PhD Microeconomic Theory, BPHD 8100-001

Chapters 5 and 10 problems

October 16, 2014

1. Consider our two-good economy with quasilinear consumer preferences. Suppose that there is one firm ($J = 1$) and one consumer ($I = 1$). The initial endowment of the numeraire is $w_m > 0$ and the initial endowment of good ℓ is 0. Let the consumer's utility function be $u(x) = \phi(x) + m$, where $\phi(x) = \alpha + \beta \ln x$ with $\alpha > 0$ and $\beta > 0$. Let the firm's cost function be $c(q) = \sigma q$ for some scalar $\sigma > 0$. Assume that the consumer receives all the profits of the firm. Both the firm and consumer act as price-takers. Normalize the price of good m to be 1 and let the price of good ℓ be p.

   a Derive the consumer's and the firm's first order conditions.

   b Derive the competitive equilibrium price and output of good ℓ.

   c How do the competitive equilibrium price and output vary with α, β, and σ?

2. Consider a firm with the cost function

$$c(q, w_1, w_2) = 2\sqrt{q^3 w_1 w_2},$$

   where w_i denotes the price of input i for $i = 1, 2$. Let p denote the output price. The production function for this cost function is:

$$f(z_1, z_2) = z_1^{1/3} z_2^{1/3}$$

   a Derive the conditional input demand functions, $z_i(w, q)$ for $i = 1, 2$.

   b Verify that the cost function in the problem is in fact the one that is associated with the production function.

   c Verify that $c(q, w_1, w_2)$ is homogeneous of degree one in w.

   d Verify that $c(q, w_1, w_2)$ and $z_i(q, w_1, w_2)$ are NOT homogeneous of degree one in q. What does this tell us about the homogeneity of the production function $f(z_1, z_2)$?

3. Assume that a firm is risk neutral with respect to profits and that if there is any uncertainty in prices, production decisions are made after the resolution of such uncertainty. Suppose that the firm faces a choice between two alternatives. In the first, prices are uncertain. In the second, prices are nonrandom and equal to the expected price vector in the first alternative. Show that a firm that maximizes expected profits will prefer the first alternative over the second.

4. Derive the cost function $c(w, q)$ and conditional factor demands $z(w, q)$ for the following single-output technology with the production function:

$$f(z) = (z_1^\rho + z_2^\rho)^{1/\rho} \text{ for } \rho \leq 1.$$

5. Consider the linear production function $f(z_1, z_2) = \alpha z_1 + \beta z_2$, where α and β are constants and $\alpha > 0$ and $\beta > 0$. Let $w_1 > 0$ and $w_2 > 0$ represent the input prices for z_1 and z_2 respectively. Suppose the firm wishes to produce $\overline{q}$.

   a. Calculate the marginal product for input z_1 and input z_2.

   b. Find the conditional factor demands, $z_1(w, q)$ and $z_2(w, q)$ for inputs z_1 and z_2.

c. Find the cost function $c(w, q)$.

6. A monopolist faces linear inverse demand $p(q) = a - bq$ and has cost $c(q) = cq + F$, where $a, b, c, F > 0$, $a > c$, and $(a - c)^2 > 4bF$.

   a Solve for the monopolist's profit-maximizing price and quantity.

   b Suppose the government requires this firm to set the price that maximizes the Marshallian aggregate surplus ("gains from trade"). What is the price the monopolist must charge? Show that the monopolist's profits are negative under this regulation. (You can use a graph to guide your answer.)