PhD Microeconomic Theory, BPHD 8100-001

Chapter 1 and 2 problems

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1. Assume $\succsim$ is rational:

   a. Prove that if $\succ$ is both irreflexive ($x \succ x$ never holds) and transitive (if $x \succ y$ and $y \succ z$, then $x \succ z$).

   b. Prove that $\sim$ is reflexive ($x \sim x$), transitive (if $x \sim y$ and $y \sim z$, then $x \sim z$), and symmetric (if $x \sim y$, then $y \sim x$)

2. Consider a rational preference relation $\succsim$. Prove that if $u(x) = u(y)$ implies $x \sim y$ and $u(x) > u(y)$ implies $x \succ y$ then $u(\cdot)$ is a utility function representing $\succsim$.

3. A consumer consumes one consumption good x and hours of leisure h. The price of the consumption good is p, and the consumer can work at a wage rate of $s = 1$. What is the consumer's Walrasian budget set?

4. Show that if the consumption set X is finite and $\succsim$ is a rational preference relation on X, then there is a utility function $u : X \to \mathbb{R}$ that represents $\succsim$.

5. Suppose $u(x_1, x_2)$ and $v(x_1, x_2)$ are utility functions. Assume that $u(x_1, x_2)$ and $v(x_1, x_2)$ are both homogeneous of degree r. Prove that $s(x_1, x_2) \equiv u(x_1, x_2) + v(x_1, x_2)$ is homogeneous of degree r.

6. A consumer lives for two periods, denoted 1 and 2, and consumes a single consumption good in each period. His wealth when born is $w > 0$. What is his (lifetime) Walrasian budget set?

7. A consumer in a two-good economy has a demand function $x(p, w)$ that satisfies Walras' law. His demand function for the first good is $x_1(p, w) = \alpha w / p_1$. Derive his demand function for the second good. Is his demand function homogeneous of degree zero?