

PhD Microeconomic Theory, BPHD 8100-001

Chapter 3 problems

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1. The n -good Cobb-Douglas utility function is given by:

$$u(x_1, \dots, x_n) = A \prod_{i=1}^n x_i^{\alpha_i}$$

where $A > 0$ and $\sum_{i=1}^n \alpha_i = 1$. Let $p_i > 0$ represent the price of good i , and let $w > 0$ be the consumer's wealth.

- a Derive the Walrasian demand functions, $x(p, w)$.
 - b Derive the indirect utility functions, $v(p, w)$.
 - c Compute the expenditure function, $e(p, u)$.
 - d Compute the Hicksian demand functions, $h(p, u)$.
2. Prove that if \succsim is rational and monotone, then it is locally nonsatiated.
3. Consider the following utility function:

$$u(x_A, x_B) = \alpha x_A^2 + \beta x_B^2$$

The consumer's income is denoted by $w > 0$. Assume $\alpha > 0$ and $\beta > 0$. The prices of goods A and B are denoted by p_A and p_B respectively.

- a A rational preference relation is one that is complete and transitive. In addition to assuming that the consumer's preference relation, \succsim , is rational we also impose desirability, convexity, and continuity assumptions on the preference relation that provide restrictions on our utility function. Based on the utility function above, is it the case that the preference relation meets our standard desirability, convexity, and continuity assumptions? Explain why or why not.
 - c Let $w = \$290$, $p_A = \$2$ and $p_B = \$5$. Also let $\alpha = \beta$. Find the consumer's optimal bundle given this income level and these prices.
4. A consumer has the following expenditure function

$$e(p, u) = \alpha_1^{-\alpha_1} \alpha_2^{-\alpha_2} \alpha_3^{-\alpha_3} p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} u$$

in the three good economy with goods x_1 , x_2 , and x_3 . Assume $\alpha_i > 0$ and $p_i > 0$ for $i = 1, 2, 3$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

- a Find the Hicksian demands for goods x_1 , x_2 , and x_3 . **Note:** You can find all of them individually or find the Hicksian demand for a general good i , whichever you prefer.
- b Derive the consumer's indirect utility function, $v(p, w)$.
- c Find the Walrasian demands for goods x_1 , x_2 , and x_3 .
- d Find the 3×3 Slutsky matrix for this expenditure function.
- e Verify that the Slutsky matrix is symmetric and that the elements along the diagonal are negative.

5. Jesse likes only two goods, green eggs and ham. However, he really likes green eggs, so much so that when comparing any bundles of green eggs and ham he first looks at the number of green eggs in the bundle. If one bundle has more green eggs than another bundle, he prefers the bundle with more green eggs regardless of how much ham is in either bundle (so if bundle one has 2 green eggs and 0 pounds of ham and bundle two has 1 green egg and 1,000,000 pounds of ham he prefers bundle one to bundle two). If both bundles have the same number of green eggs then he looks to see how much ham is in the bundle and prefers the bundle with more ham. Additionally, Jesse is willing to accept bundles with fractions of green eggs and ham (so a bundle with 1.5 green eggs is better than a bundle with 1.3 green eggs, etc.). Draw (or describe) an indifference set for Jesse's preferences. Are these preferences convex and/or locally nonsatiated? Explain.
6. Show that if for every x the upper and lower contour sets $\{y \in \mathbb{R}_+^L : y \succsim x\}$ and $\{y \in \mathbb{R}_+^L : x \succsim y\}$ are closed, then \succsim is continuous according to the following definition from class:

Definition 1 *The preference relation \succsim on X is continuous if it is preserved under limits. That is, for any sequence of pairs $\{(x^n, y^n)\}_{n=1}^\infty$, with $x^n \succsim y^n \forall n$, $x = \lim_{n \rightarrow \infty} x^n$, and $y = \lim_{n \rightarrow \infty} y^n$, we have $x \succsim y$.*

Hint: If you cannot show this generally you may assume that \succsim is monotone which may be an easier proof.

7. Bob consumes ice cream cones (x_1) and hamburgers (x_2). His utility function is

$$u(x_1, x_2) = (x_1)^{\frac{1}{2}}(x_2)^{\frac{1}{2}}$$

Bob's income is \$100. The price of each hamburger is \$2. The price of ice cream depends on the quantity that Bob consumes. Specifically, he can buy the first ten ice cream cones at the price of \$2 each. For each additional ice cream cone there is a discount, and Bob has to pay only \$1 each.

Derive Bob's budget constraint and compute his optimal consumption plan.

8. Consider the following utility functions: $u(x_1, x_2) = \sqrt{x_1 x_2}$ and $v(x_1, x_2) = \ln(x_1) + \ln(x_2)$. Verify that u and v have the same indifference curves and the same marginal rate of substitution. Explain why.