Chapter 5 Notes

These notes correspond to chapter 5 of Mas-Colell, Whinston, and Green.

1 Production

We now turn from consumer behavior to producer behavior. For the most part we will examine producer behavior in isolation, leaving the study of general equilibrium for the next course. We begin with a discussion of the producer's objectives and develop some assumptions underlying producer behavior. We will then discuss the producer's problem and study the specific case of cost and supply for a production technology that produces a single output.

Generally speaking we will consider our producers to be firms, although it should be noted that the theory developed applies equally to all types of producers, whether they are called firms, production units, families, etc. There may be additional assumptions/restrictions that one desires to make when discussing firms that are not controlled by agents that no not have identical preferences. The theory developed here applies directly to a firm composed of a single individual or agents with identical preferences. Whether these assumptions are applicable for other models of firms, such as partnerships, corporations with owners and managers, etc., is a decision each researcher needs to make on his or her own.

The firm is an interesting economic "agent". There are many potential questions one can ask about the firm. Such questions could be:

1. Who owns the firm? Does the identity of the owner change the firm's objectives?
2. Who manages the firm? If the owner and the manager are different agents, how does this affect the firm's behavior?
3. How is the firm organized? Do different organizational forms of the firm promote or inhibit efficiency?
4. What can the firm do?

These are interesting and important questions that an individual could turn into a long and fruitful academic career, and by no means is this an exhaustive list. Our primary focus will be on the 4^{th} item, What can the firm do? In particular, we will begin by assuming that the firm can transform inputs into outputs. The firm's goal is to make profits – specifically, the firm's goal is to *maximize* profits. It is possible that the firm has other goals – many firms set profit targets or sales targets (in $) or sales targets (in quantity of items sold) or wish to maximize sales. We will focus on profit maximization as (1) it provides a close approximation to the goals of many firms; (2) it is consistent with utility maximization if the firm is controlled by a single individual or agents with identical preferences; and (3) it allows us to use the tools developed in the study of consumer theory to solve the firm's problem.

Now, if you were to start a business one thing you would want to know is exactly *how* your firm would transform inputs into outputs (and ultimately profits). In the theory constructed, the *how* is assumed to be simply a black-box of production. We do not know how, just that there is some technology that exists that allows the firm to transform inputs into outputs. Thus, our view of the theory is fairly accurately represented by the Underpants Gnomes in South Park.[1] The Underpants Gnomes have a business plan:

- Phase 1: Collect underpants

[1] Episode 217, titled Gnomes, originally aired on 12/16/1998.

- Phase 2: (Gnome shrugs shoulders, suggesting he does not know)
- Phase 3: Profit

Well, they skip the stage where they turn the inputs into outputs, but for the most part this is dead on. Our firms transform inputs (underpants, phase 1) into outputs to make profits (phase 3), and the exact process is unknown (phase 2). We will be slightly (but not much) more formal than shrugging our shoulders when discussing the production process, but will simply specify it as a "production function", and will list certain properties of that function. Again, should you attempt to enter the business world, most lending agents will want a little more than a shrug of the shoulders or that you have some unspecified production function. However, for general theoretic purposes the generic "production function" will suffice.

While the Underpants Gnomes portrayal of the theory is a little harsh, it is important to note that the theory does not discuss the *how* of production. It is also important to note that given some minimum assumptions about firms, which in essence is what we will discuss, the economy will be able to obtain a competitive equilibrium outcome (we will discuss this in chapter 10 and you will also see this next semester in discussing general equilibrium). Thus, there is beauty in the simplicity of the theory in that it allows for equilibrium to be achieved with minimal assumptions.

2 Production Sets

Consider an economy with L commodities. A production vector (also input-output vector, netput vector, or production plan) is a vector $y = (y_{1,...,}y_L) \in \mathbb{R}^L$ that describes the net outputs of the L commodities from a production process. In the most general notation, it is assumed that inputs are negative numbers (a firm uses 7 units of y_1, so 7 units are consumed) and that outputs are positive numbers (a firm makes 5 units of y_2, so 5 units are created). It could be that $L = 5$ so $y \in \mathbb{R}^5$ and $y = (7, -4, 2, 0, -6)$. In this case, the firm will use 4 units of y_2 and 6 units of y_5 (net) in the production process and will create 7 units of y_1 and 2 units of y_3 (net). The entry for $y_4 = 0$, so that there is no *net* production of y_4. Note that the firm may use 9 units of y_2 in the production process, but that the production process returns 5 of those units. Our vector y simply provides the net production of the L commodities.

The first question to ask is which production vectors are possible. This is the analog to the consumer's consumption set – which consumption bundles are possible. The set of all production vectors that constitute feasible plans for the firm is the production set, denoted $Y \subset \mathbb{R}^L$. Any $y \in Y$ is possible, and any $y \notin Y$ is not. Note that because we are allowing for inputs and outputs that we do not make the restriction that Y is constrained to be positive as we did in consumer theory.

Like the consumer, the firm faces constraints. These constraints may be technological constraints, legal constraints, pre-commitments restricted by contracts – there are any number of constraints the firm may face. We can describe the production set Y by the transformation function $F(\cdot)$, where $Y = \left\{y \in \mathbb{R}^L : F(y) \leq 0\right\}$ and $F(y) = 0$ if and only if y is an element of the boundary of Y. The set of boundary points of Y, $\left\{y \in \mathbb{R}^L : F(y) = 0\right\}$ is known as the transformation frontier. If $F(\cdot)$ is differentiable, and the production vector $\overline{y}$ satisfies $F(\overline{y}) = 0$, then for any commodities ℓ and k, the marginal rate of transformation of y_ℓ for y_k is:

$$MRT_{\ell k} = \frac{\partial F(\overline{y})/\partial y_\ell}{\partial F(\overline{y})/\partial y_k}$$

The marginal rate of transformation is simply a measure of how much the net output of y_k can increase if the firm decreases the net output of y_ℓ by one unit.

2.1 Some properties of production sets

The following is a list of some standard properties of production sets. It is not a complete list, nor is it meant to imply that all of the properties hold for all production sets.

1. Y is nonempty – just like the consumer's problem is uninteresting if the consumer has no wealth to spend, the firm's problem is uninteresting if it cannot do anything. This assumption simply states that the firm needs to be able to do something in order for us to study it.

2. Y is closed – we know that a set is closed if its complement is open. Essentially, the closedness of the production set means that it includes the transformation frontier.

3. No free lunch – We cannot have $y \in Y$ and $y \geq 0$. If both $y \in Y$ and $y \geq 0$ then this would mean that the firm could produce some output without using any inputs. Even the Underpants Gnomes needed underpants.

4. Possibility of inaction – We can have $0 \in Y$. Thus, the firm is allowed to be inactive. This seems to be a reasonable assumption if the firm has not yet undertaken any commitments to costly activities and is only thinking about the possibility of becoming a firm. However, it may not be as reasonable if the firm has already incurred some costs – the firm may still decide to be inactive, but it has already made some commitments.

5. Free disposal – holds if the absorption of any additional amounts of inputs without any reduction in output is always possible. The extra amount of inputs can be disposed of or eliminated at no cost.

6. Irreversibility – If $y \in Y$ and $y \neq 0$, then $-y \notin Y$. If I make outputs using inputs, I cannot reverse the production process and turn the output back into the same amount of original inputs.

7. Nonincreasing returns to scale – If Y exhibits nonincreasing returns to scale, for any $y \in Y$, then $\alpha y \in Y$ for $\alpha \in [0,1]$. This means that the production process can be scaled down.

8. Nondecreasing returns to scale – If Y exhibits nondecreasing returns to scale for any $y \in Y$, then $\alpha y \in Y$ for $\alpha \geq 1$. This means that the production process can be scaled up.

9. Constant returns to scale – If Y exhibits constant returns to scale for any $y \in Y$, then $\alpha y \in Y$ for $\alpha \geq 0$. The production process can be scaled up or down.

10. Additivity (free entry) – Let $y, y' \in Y$. Then $y + y' \in Y$, or $Y + Y \subset Y$. This implies that if $y \in Y$, then $ky \in Y$ for all $k = \mathbb{Z} > 0$, where $\mathbb{Z}$ is the set of integers. From an economic viewpoint, additivity means that a firm with one production plant can set up a second production plant and that the second plant will not interfere with the first. Also, if one firm produces y and the other produces y', then aggregate production is $y + y'$. So production sets must satisfy additivity when free entry is possible.

11. Convexity – We know what convexity is. The production set Y is convex. If $y, y' \in Y$ and $\alpha \in [0,1]$, then $\alpha y + (1-\alpha) y' \in Y$. Convexity incorporates two ideas about production possibilities. Number one is nonincreasing returns. If inaction is possible, then convexity implies nonincreasing returns because $\alpha y + (1-\alpha) 0 \in Y$. Also, balanced plans are more productive than imbalanced plans, so that if y and y' produce the same amount of output, some convex combination of y and y' will produce at least as much output as y and y'. Again, this is similar to the consumer typically choosing mixed bundles rather than bundles of predominantly one type of good.

12. Y is a convex cone – This occurs when Y is convex and also satisfies constant returns to scale. Y is a convex cone if for any production vector $y, y' \in Y$ and constants $\alpha \geq 0$ and $\beta \geq 0$, we have $\alpha y + \beta y' \in Y$.

Proposition 1 *The production set Y is additive and satisfies the nonincreasing returns condition if and only if it is a convex cone.*

Proof. If the production set Y is a convex cone, then it is additive. Let $y, y' \in Y$.

Statement	Reason
1. $y + y' \in Y$	1. Definition of convex cone
2. Y is additive	2. From 1 and def. of additive

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Proof. If the production set Y is a convex cone, then it satisfies the nonincreasing returns condition. Let $y \in Y$.

Statement	Reason
1. $\alpha y \in Y$	1. Definition of convex cone
2. Y satisfies nonincreasing returns	2. From 1 and def. of nonincreasing returns

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Proof. If Y is additive and satisfies the nonincreasing returns condition, then it is a convex cone (or, $y, y' \in Y$ and α and β are positive constants, then $\alpha y + \beta y' \in Y$)

Statement	Reason
1. $k \in \mathbb{Z}$ such that $k > \max(\alpha, \beta)$	1. We know that there is no greatest integer
2. $ky \in Y, ky' \in Y$	2. Additivity
3. $\alpha y = \left(\frac{\alpha}{k}\right) ky$, $\beta y = \left(\frac{\beta}{k}\right) ky$	3. Arithmetic
4. $\frac{\alpha}{k} < 1$, $\frac{\beta}{k} < 1$	4. From 1
5. $\alpha y \in Y$, $\beta y' \in Y$	5. Nonincreasing returns
6. $\alpha y + \beta y' \in Y$	6. Additivity

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This result should be considered carefully. Nonincreasing returns states that firms can scale production processes down. Additivity states that the operation of additional plants do not interfere with one another. If both of these hold then convexity is obtained. Note that this does not say that a convex combination of the technology is "better" than one of the extreme technologies, just that it is available in the production set.

3 Profit Maximization (PMP)

The firm's goal is to maximize profits. For now, we assume that firms are price-taking, meaning that the prices for the L commodities (both inputs AND outputs) are given and that the firm's behavior does not affect these prices. The firm's profit is then:

$$py = \sum_{\ell=1}^{L} p_\ell y_\ell.$$

The firm's profit maximization problem is then:

$$\max_y \; py \text{ s.t. } y \in Y$$

or

$$\max_y \; py \text{ s.t. } F(y) \leq 0.$$

The profit function, $\pi(p)$, associates to every p the amount $\pi(p) = \max\{py : y \in Y\}$. We also have the supply correspondence at p, $y(p)$, which is the set of profit-maximizing vectors $y(p) = \{y \in Y : py = \pi(p)\}$. If the transformation function is differentiable then we can find first-order conditions to characterize the solution to the consumer's problem. To do this we can set up the Lagrangian as we did for consumer theory:

$$\max_y \; \mathcal{L}(y, \lambda) = py + \lambda(-F(y))$$

The first-order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial y_\ell} = p_\ell - \lambda \frac{\partial F(y^*)}{\partial y_\ell} = 0 \text{ for all } \ell = 1, ..., L$$

Alternatively, the first-order condition can be rearranged so that:

$$p_\ell = \lambda \frac{\partial F(y^*)}{\partial y_\ell}$$

The ratio of the first-order condition for commodities ℓ and k is:

$$\frac{p_\ell}{p_k} = \frac{\partial F(y^*)/\partial y_\ell}{\partial F(y^*)/\partial y_k} = MRT_{\ell k}$$

Note that this condition is similar to the one found in consumer theory that equates the marginal rate of substitution with the price ratio when the consumer is at an optimum (and the nonnegativity constraints are not binding).

3.1 The distinct output case

The previous discussion applies to the general theory of production. In most cases the researcher (as well as the real-world firm) is concerned with the case of distinct inputs and distinct outputs. We retain the world of L commodities, and assume that there are M outputs and $L-M$ inputs. We denote outputs as the vector $q_M \in \mathbb{R}_+^M$ and inputs as the vector $z_{L-M} \in \mathbb{R}_+^{L-M}$. Note that in the case with distinct inputs and outputs that both the outputs AND the inputs are assumed to be nonnegative, so $q = (q_1, ..., q_M) \geq 0$ and $z = (z_1, ..., z_{L-M}) \geq 0$.

The most basic case is that of multiple inputs and a single output ($M = 1$). This case can be described by the production function, $f(z)$. The production function describes the maximum output of q that can be produced using the z inputs. In the simplest economic models, the inputs are given generic identities like "capital" and "labor". Slightly more complex models may allow for different types of capital (old and new technologies), or different types of labor (perhaps skilled and unskilled laborers). But in the general theories of economics specific factors of production are typically not specified.

If output is held constant at $\overline{q}$, then the marginal rate of technical substitution (MRTS) of z_ℓ for z_k at $\overline{z}$ is:

$$MRTS_{\ell k} = \frac{\partial f(\overline{z}) / \partial z_\ell}{\partial f(\overline{z}) / \partial z_k}.$$

The MRTS is a measure of how much of z_k is needed to keep output at $\overline{q} = f(\overline{z})$ when z_ℓ is marginally decreased.

When Y is a single-output technology, we can write:

$$\max_{z \geq 0} \; pf(z) - wz$$

where p is a scalar denoting the price of the output good and w is a vector of input prices. Finding the first-order conditions we see that:

$$p\frac{\partial f(z)}{z_\ell} - w_\ell \leq 0$$

and when $z_\ell > 0$,

$$\begin{aligned} p\frac{\partial f(z)}{z_\ell} &= w_\ell \\ \frac{w_\ell}{p} &= \frac{\partial f(z)}{z_\ell} \end{aligned}$$

Note that $\frac{\partial f(z)}{z_\ell}$ is simply the marginal product of the input z_ℓ, or MP_{z_ℓ}. This result states that the MP_{z_ℓ} is equal to the relative price of the input to the output. Alternatively, if we find the ratio of the first-order conditions for two inputs, we get:

$$\frac{w_\ell}{w_k} = \frac{\partial f(z^*) / \partial z_\ell}{\partial f(z^*) / \partial z_k}.$$

Note that $\frac{\partial f(z^*)/\partial z_\ell}{\partial f(z^*)/\partial z_k}$ is the marginal rate of technical substitution of the inputs, or alternatively the ratio of marginal products. We can rewrite this as:

$$\frac{MP_k}{w_k} = \frac{MP_\ell}{w_\ell}.$$

If a firm is optimizing *and* using the inputs,[2] then their ratio of marginal products to price must be equal. This is similar to the consumer problem where the ratio of marginal utilities to price are equal at the optimum.

Finally, if Y is convex then the first-order conditions are necessary and sufficient conditions for determining a solution.

Proposition 2 *Suppose that $\pi(\cdot)$ is the profit function of production set Y and that $y(\cdot)$ is the associated supply correspondence. Assume that Y is closed and satisfies free disposal.*

[2] If an input is not used then this is akin to a corner solution in consumer theory and this result need not hold.

1. *$\pi(\cdot)$ is homogeneous of degree one in p*

2. *$\pi(\cdot)$ is convex in p*

3. *If Y is convex, then $Y = \{y \in \mathbb{R}^L : py \leq \pi(p) \text{ for all } p >> 0\}$*

4. *$y(\cdot)$ is homogeneous of degree zero in prices*

5. *If Y is convex, then $y(p)$ is a convex set for all p. Moreover, if Y is strictly convex, then $y(p)$ is single-valued (if nonempty).*

6. *(Hotelling's Lemma) If $y(\overline{p})$ consists of a single point, then $\pi(\cdot)$ is differentiable at $\overline{p}$ and $\nabla\pi(\overline{p}) = y(\overline{p})$.*

7. *If $y(\cdot)$ is a function differentiable at $\overline{p}$, then $Dy(\overline{p}) = D^2\pi(\overline{p})$ is a symmetric and positive semidefinite matrix with $Dy(\overline{p})\overline{p} = 0$.*

Points 1, 2, and 4 are results that are fairly similar to those in the consumer's problem. Result 1 states that if all prices double, then profit will double (technically, if we change all prices by the same percentage, then profits will change by the same percentage). Result 2 states that convex combinations of prices produce profits at least as high as the extremes. Result 4 states that if all prices change by the same percentage, then supply will remain unchanged. Result 5 provides results for the supply function based upon the convexity of the production set. Result 3 provides an alternative description of the technology, similar to the indirect utility function, $v(p, w)$, providing a description of preferences. We can then use result 6 to directly calculate supply (similar to using Roy's Identity to find demand functions from the indirect utility function) from the profit function. Result 7 follows from result 6 and provides the law of supply – holding all other prices constant, supply of a commodity will change in the same direction as its price. Note that this is unambiguous, as opposed to the case of consumer theory where the demand for a good may increase if its price increases (Giffen goods). Since there are no budget constraints (generally) in producer theory, there are no wealth effects to be concerned about and this matrix of second derivatives is akin to the matrix of second derivatives of the expenditure function, which is the substitution matrix in consumer theory. Thus, the matrix of second derivatives of the profit function, $D^2\pi(\overline{p})$, may be regarded as a substitution matrix.

4 Cost minimization (CMP)

The "dual" problem for the producer is the cost-minimization problem. The producer's goal with this problem is to choose an output level and then determine the cost-minimizing amount of inputs based upon the prices of the inputs and the production technology that will produce this level of output. Since this problem is just the dual of the profit-max problem, why study cost minimization?

1. There are a number of useful results that follow from the cost-minimization problem.

2. If a firm is not a price-taker in the output market then we cannot use the profit function for analysis. But if the firm remains a price-taker in the input market, we can use results from the cost minimization problem.

3. When Y has nondecreasing returns to scale, the value function and optimizing vectors of the CMP are better behaved than $\pi(p)$ and $y(p)$.

Our focus is on the single-output case. Let z be a nonnegative vector of inputs with input prices given by w and $f(z)$ be the production function that produces output q. The firm's problem is then:

$$\min_{z \geq 0} \; wz \text{ s.t. } f(z) \geq q.$$

The optimized value of the CMP is the cost function, $c(w, q)$. The optimizing set of input choices, $z(w, q)$, is known as the conditional factor demands correspondence. Note that this optimizing set of input choices is conditional on the amount of output produced, q. The Lagrangian can then be constructed:

$$\min_{z \geq 0} \; \mathcal{L}(z, \lambda) = wz + \lambda(q - f(z))$$

and we can use the first-order conditions to solve for the cost function and conditional factor demands. The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial z_\ell} = w_\ell - \lambda \frac{\partial f(z^*)}{\partial z_\ell} = 0 \text{ for all } \ell = 1, ..., L$$

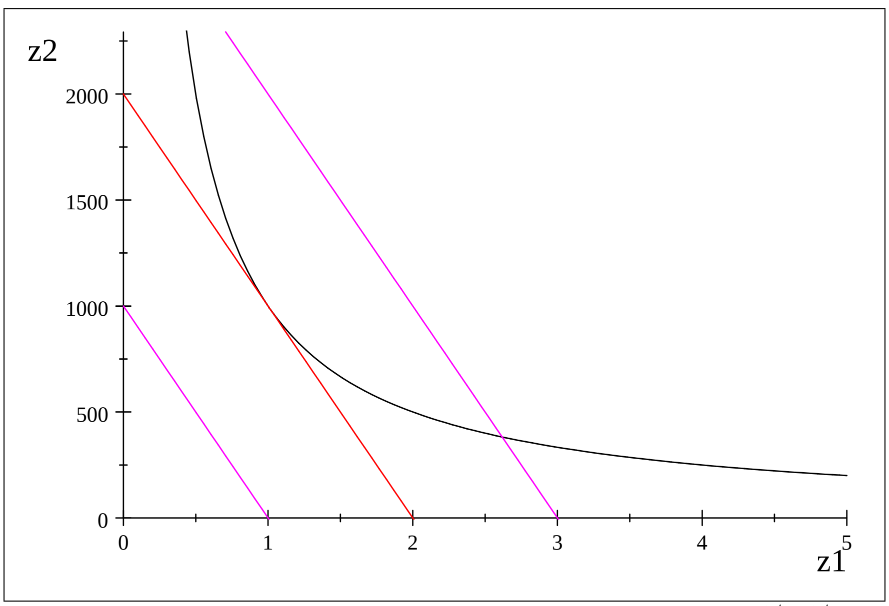
Rearranging we find

$$w_\ell = \lambda \frac{\partial f(z^*)}{\partial z_\ell}$$

And taking the ratio of the first-order conditions for commodities ℓ and k we find:

$$\frac{w_\ell}{w_k} = \frac{\partial f(z^*)/\partial z_\ell}{\partial f(z^*)/\partial z_k} = MRTS_{\ell k}.$$

It seems like overkill to keep repeating this statement, but these equality conditions ($MRTS = \frac{w_\ell}{w_k}$, $MRS = \frac{-p_1}{p_2}$, $MRT = \frac{p_\ell}{p_k}$) from the consumer and producer problems are extremely important for our analysis. Again, this only holds for the inputs the firms use, so if there is some input z_i such that $z_i = 0$ in the firm's problem, then these equality conditions do not hold. Figure 4 shows the isoquant (meaning "same quantity") for $f(z_1, z_2) = 10$ (the curved black line) assuming the firm's production function is $f(z_1, z_2) = z_1^{1/3} z_2^{1/3}$. The input prices are assumed to be $w_1 = 1000$ and $w_2 = 1$. The cost for the line below the isoquant (purple line) is 1000 while the cost for the purple line above the isoquant is 3000. The cost for the red line is 2000. The idea in this problem is similar to the consumer's expenditure minimization problem – fix a level of utility and find the lowest level of expenditure. The firm is merely fixing a quantity level and finding the lowest cost at which it can produce that quantity.



Isoquant for the Cobb-Douglas production function of $10 = z_1^{1/3} z_2^{1/3}$

For the mechanics of the problem, the conditional factor demands can be found by solving the first-order conditions for the input levels in terms of w and q (do not forget the first-order condition for λ, which I have not been including because we "know" we need to find that condition). To find the cost function, simply take the factor demands, which are functions of w and q, and substitute them into wz, so that $c(w, q)$ would look something like $w_1 z_1(q, w) + w_2 z_2(q, w)$.

Proposition 3 *Suppose that $c(w, q)$ is the cost function of a single output technology Y with production function $f(\cdot)$ and that $z(w, q)$ is the associated conditional factor demand correspondence. Y is closed and satisfies free disposal.*

1. *$c(\cdot)$ is homogeneous of degree one in w and nondecreasing in q*

2. *$c(\cdot)$ is a concave function of w*

3. *If the sets $\{z \geq 0 : f(z) \geq q\}$ are convex for every q, then $Y = \{(-z, q) : wz \geq c(w, q) \text{ for all } w >> 0\}$.*

4. *$z(\cdot)$ is homogeneous of degree zero in w*

5. *If the set $\{z \geq 0 : f(z) \geq q\}$ is convex then $z(w, q)$ is a convex set. Moreover, if $\{z \geq 0 : f(z) \geq q\}$ is strictly convex then $z(w, q)$ is single-valued.*

6. *(Shepard's Lemma) If $z(\overline{w}, \overline{q})$ consists of a single point, then $c(\cdot)$ is differentiable with respect to w at $\overline{w}$ and $\nabla_w c(\overline{w}, q) = z(\overline{w}, q)$*

7. *If $z(\cdot)$ is differentiable at $\overline{w}$, then $D_w z(\overline{w}, q) = D_w^2 c(\overline{w}, q)$ is a symmetric and negative semidefinite matrix with $D_w z(\overline{w}, q)\, \overline{w} = 0$.*

8. *If $f(\cdot)$ is homogeneous of degree one, then $c(\cdot)$ and $z(\cdot)$ are homogeneous of degree one in q.*

9. *If $f(\cdot)$ is concave, then $c(\cdot)$ is a convex function of q.*

The first property states that if input prices double then costs will double. Also, costs either remain the same or increase as quantity of the output increases.

The second property states that the cost function is concave in w.

The third property provides some results on the production set Y under convexity of the production function.

The fourth property states that if all input prices double, then the cost-minimizing conditional factor demands remain the same for a given level of output.

The fifth property states that the conditional factor demands are convex if the production function is convex.

The sixth property states that the conditional factor demands are the derivatives of the cost function with respect to the input prices. This is similar to Roy's Identity and Hotelling's Lemma, where the Walrasian demand functions and the supply function can be derived from the indirect utility function and the profit function.

The seventh property provides some results on the Hessian matrix of the cost function. In particular, own input price effects are negative, so that if an input price increases, the conditional factor demand will decrease. Also, the cross input price derivatives are equal (symmetry) and Euler's formula holds.

The eighth property provides states results about the cost function and the conditional factor demands given the homogeneity of the production function. If doubling inputs leads to a doubling of output in the production function, then doubling of output leads to a doubling of cost as well as a doubling of the conditional factor demands.

The ninth property provides results on the convexity of the cost function based on the concavity of the production function.

Now that we have defined the cost function, the firm's profit maximization problem can be restated.

$$\max_{q \geq 0} \; pq - c(w, q)$$

This is the "traditional specification" of the firm's profit-maximization problem. Note that pq is simply revenue and that $c(w, q)$ is the firm's cost given input prices and its choice of q. The necessary FOC for a level of q to be optimizing is:

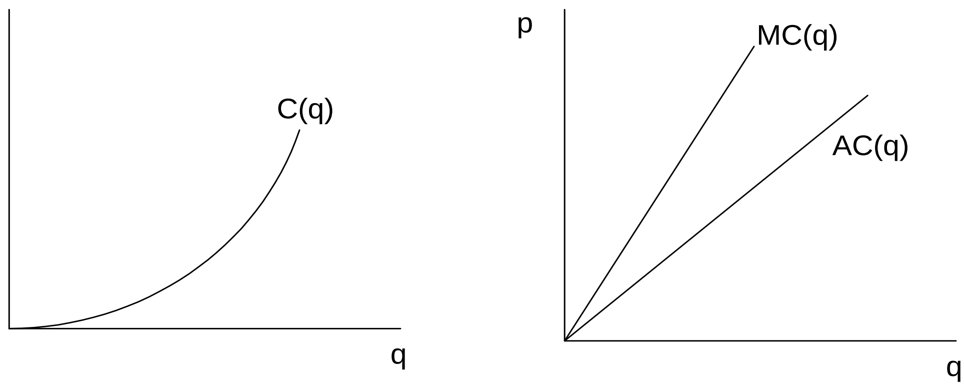
$$p - \frac{\partial c(w, q^*)}{\partial q} \leq 0.$$

Thus price must be less than or equal to marginal cost, and if $q^* > 0$ then price equals marginal cost. PLEASE REMEMBER that this is only true when the firm's output decision has no impact on the price of the output (i.e. the firm is a price-taker in the output market). Alternatively, we can say that the firm's profit maximizing output occurs at the point where marginal revenue, p, equals marginal cost, $\frac{\partial c(w, q^*)}{\partial q}$.

Note that right now we are not discussing any notion of equilibrium, thus the firm may be making positive, negative, or zero profit.[3]

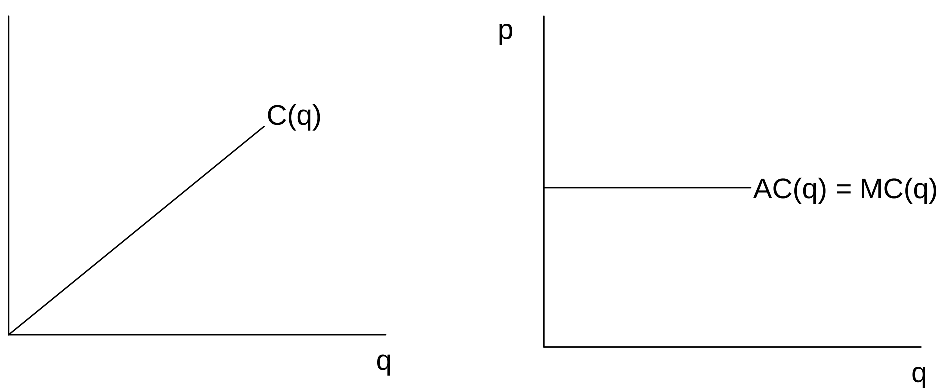
Now, consider the cost function, $c(w, q) = C(q)$, of the single output case with fixed input prices. Also, consider the average cost function, $AC(q) = \frac{C(q)}{q}$ and the marginal cost function $MC(q) = \frac{dC(q)}{dq}$. The average and marginal cost functions will be different functional forms depending on the assumptions of our production technology.

First consider the case of a convex production set where there are strictly decreasing returns to scale. Note that inactivity is possible. The firm's cost function may look like the left-hand side of figure 4 with $AC(q)$ and $MC(q)$ on the right-hand side. Note that the supply at a particular price is given by the marginal cost function.



Strictly decreasing returns to scale.

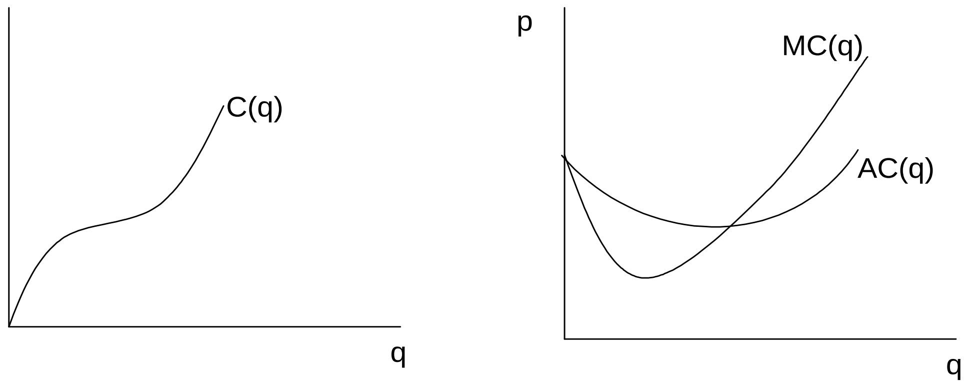
Next consider the case of constant returns to scale. In this case, the firm's cost function is a linear function since when we double output we double cost (property 8 of the cost function). Then $AC(q) = MC(q)$ and both are equal to some constant parallel to the x-axis, as in figure 4. Note that the supply is zero until the price reaches that constant because the firm's profit when price is below that constant would be negative for an quantity choice of the firm. Once price has surpassed that constant then the supply is infinity.



Constant returns to scale.

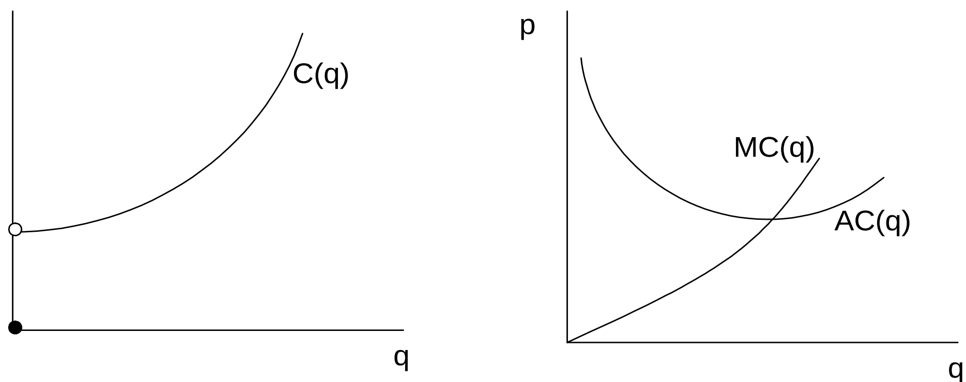
Now consider the case of a nonconvex technology. This is the typical case considered in the principles of microeconomics classes as depicted in figure 4. Marginal cost begins below average cost then crosses average cost at the minimum of average cost and then remains above average cost. Supply is given by 0 until price equals the minimum of average cost and then by the marginal cost function.

[3] I will discuss the zero-profit condition a little more in detail in chapter 10. It is my belief that this condition and the concept of rationality discussed in chapter 1 are two of the most misunderstood concepts of basic economics.



A nonconvex technology.

It is also possible that there are nonsunk setup costs (usually called fixed costs) of operation, depicted in figure 4. The firm does not have to pay these costs unless it enters into the production stage, and once the firm enters into the production stage it pays this cost once. Note that these costs do not depend on the amount of output produced. For a strictly convex variable cost production technology, the picture is a "combination" of the case with strictly decreasing returns to scale and the nonconvex production technology. Again, supply is zero until price equals the minimum of average cost and then it is equal to the firm's marginal cost.



A technology with fixed costs.