Consumer Choice[*]

A market economy is defined as the process by which a consumer may acquire goods and services which are available at known prices. The consumer is the most basic element in a market economy and the most natural starting point for its discussion.

1 Commodities

What does a consumer do?

A consumer chooses consumption levels of goods and services, or commodities, available for purchase. We are going to assume that the number of commodities is finite, and equal to L. We will index the commodities using $\ell = 1, ..., L$. We will say that a commodity vector (or bundle) is a list of amounts of different commodities: $x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_L \end{bmatrix}$. Each x_i can be positive or negative, although negative x_i will typically refer to inputs in a process. x is a point in $\mathbb{R}^L$, the commodity space. We use the commodity vector to represent an individual's consumption level, and the ℓth entry of x is the quantity of commodity ℓ consumed. We will also call the commodity vector the consumption bundle.

2 Consumption Set

The consumption choices a consumer may make are limited by physical constraints (as well as monetary constraints, which we will discuss momentarily). The consumption set is a subset of the commodity space $\mathbb{R}^L$, denoted by $X \subset \mathbb{R}^L$. The elements of X are the consumption bundles that the individual can consume given the physical constraints of the environment. The simplest form of consumption set is $X = \mathbb{R}^L_+ = \left\{ x \in \mathbb{R}^L : x_\ell \geq 0 \text{ for } \ell = 1, 2, ..., L \right\}$. This is simply the set of all nonnegative bundles of commodities. If $L = 2$, then this set is simply $\mathbb{R}^2_+$, or quadrant I of the Cartesian Plane. To build intuition we will work with examples that typically assume $L = 2$ as it is much easier to draw a picture using $\mathbb{R}^2$ than it is to draw one with more than 2 dimensions.

An important feature of the commodity space $\mathbb{R}^L_+$ is that it is a convex set. Formally we define a convex set as follows:

Definition 1 *The set $A \subset \mathbb{R}^N$ is __convex__ if $x'' = \alpha x + (1 - \alpha) x' \in A$ whenever $x, x' \in A$ and $\alpha \in [0, 1]$.*

Basically, if the weighted average of any two consumption bundles in the set X is also in the set X, then the set X is convex. Using $\mathbb{R}^2_+$ we can think about taking any two points in the set and drawing a line between them. All points on that line are also in $\mathbb{R}^2_+$.

3 Competitive Budgets

Generally speaking, a consumer's consumption set is limited by a variety of constraints (physical, temporal, etc.). However, the most common constraint that we will impose is a budget constraint, meaning the

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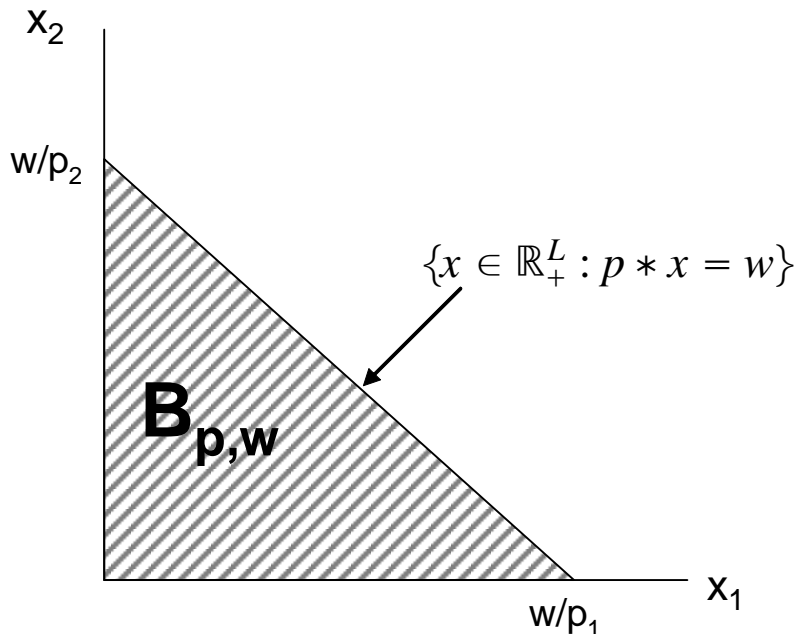[*]Correspond to chapter 2 of Mas-Colell, Whinston, and Green.

Figure 1: A Walrasian budget set for 2 goods.

consumer is constrained by what he or she can afford. Suppose that the L commodities are traded in \$ prices that are publicly quoted. Prices are represented by a price vector, $p = \begin{bmatrix} p_1 \\ p_2 \\ . \\ . \\ . \\ p_L \end{bmatrix} \in \mathbb{R}^L$. The elements of p can be less than zero. In this case, someone would be paying you to consume an item, which usually occurs if the item was an "economic bad". Typically, we will assume that $p >> 0$, meaning that each individual price is greater than 0. Consumption bundle affordability depends on two things

1. Market prices, p

2. Wealth, w (this is a scalar – you only get one wealth)

The consumption bundle $x \in \mathbb{R}^L_+$ is affordable if total cost is less than wealth, or $p \cdot x \leq w$. Note that this is written $p \cdot x \leq w$ and not $p * x \leq w$ because we want the inner (or dot) product of the two $Nx1$ vectors. We could write this as $p' * x \leq w$ or $p^T * x \leq w$, whichever way you prefer to write the transpose of the vector. Note that the inner product of $p \cdot x = p_1 * x_1 + p_2 * x_2 + ... + p_L * x_L$. It is highly probably in class that I use $*$ and $\cdot$ interchangeably. The set of feasible bundles, which we will call the Walrasian or competitive budget set, is defined as $B_{p,w} = \{x \in \mathbb{R}^L_+ : p \cdot x \leq w\}$. The consumer's problem is to choose a consumption bundle from that budget set (precisely how the consumer goes about making that choice will be discussed in chapter 3). A Walrasian budget set for a 2-good economy is shown in Figure 1. Note that the budget set includes the shaded area as well as the budget line, which is the set where expenditures exactly equal wealth, $\{x \in \mathbb{R}^L_+ : p \cdot x = w\}$. This leads to our next definition of the budget hyperplane.

Definition 2 *The budget hyperplane (budget line when $L = 2$) is the set $\{x \in \mathbb{R}^L_+ : p \cdot x = w\}$, which determines the upper boundary of the budget set (the consumer has no additional wealth to spend).*

Note that the slope of the budget line, $-\frac{p_1}{p_2}$, captures the rate of exchange of the two commodities. Also note that as the price of any good decreases, the entire budget set expands. Note that the Walrasian budget set is convex, and will be convex if the consumption set (which we typically assume is $X = \mathbb{R}^L_+$) is convex.

4   Demand Functions

The consumer's Walrasian (or market, or ordinary, or what some texts call Marshallian) demand correspondence $x\,(p,w)$ assigns a set of chosen consumption bundles for each price-wealth pair. Since it is a correspondence it can be multi-valued, which means that for specific prices and wealth, $\overline{p}$ and $\overline{w}$, the consumer can assign multiple consumption bundles. When $x\,(p,w)$ is single-valued, it is a demand function.

Definition 3 *The Walrasian demand correspondence $x\,(p,w)$ is <u>homogeneous of degree zero</u> if $x\,(\alpha p, \alpha w) = x\,(p,w)$ for any p, w and $\alpha > 0$.*

Homogeneity of degree zero tells us that changing prices and wealth in the same proportion will not affect the consumer's chosen consumption bundle. Thus, if a consumer's wealth doubles and ALL prices also double, then the consumer will choose the same consumption bundle he did when wealth and prices had not doubled.
This is a good place to mention homogeneity in general.

Definition 4 *A real-valued function is called homogeneous of degree k if $f\,(tx) = t^k f\,(x)\ \forall t > 0$.*
   Homogeneous of degree 1 if: $f\,(tx) = tf\,(x)\ \forall t > 0$
   Homogeneous of degree 0 if: $f\,(tx) = f\,(x)\ \forall t > 0$

Definition 5 *The Walrasian demand correspondence $x\,(p,w)$ satisfies Walras' law if for every $p >> 0$ and $w > 0$, we have $p * x = w\ \forall x \in x\,(p,w)$.*

Walras' law is satisfied if the consumer spends all of his wealth. This does not necessarily mean that a consumer gets a paycheck and then spends all of that paycheck on durable and non-durable goods. We can allow for goods to be assets (like a stock or bond, or even a savings account or a can of cash buried in the ground), but the point is that the consumer must do something with all of his wealth. While this is an assumption we are making now, it holds under very general circumstances, which we will discuss later.
For the comparative statics section, assume that $x\,(p,w)$ is single-valued, so that we have a demand function. Also, $x\,(p,w)$ will be assumed to be continuous and differentiable (we will discuss why in detail in chapter 3).

5   Parting comments

1. You can read section 2.F of Mas-Colell on your own, It is about WARP and the law of demand.

2. Remember that p refers to a vector of prices and that if we are discussing an individual price we will label it p_k or p_1 or p_2, etc. Same thing for $x\,(p,w)$. This will refer to the demand function for all goods $(x_1, x_2, ..., x_L)$. If we want to specify a demand function for a particular good, then we will write it in the following forms, $x_1\,(p,w)$ or $x_\ell\,(p,w)$.

3. If you see some equation that has a lot of terms in it (like the $\sum$ equations that are full of partial derivatives) it may be helpful to write them out for 2 or 3 goods to get a feeling for what exactly the math is saying rather than just glossing over the equation and saying, "yeah, looks right". Combining this bullet point with the previous should also help in obtaining the formulas for the partial derivatives with respect to α, p_k, and ω.

4. Remember that we really have not, as of yet, solved any problems. All we have done is discuss some basic assumptions of preference relations (completeness and transitivity) in chapter 1 and some basic elements of consumer choice (consumption sets, budget sets, and demand functions) and underlying assumptions (convexity, homogeneity of degree zero, and Walras' law) and their implications (all the

mathematical results on comparative statics) in chapter 2. We will solve problems in chapter 3 (specifically utility maximization problems), although we will likely need one more day of definitions and propositions before solving problems.