PhD Microeconomic Theory, BPHD 8100-001

Chapter 6 problems

September 18, 2014

1. Consider the following axiom, known as the betweeness axiom:

$$\text{For all } L, L' \text{ and } \lambda \in (0,1), \text{ if } L \sim L', \text{ then } \lambda L + (1-\lambda) L' \sim L.$$

Suppose that there are 3 possible outcomes.

a Show that a preference relation on lotteries satisfying the independence axiom also satisfies the betweeness axiom.

b Using a simplex representation for lotteries similar to the one in Figure 6.B.1(b) (of the textbook – page 169), show that if the continuity and betweeness axioms are satisfied, then the indifference curves of a preference relation are straight lines. Conversely, show that if the indifference curves are straight lines, then the betweeness axiom is satisfied. Do these straight lines need to be parallel?

c Using (b), show that the betweeness axiom is weaker (less restrictive) than the independence axiom.

d Using Figure 6.B.7, show that the choices of the Allais paradox are compatible with the betweeness axiom by exhibiting an indifference map satisfying the betweeness axiom that yields the choices of the Allais paradox.

2. Consider the quadratic Bernoulli utility function $u(w) = a + bw + cw^2$.

a What restrictions, if any, must be placed on paramters a, b, and c for this function to display risk aversion and follow the standard properties of a Bernoulli utility function?

b Over what domain of wealth can a quadratic Bernoulli utility function be defined?

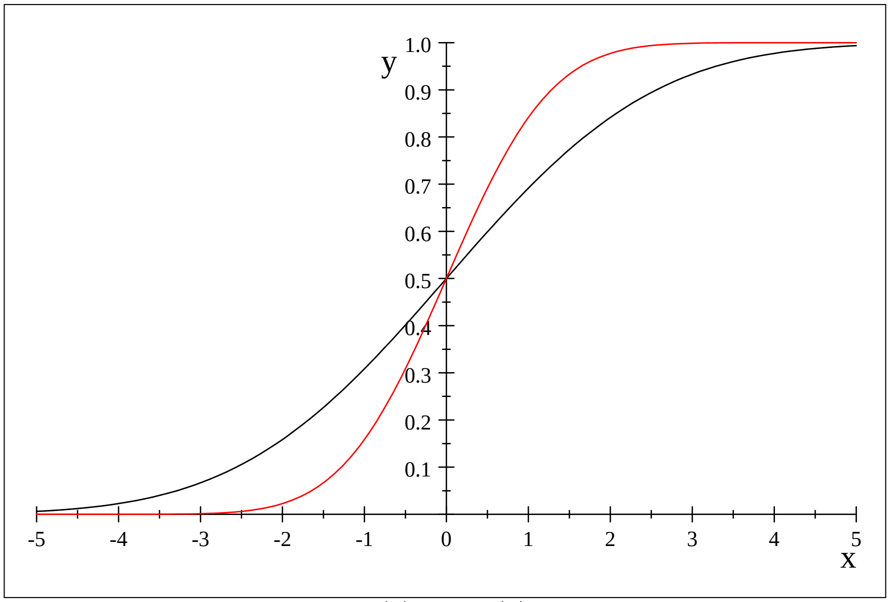
c Given the lottery

$$L = \left(\frac{1}{2}, \frac{1}{2}\right)$$

over $w + h$ and $w - h$, show that the certainty equivalent, $C(F, u)$, is less than the expected value of the gamble, $E(L)$.

d Show that this function, satisfying the restrictions in part **a**, cannot represent preferences that display decreasing absolute risk aversion.

3. Consider two distribution functions, $F(x)$ and $G(x)$. Both distribution functions are normal distributions with mean 0. Distribution $F(x)$ has variance $\frac{1}{2}\sigma^2$ and distribution $G(x)$ has variance σ^2, with $\sigma > 1$. Thus, under $F(x)$ we have $x \sim N\left(0, \frac{1}{2}\sigma^2\right)$ and under $G(x)$ we have $x \sim N\left(0, \sigma^2\right)$. Figure 3 shows $F(x)$ and $G(x)$, where $F(x)$ is the red line and $G(x)$ is the black line. Note that for $(-\infty, 0)$ we have that the red line is always below the black line, and for $(0, \infty)$ we have that the red line is always above the black line.



F(x) and G(x)

a Does either distribution first order stochastically dominate the other? Explain.

b Does either distribution second order stocahstically dominate the other? Explain.

4. Suppose that an individual has a Bernoulli utility function $u(x) = \sqrt{x}$.

   a. Calculate the Arrow-Pratt coefficients of absolute and relative risk aversion at the level of wealth $w = 5$.

   b. Calculate the certainty equivalent and the probability premium for a gamble $\left(16, 4; \frac{1}{2}, \frac{1}{2}\right)$ (half of the time the individual receives 16 and the other half 4).

   c. Calculate the certainty equivalent and the probability premium for a gamble $\left(36, 16; \frac{1}{2}, \frac{1}{2}\right)$.

   d. Compare the results from parts **b** and **c** and interpret.

5. Prove that if $F(\cdot)$ first-order stochastically dominates $G(\cdot)$, then the mean of x under $G(\cdot)$, $\int x dG(x)$, is less than or equal to the mean of x under $F(\cdot)$, $\int x dF(x)$. Also, provide an example where $\int x dF(x) > \int x dG(x)$ but $F(\cdot)$ does not first-order stochastically dominate $G(\cdot)$.

6. Suppose that an individual has a Bernoulli utility function $u(w) = 7 + w^{1/3}$.

   a Calculate the Arrow-Pratt coefficient of absolute risk aversion, $r_a(w)$.

   b Does this individual have increasing, constant, or decreasing absolute risk aversion (IARA, CARA, or DARA)? Explain how you know.

   c Calculate the certainty equivalent for the lottery $L = \left(\frac{1}{2}, \frac{1}{2}\right)$ over the outcomes 64 and 27 using the Bernoulli utility function provided.

7. Assume an individual has decreasing absolute risk aversion (DARA). The individual has an amount w to put into a risky asset. The risky asset has N potential rates of return r_i (the outcomes) with probability p_i for $i = 1, ..., N$. Let β be the amount of wealth to be put into the risky asset, so that final wealth (once an outcome is realized) is:

$$(w - \beta) + (1 + r_i)\beta = w + \beta r_i$$

   Show that the optimal amount invested into the risky asset is increasing in w, or $\frac{d\beta^*}{dw} > 0$.