

# PhD Microeconomic Theory, BPHD 8100-001

Chapters 7, 8, and 9 problems

November 3, 2014

1. Consider a developer who wishes to purchase  $k$  parcels of land. If the developer purchases all  $k$  parcels, the developer receives a payment of  $D$ . If the developer does not purchase all  $k$  parcels, the developer receives a payment of 0. The developer must purchase each parcel of land from the landowner who owns the land.

Consider  $k$  landowners who each own a parcel of land. That parcel has value of  $v_i$  to the landowner, where  $v_i \sim U[0, \frac{D}{k}]$ . The individual landowners know their own value for the land but the developer does not. Also, the landowners do NOT know the values of other landowners.

The game can be modeled as a sequential game. The developer makes an offer  $w_i$  to each landowner. Each landowner only observes his own  $w_i$  and must make a decision to accept or reject that  $w_i$ . If all  $k$  landowners accept their own offer  $w_i$ , then the landowners each receive  $w_i$  as a payment from the developer; the developer pays an amount  $\sum_{i=1}^k w_i$ , and the developer receives a payment of  $D$ . If ANY landowner chooses to reject  $w_i$ , then the developer makes no payment to any landowner and acquires no parcels of land – the developer receives 0 but pays 0. The landowners, who still own their land, receive  $v_i$ .

For simplicity, assume the seller sets  $w_i = w_j$  for all  $i, j$ . The developer maximizes expected utility, and receives  $D - kw$  if aggregation is successful (which occurs only if all  $k$  landowners accept the offer) and 0 if not. Note that  $\Pr(\tilde{v} > v)$  for the uniform distribution  $U[0, \frac{D}{k}]$  is  $\frac{\tilde{v}}{\frac{D}{k}}$ .

Assume the developer is risk neutral. Find a subgame perfect Nash equilibrium to this game with  $k$  landowners. Be sure to set up the developer's expected utility function correctly.

2. Consider a simultaneous game between a goalie and a player trying for a penalty kick in soccer. The goalie can choose to guard the left or right side of the goal and the kicker can choose to kick to the left or right side of the goal. If both choose left then the kicker receives 58 and the goalie receives 42. If both choose right then the kicker receives 70 and the goalie receives 30. If the kicker chooses left and the goalie chooses right then the kicker receives 95 and the goalie receives 5. If the kicker chooses right and the goalie chooses left then the kicker receives 93 and the goalie receives 7. Draw the strategic form for this game and find all pure and mixed strategy Nash equilibria for this game.
3. Consider two firms who compete by simultaneously choosing prices (a Bertrand game). If firms 1 and 2 choose prices  $p_1$  and  $p_2$ , respectively, the quantity that consumers demand from firm  $i$  is

$$q_i(p_i, p_j) = a - p_i + bp_j, \text{ with } 0 < b < 2.$$

Assume that there are no fixed costs and that marginal cost is constant and equal to  $c$ , where  $a > c > 0$ . Prices must be nonnegative ( $p_1 \geq 0, p_2 \geq 0$ ) and firms wish to maximize profit.

- a Find the best response functions for firms 1 and 2.
  - b Find the pure strategy Nash equilibrium to this game.
  - c Explain why  $b < 2$ .
4. Show that any strictly dominant strategy in game  $[I, \{\Delta(S_i), \{u_i(\cdot)\}\}]$  must be a pure strategy.

5. Suppose that we have two players, Mortimer and Hotspur, who are charged with a task of dividing \$100. In the first stage of the game, Mortimer offers Hotspur an amount of the \$100, of which Hotspur would receive the offer and Mortimer would receive  $(\$100 - offer)$ . (**Note:** All offers must be greater than \$0 and less than the amount of the pie.) This phase is sequential, so Hotspur observes this offer and can choose to accept or reject the offer. If Hotspur accepts, the game ends and Hotspur receives the offer and Mortimer receives  $(\$100 - offer)$ . If Hotspur rejects, the game ends and both players receive \$0.

**a** Assume that both players have a Bernoulli utility function such that  $u(x) = x$ . Assume that offers can only be made in dollar increments (\$1, \$2, \$3, etc.). Find a subgame perfect Nash Equilibrium (SPNE) to this game.

*Now, consider a modified version of the game.* The beginning of the game is still the same (Mortimer makes an offer, Hotspur observes this offer and can accept or reject, if he accepts the game ends). However, now if Hotspur rejects, the game continues to a second phase, where the amount of money shrinks to \$90. Hotspur now has the chance to make an offer to Mortimer. The second phase is sequential so Mortimer observes this offer and can either accept or reject the offer. If Mortimer accepts, the game ends and Mortimer receives the offer and Hotspur receives  $(\$90 - offer)$ . If Mortimer rejects, the game ends and each receives \$0.

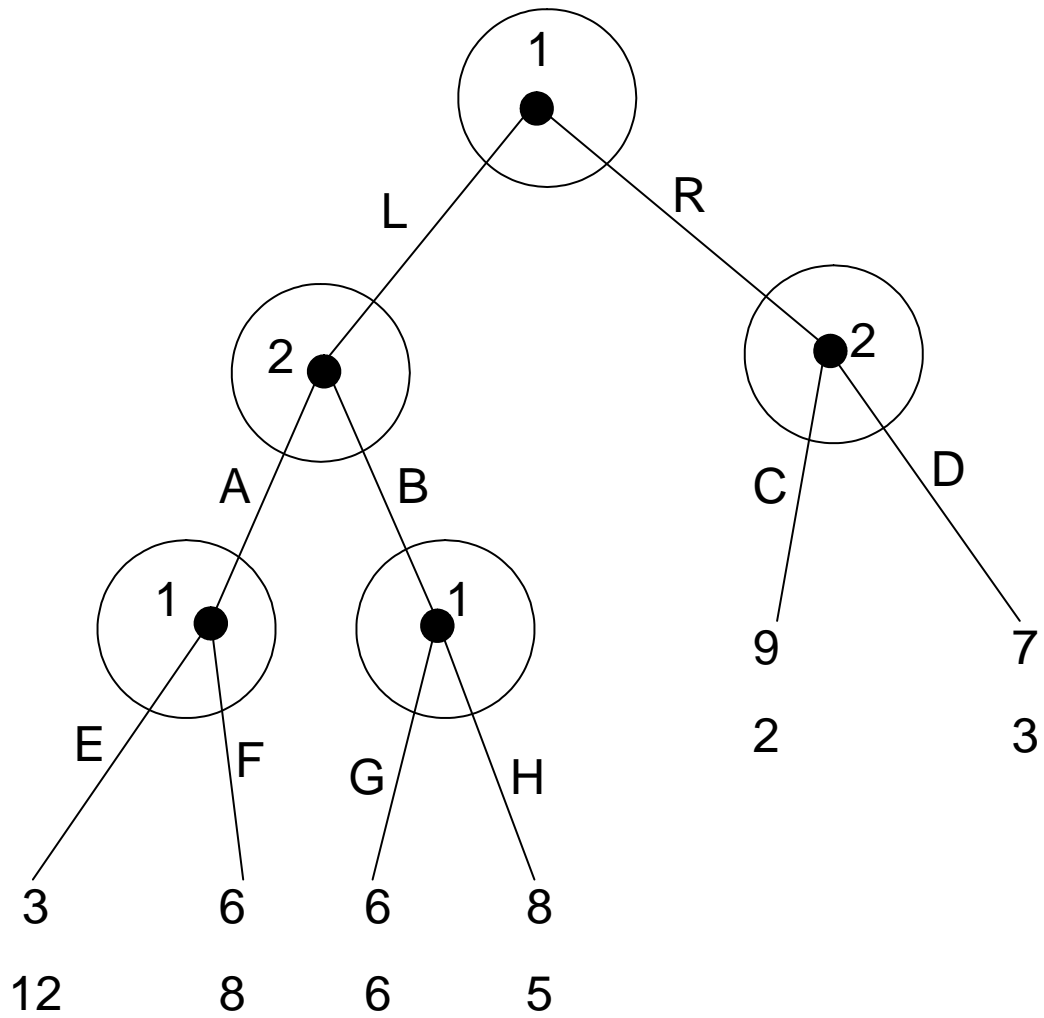
**b** Assume that both players have a Bernoulli utility function such that  $u(x) = x$ . Assume that offers can only be made in dollar increments (\$1, \$2, \$3, etc.). Find a subgame perfect Nash Equilibrium (SPNE) to the entire game.

**c** What is the outcome to the SPNE that you found in part **b**?

6. Find all pure and mixed strategy Nash equilibria to the following game:

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Player 1	<i>T</i>	2, 0	1, 1	4, 2
	<i>M</i>	3, 4	1, 2	2, 3
	<i>B</i>	1, 3	0, 2	3, 0

7. Consider the extensive form game in the following game. Note that player 1's payoffs are listed first.



- a Find the pure strategy Nash Equilibrium (equilibria) of this sequential game.
- b What is the subgame perfect Nash Equilibrium of this game?