These notes essentially correspond to chapter 11 of the text.

1 Monopoly

A monopolist is defined as a single seller of a well-defined product for which there are no close substitutes. In reality, there are very few "true" monopolists; however, people sometimes consider firms with a large market share (such as Microsoft) a monopolist. We will focus on the implications of the "true" monopolist.

In the perfectly competitive market, the market demand curve is downward sloping, and the firm's demand curve is horizontal (perfectly elastic). In a monopoly, the market demand curve is also downward-sloping – however, since there is only a single seller in the market, the market demand curve is also the monopolist's demand curve. The monopolist's downward-sloping demand curve has some implications for the monopolist's MR.

1.1 Deriving MR for monopolist

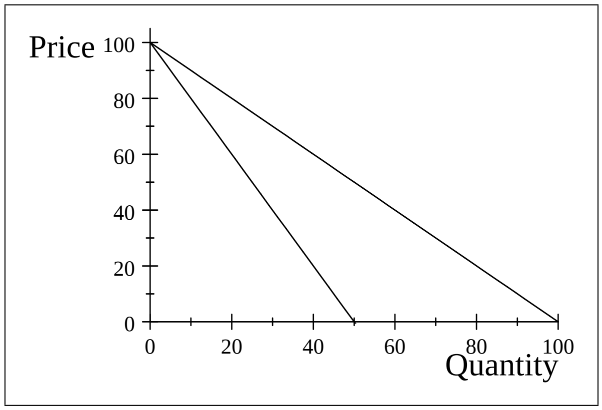
We will derive the monopolist's MR by example first, and then through a formal mathematical derivation.

1.1.1 Deriving MR by example

Suppose that the monopolist faces the following inverse demand function, $P(Q) = 100 - Q$. The monopolist's TR function is found by multiplying price and quantity, so that $TR = P(Q) * Q = (100 - Q) * Q$ in this example. We can now fill out the table below for the given quantities. The price is found by plugging the different quantity levels into the inverse demand function. Total revenue is found by multiplying price and quantity. Recall that MR is just the increase in TR from one unit to the next (which is how we found MC in chapter 7, except we looked at the increase in TC from one unit to the next).

Quantity	**Price**	**TR**	**MR**
0	100	0	–
1	99	99	99
2	98	196	97
3	97	291	95
4	96	384	93
5	95	475	91

If we were to plot the price and quantity pairs, we would get the firm's demand curve. If we were to plot the MR and quantity pairs, we would get the firm's MR. Plotting the two relationships gives us:



The MR is the steeper of the two lines, and lies inside the demand curve. Notice that the MR of the 2^{nd} unit is \$97 even though the price is \$98. The reason that $MR < P$ is because if the monopolist wishes to sell an additional unit, it needs to lower the price on EVERY unit sold. Thus, the first unit that was initially sold for \$99 brought in additional revenue of \$99. To sell 2 units, the monopolist must lower the price to \$98. The second unit brings in additional revenue of \$98, but the 1^{st} unit must now also be sold for \$98, which is a loss of \$1 in revenue. Thus, the total additional revenue generated by the second unit is $\$98 - \$1 = \$97$. So, the MR for a monopolist will fall faster than the demand curve. Recall that in a perfectly competitive market the MR and demand curves were the same curves.

1.1.2 Deriving a MR function

We can also derive a monopolist's MR as a function of quantity. I will work through the steps for deriving a monopolist's MR function for a linear inverse demand function (we will only work with linear inverse demand functions). At the end of the derivation, I will give you a rule that you can use to find the monopolist's MR function for any linear inverse demand function.

Recall that $TR = P(Q) * Q$ which equals $(a - bQ) * Q$ for a general linear inverse demand function with intercept a and slope $(-b)$. We want to see how total revenue changes for a very small, essentially zero, change in quantity. Start with the total revenue of some quantity Q.

$$TR(Q) = (a - bQ) * Q = aQ - bQ^2$$

Now, find the total revenue for $Q + h$, which h is some positive, albeit very small, amount.

$$\begin{aligned} TR(Q+h) &= (a - b*(Q+h)) * (Q+h) = \\ (a - bQ - bh) * (Q+h) &= aQ - bQ^2 - bQh + ah - bQh - bh^2 \end{aligned}$$

So, $TR(Q) = aQ - bQ^2$ and $TR(Q+h) = aQ - bQ^2 - 2bQh + ah - bh^2$ (notice that I combined the 2 $(-bQh)$ terms into $(-2bQh)$). The definition for MR says that it is the change in TR divided by the change in quantity. The change in TR is:

$$\Delta TR = \left(aQ - bQ^2 - 2bQh + ah - bh^2\right) - \left(aQ - bQ^2\right)$$

This simplifies to:

$$\Delta TR = \left(-2bQh + ah - bh^2\right)$$

Since we began by producing Q and we have now increased production to $Q+h$, the change in quantity is $Q+h-Q=h$. So our MR is:

$$MR(Q) = \frac{\Delta TR}{\Delta Q} = \frac{\left(-2bQh + ah - bh^2\right)}{h}$$

Simplifying, we are left with:

$$MR(Q) = (-2bQ + a - bh)$$

There is one final step. Recall that we wanted h to be a very small number, essentially zero. If h is essentially zero, then the term $(-bh) = (-b * 0) = 0$ and it drops out. This leaves us with (I will rearrange so that the a term is first):

$$MR(Q) = a - 2bQ$$

For those of you that have had calculus, the MR function is simply the derivative of the TR function with respect to quantity. So:

$$\frac{dTR(Q)}{d(Q)} = \frac{d\left(aQ - bQ^2\right)}{dQ} = a - 2bQ$$

Either way gets you the same answer, that the MR function is $a - 2bQ$ for a linear inverse demand function of the form $a - bQ$. This is our rule:

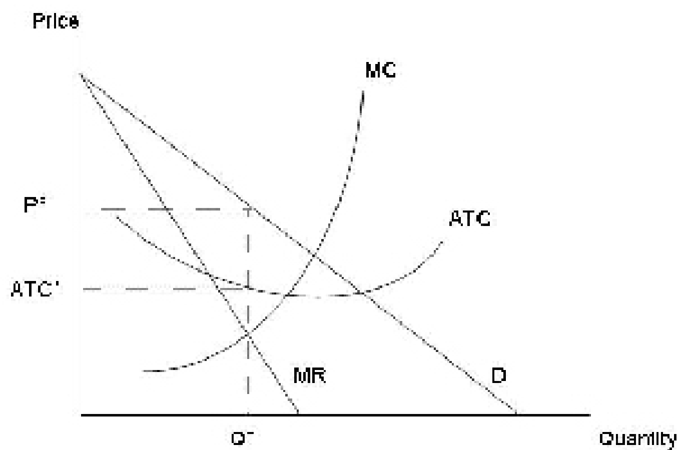
RULE If the inverse demand function is a linear inverse demand function of the form $P(Q) = a - bQ$, then the marginal revenue function is: $MR(Q) = a - 2bQ$.

1.2 Profit maximization for a monopolist

We will use two methods to find the monopolist's maximum profit. The first is a graphical method and the second is a mathematical method.

1.2.1 Profit maximization – graphically

The steps to finding the monopolist's profit-maximizing price and quantity are similar to those for the perfectly competitive firm. A picture is shown below and the steps are described following the picture.



1. The first step is to find the quantity that corresponds to the point where $MR = MC$. This is Q^* in the picture.

2. The second step is to find the price that corresponds to the quantity that corresponds to the point where $MR = MC$. The firm finds this price by finding the price on the DEMAND curve that corresponds to its profit-maximizing quantity. This is shown in the picture as P^*.

3. Find the firm's total revenue at the profit-maximizing price and quantity. Since this is just the price times the quantity it is $(P^*) * (Q^*)$.

4. Now, find the ATC that corresponds to the profit-maximizing quantity. This is shown as ATC^* in the picture.

5. Find the firm's total cost at the profit-maximizing price and quantity. This is $TC = (ATC^*) * (Q^*)$.

6. The firm's profit is then $TR - TC$. Alternatively, the firm's profit can be written as $\Pi = (P^*) * (Q^*) - (ATC^*) * (Q^*) = (P^* - ATC^*) * (Q^*)$. When written this way, it is easy to see that the profit the firm earns is simply the rectangle outlined by the dotted lines in the picture from P^*

to ATC^* and over to Q^*. So profit is simply the area outlined by that rectangle.

1.2.2 Profit maximization – mathematically

We will follow the same basic steps to determine the profit-maximizing price and quantity mathematically. We will need a few pieces of information: the monopolist's inverse demand function, marginal revenue function, marginal cost function, and either the average total cost function or the total cost function. Assume that the inverse demand function is: $P(Q) = 24 - Q$. This means that the marginal revenue function is: $MR(Q) = 24 - 2Q$. Suppose that the monopolist's marginal cost function is: $MC(Q) = 2Q$, and that the monopolist's average total cost function is: $ATC(Q) = Q + \frac{12}{Q}$.

1. The first step is to find the quantity that corresponds to the point where $MR = MC$. Since $MR(Q) = 24 - 2Q$ and $MC(Q) = 2Q$, we set $MR(Q) = MC(Q)$. This gives us:

$$24 - 2Q = 2Q$$

$$24 = 4Q$$

$$Q = 6$$

2. The second step is to find the price that corresponds to the quantity that corresponds to the point where $MR = MC$. We know that when $MR = MC$, the firm's quantity is 6. The price that the firm will sell 6 units at is found by plugging the quantity into the inverse demand function, which is $P(Q) = 24 - Q$.

$$P(6) = 24 - 6 = 18$$

   Thus, the firm will sell 6 units at a price of 18.

3. Now, total revenue is simply price times quantity, or $\$18 * 6 = \$108$.

4. Now, find the ATC that corresponds to the profit-maximizing quantity. We can use the ATC function, which is $ATC(Q) = Q + \frac{12}{Q}$. So:

$$ATC(6) = 6 + \frac{12}{6} = 6 + 2 = 8$$

   Thus, the average total cost of producing 6 units is $8.

5. The monopolist's total cost is just ATC times Q, or $\$8 * 6 = \$48$.

6. The monopolist's profit is then $TR - TC$, which is just $\$108 - \$48 = \$60$.

Notice that the steps to find maximum profits are the same in either method. In one method a picture is used and in another method functions are used.

2 Monopolist and Price Elasticity of Demand

We can determine how much market power a monopolist has if we calculate the monopolist's PED at its profit-maximizing quantity. However, we will need to rewrite the MR in terms of PED. We know that, for a linear inverse demand function, $MR(Q) = a - 2bQ$. Use the following steps to rewrite the MR.

$$MR(Q) = a - 2bQ = a - bQ - bQ$$

We know that $P(Q) = a - bQ$, so:

$$MR(Q) = P - bQ$$

We also know that $(-b) = \frac{\Delta P}{\Delta Q}$ (where it is understood that $\frac{\Delta P}{\Delta Q}$ is negative). So:

$$MR(Q) = P + \frac{\Delta P}{\Delta Q}Q$$

Multiply both sides by "one" (this is one of the tricks). On the left-hand side, we will choose our "one" to be 1, and on the right-hand side we will choose our "one" to be $\frac{P}{P}$.

$$MR(Q) = \frac{P}{P}\left(P + \frac{\Delta P}{\Delta Q}Q\right)$$

Distribute:

$$MR(Q) = \frac{P}{P}P + \left(\frac{P}{P}\right)\frac{\Delta P}{\Delta Q}Q$$

Now rewrite as:

$$MR(Q) = P + P\frac{\Delta P}{\Delta Q}\frac{Q}{P}$$

Factor out the P:

$$MR(Q) = P\left(1 + \frac{\Delta P}{\Delta Q}\frac{Q}{P}\right)$$

Recall that price elasticity of demand is equal to $\left(\frac{\Delta Q}{\Delta P} * \frac{P}{Q}\right)$. Notice that the last term is simply the reciprocal of the PED. So:

$$MR(Q) = P\left(1 + \frac{1}{PED}\right)$$

where PED is negative (and NOT the absolute value).

This formula has some interesting implications for the monopolist's pricing decision. First, notice that if $PED = -1$ then $MR = 0$. If $0 > PED > -1$, then $MR < 0$. If $-1 > PED > -\infty$, then $MR > 0$. This suggests that the monopolist will never price on the inelastic portion of its demand curve, as the monopolist will actually be losing revenue by choosing a price on the inelastic portion of the demand curve.

Recall that the elasticity for a linear demand curve depends on the particular point chosen along the demand curve. As we move up the demand curve (higher prices), demand becomes more elastic. As we move down the demand curve (towards a 0 price), demand becomes more elastic. For linear demand curves, the quantity level halfway between 0 and the point where the demand curve crosses the quantity axis is the point that corresponds to $PED = -1$. Thus, the MR at this point is zero.

2.1 Price-cost Markup and Market Power

In a perfectly competitive market, when a firm chooses its quantity it sets $MR = MC$. However, since the firm's MR is the same as the price in the market, the firm is in essence charging a price equal to its marginal cost, or $P = MC$. With a monopolist, the price charged by the firm is above the MC of production for that unit. We can use the price-cost markup as an indicator of a monopolist's market power. The price-cost markup formula is:

$$\frac{P - MC}{P}$$

The formula will range from 0 to 1: If $P = MC$, as in the perfectly competitive firm, then the price-cost markup will equal 0; as the monopolist increases its price above marginal cost (effectively making marginal cost very small relative to price), then the price-cost markup will tend to 1. So, the closer the number is to 0 the less market power the firm has.

2.2 Lerner Index and Market Power

An alternative method of determining market power is to look at the Lerner Index. Recall that $MR = P\left(1 + \frac{1}{PED}\right)$. At the profit-maximizing quantity, $MR = MC$, so $P\left(1 + \frac{1}{PED}\right) = MC$. We can use a few algebra maneuvers to show that $\frac{P-MC}{P} = \frac{-1}{PED}$ at the profit-maximizing quantity.

Start with:

$$P\left(1 + \frac{1}{PED}\right) = MC$$

Then:

$$\left(1 + \frac{1}{PED}\right) = \frac{MC}{P}$$

Multiply both sides by negative one:

$$-1 - \frac{1}{PED} = -\frac{MC}{P}$$

Now, add "one" to both sides. I will add the number 1 to the left-hand side, and the term $\frac{P}{P}$ to the right-hand side. But I am really just adding one to both sides.

$$1 - 1 - \frac{1}{PED} = \frac{P}{P} - \frac{MC}{P}$$

Simplify to get:

$$\frac{-1}{PED} = \frac{P - MC}{P}$$

Thus, at the profit-maximizing quantity, the price-cost markup is simply the reciprocal of the monopolist's price elasticity of demand at that quantity. This is known as the Lerner Index, and it measures market power in the same manner as the price-cost markup. Note that the more elastic demand is at the monopolist's profit-maximizing quantity, the less market power the firm has. Also note that this relationship only holds at the firm's profit-maximizing quantity, since we assumed that $MR = MC$ when we derived the fact that $\frac{-1}{PED} = \frac{P-MC}{P}$.

3 Monopolies and Social Welfare

It was suggested that one reason to use the perfectly competitive market was that it provided a benchmark model for markets to reach. We can now compare the welfare properties of the monopoly with those of the perfectly competitive market.

There are quite possibly more definitions for the term "efficient" in economics than there are for any other term. We can define efficiency as a market situation where all the gains from trade are captured. Recall the partial equilibrium analysis of a tax from chapter 3. When a tax was imposed on the market there were some trades that were previously made that were no longer possible. This loss to society from trades that were not made is called deadweight loss. What we will show is that the perfectly competitive market contains no deadweight loss, while the monopoly market does. However, we will need to define a few terms first.

Consumer Surplus Consumer surplus is the difference between a consumer's maximum willingness to pay for a unit of the good (the height of the demand curve) and the price actually paid for the good. Thus, if the most a consumer is willing to pay is \$10 for a good and the consumer only pays \$3, then the consumer has \$7 in consumer surplus. If we were to look at the consumer surplus on a graph, it would be the entire area under the demand curve but above the price paid.

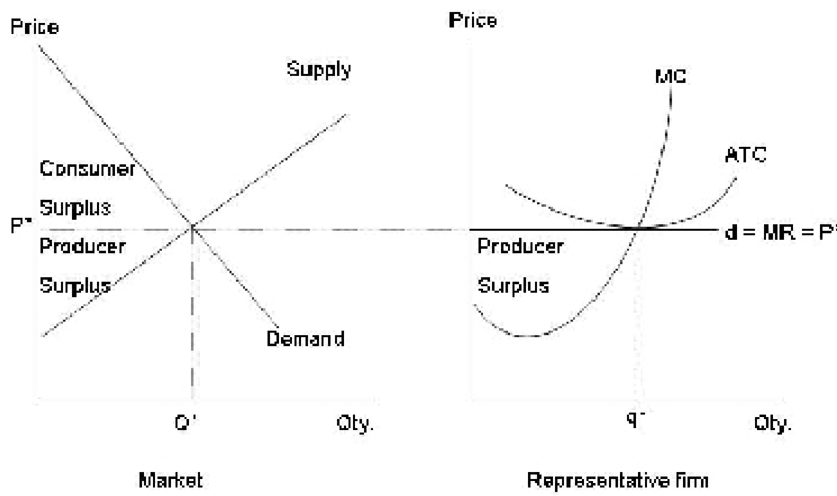
Producer Surplus Producer surplus is the difference between the price a producer pays and the producer's minimum willingness to sell that unit of the good (given by the height of the MC). Thus, if the minimum a producer is willing to sell a unit for is $1 and the producer sells it for $3, then the producer receives $2 in producer surplus (note that this is different from profit, as producer surplus only focuses on marginal costs, which do not consider fixed costs, whereas profit takes fixed costs into account). If we were to look at producer surplus on a graph, it would be the entire area below the price of the good but above the MC (or supply curve).

Gains from trade The possible gains from trade in the market is the area under the demand curve but above the MC (or supply curve). The actual (or realized) gains from trade in the market is the sum of the consumer surplus and producer surplus. If the actual gains from trade equals the possible gains from trade, then we say that the market is efficient.

Deadweight loss We've already covered deadweight loss, but I bring it up as a refresher. The deadweight loss is essentially the difference between the possible gains from trade and the actual gains from trade. It is the loss in efficiency that occurs because there is some feature of the market that keeps the market from trading the efficient quantity.

3.1 Welfare and Perfect Competition

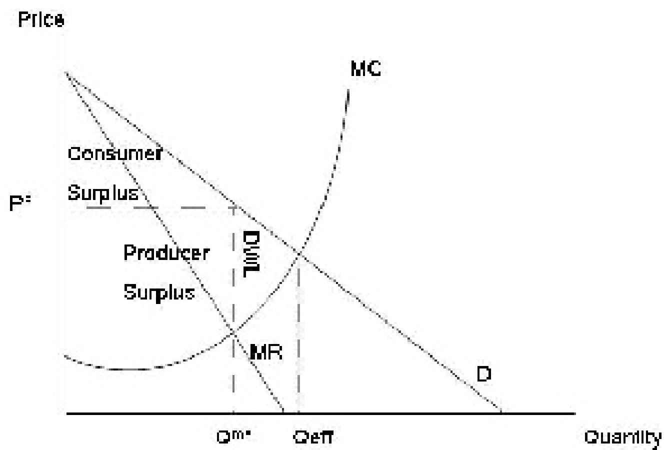
The picture below shows a perfectly competitive market in LR equilibrium (however, the analysis provided also pertains to perfectly competitive markets where firms are earning positive profits or losses).



In the competitive market, there is no deadweight loss. The market is perfectly efficient, as all the gains from trade in both the market and the firm pictures have been captured.

3.2 Welfare and monopoly

The picture below shows the welfare effects of a monopoly.



Notice that in the monopoly market the efficient quantity ($Qeff$) is not the same as the monopolist's profit-maximizing quantity (Q^{m*}). This is because the efficient quantity is found at the point where society's marginal benefit (the demand curve) equals society's marginal cost (the monopolist's MC), while the monopolist looks at its own marginal benefit (which is the MR curve) and finds the quantity that sets its own marginal benefit equal to MC. Since the monopolist's marginal benefit curve is not the same as society's marginal benefit curve, the market is inefficient, and deadweight loss (DWL) results.

The fact that deadweight loss results in a monopoly is the reason that monopolies are considered bad (well, at least that's why economists consider monopolies bad). Of course, since monopolies are so bad, why then do they exist?

3.3 Reasons monopolies exist

There are two major reasons why monopolies exist, which can be broken into a few subcategories. Those reasons are cost advantages and government actions

3.3.1 Cost Advantages

1. Control a key input

   One reason that a monopoly may exist is that a firm may control a key input needed in the production of a product. In the diamond market, DeBeers owned 80% of the world's diamond supply at one point in time. Thus, if someone wanted diamonds, they had to go through DeBeers.

2. Superior technology/production technique

   It can also be the case that one firm has a better production technology or technique than other firms. If this is the case, that firm will be able to charge a lower price than the other firms and, if it can charge a low enough price while still maintaing profits, it should be able to drive the other firms from the market, creating a monopoly or at least a near-monopoly.

3. Natural monopoly

   A natural monopoly exists when the LRATC for a representative firm in an industry is decreasing throughout the entire range of relevant demand. In this case, the larger a firm becomes the the lower the per-unit costs it experiences (there are no diseconomies of scale). Thus, a single firm will have lower production costs than 2 or more firms.

3.3.2 Government Actions

1. Government monopolies

   There are some industries, such as the post office, that are run by the government and protected from competition. These industries are monopolies because the government has deemed them as monopolies.

2. Licensing

   In most cases the government does not license monopolies, but it does require licenses (liquor licenses, medallions for New York City taxicabs) that protect firms from competition.

3. Patents

   Patents are used to protect "inventors" from having their creative work stolen/copied by others. The government grants the inventor a patent that gives him monopoly power over his product for a specified time period.

3.4 Government actions that reduce market power

The government attempts to reduce market power because firms with more market power tend to cause larger deadweight loss in the market. The government can reduce market power through a few methods.

1. Remove artificial restrictions in the market

   Any government action that creates market power could be removed in order to reduce market power.

2. Increase competition through antitrust laws

   The antitrust laws were created to reduce market power. The government prosecutes firms for various forms of anti-competitive behavior in an effort to reduce market power.

3. Price or profit regulation

   If we look at the monopolist's picture, we can see what the price should be that will allow the efficient quantity to be traded in the market. Thus, the government could force the monopolist to price at this level, increasing efficiency. Of course, finding this price in a theoretical model is much easier than it is in the real-world.

4 The monopolist's LR equilibrium

Recall that positive economic profits attract other firms to enter the market when the market is perfectly competitive. However, when the market is a monopoly, the monopolist is protected by some entry barrier. Since the monopolist is protected by an entry barrier other firms cannot enter into the industry – thus they cannot take away the monopolist's economic profit. This means that the monopolist's LR equilibrium, if its entry barriers stay intact, will look exactly like its short-run equilibrium, even if positive economic profits are being made. The primary difference between the monopolist and the perfectly competitive market in the LR is that the monopolist can sustain economic profits in the LR while the perfectly competitive firm cannot.