference relation is not necessarily complete or transitive. In the preference relation approach, we assume that individuals have preferences over goods. However, in the choice rule approach, at least under WARP, we must observe a choice being made between two goods in order to determine which is preferred. Therefore, if two goods are not an element of the same budget set, then we cannot be certain which the individual prefers, as the choice rule approach makes no assumptions about goods not chosen.

Also, it is possible for two goods to be elements of the same budget set, but if a third good is chosen, then all we know is that the third good is revealed at least as good as the other two goods, and we have no information as to how the individual values the other two goods.

5 Relationship between \succsim and \succsim^*

1. If a decision maker has rational preference relation \succsim , do her decisions when facing choices from budget sets in β necessarily generate a choice structure that satisfies WARP?

YES.

Proposition 8 *Suppose that \succsim is a rational preference relation. Then the choice structure generated by \succsim , $(\beta, C^*(\cdot, \succsim))$, satisfies WARP.*

Proof. If \succsim is a rational preference relation, and for some $B \in \beta$ we have $x, y \in B$ and $x \in C^*(B, \succsim)$, then $(\beta, C^*(\cdot, \succsim))$ satisfies WARP.

Statement	Reason
1. $x \succsim y$	1. Definition of $C^*(B, \succsim)$
2. $x, y \in B'$; $y \in C^*(B', \succsim)$	2. New assumption
3. $y \succsim z \forall z \in B'$	3. Definition of $C^*(B', \succsim)$ ■
4. $x \succsim z \forall z \in B'$	4. Transitivity of \succsim
5. $x \in C^*(B', \succsim)$	5. From 2 and 4
6. WARP is satisfied	6. From 5 and definition of WARP

Thus, the rationality of \succsim is enough to guarantee that WARP is satisfied. Intuitively, if x is at least as good as y , then if x and y are both available the individual's choice rule must include x if it includes y .

2. If an individual's choice behavior for a family of budget sets in β is captured by a choice structure $(\beta, C(\cdot))$ that satisfies WARP, is there necessarily a rational preference relation that is consistent with these choices?

MAYBE.

Definition 9 *Given a choice structure $(\beta, C(\cdot))$, we say that the rational preference relation \succsim rationalizes $C(\cdot)$ relative to β if $C(B) = C^*(B, \succsim)$ for all $B \in \beta$, that is, if \succsim generates the choice structure $(\beta, C(\cdot))$*

The rational preference relation \succsim rationalizes choice rule $C(\cdot)$ on β if the optimal choices generated by \succsim coincide with $C(\cdot)$ for all budget sets in β . WARP must be satisfied if there is to be a rationalizing preference relation.

Proposition 10 *If $(\beta, C(\cdot))$ is a choice structure such that WARP is satisfied and β includes all subsets of X of up to 3 elements, then there is a rational preference relation \succsim that rationalizes $C(\cdot)$ relative to β . That is, $C(B) = C^*(B, \succsim)$ for all $B \in \beta$.*

Proof. I have to save something for you all to do. ■