These notes essentially correspond to chapter 13 of the text.

1 Oligopoly

The key feature of the oligopoly (and to some extent, the monopolistically competitive market) market structure is that one firm's decision depends on the other firms' decisions. In other words, firm behavior is mutually interdependent. Note that in a monopoly there is no other firm on which behavior can depend, and in perfect competition no firm can affect the market price on its own, so firms do not have to worry about how much other firms produce as there will be no effect on the market price. We typically assume that oligopolies are small in number (while monopolistic competitors are larger) and that oligopolies are protected by some entry barrier (while free entry can occur under monopolistic competition). Products may either be identical or homogeneous in an oligopoly. OPEC is an oligopoly that produces oil (or petroleum if you want to be more precise), which is a fairly homogeneous product, while historically the big three auto manufacturers were an oligopoly that produce differentiated products (I say historically because they have less market power due to the recent influx of imports – they still produce differentiated products). We will use a new tool because of this mutual interdependence – game theoretic analysis, which essentially studies the decisions agents make in different environments.

2 Intro to game theory

Although it is called game theory, and most of the early work was an attempt at "solving" actual games (like Chess), the tools used in game theory can be applied to many economic situations (how to bid in an auction, how to bargain, how much to produce in a market setting, etc.). A game consists of the following four items:

1. Players – the agents (firms, people, countries, etc.) who actively make decisions

2. Rules – the procedures that must be followed in the game (knights must move in an L-shaped pattern in Chess, three strikes and you're out in baseball, a firm cannot produce a quantity less than 0 – these are all rules); may also include timing elements (white moves first in Chess then player's alternate moves, one firm may produce first and the other firm may observe this production before it makes a quantity decision,

3. Outcomes – what occurs once all decisions have been made (in a winner/loser game like Chess or baseball, the outcome is a win or a loss or perhaps a tie, while in a market game the outcome is more like a profit level)

4. Payoffs – the value that the player assigns to the outcome (in most of our examples outcomes and payoffs will be identical, as the outcomes will be dollars and players will just translate

2.1 Solution Concept

Our goal will be to "solve" these games. Although there are a variety of solution methods, the one we will use is the Nash Equilibrium concept (yes, named after that guy Nash in the movie). A Nash Equilibrium is a set of strategies such that no one player can change his strategy and obtain a higher payoff given the strategy the other player(s) is (are) currently using.

A strategy is a complete plan of play for the game. Suppose we were trying to solve the game of Chess (if you ever actually solve Chess you will become famous, at least within the mathematics community). There are two players, and the player with the white pieces moves first. One piece of the white pieces player's strategy might be, "move king side knight to square X to start the game". However, this is not a complete strategy – you need to write down what you will do for every possible move that you will make. By contrast, look at the beginning of the black pieces player's strategy. There are 20 possible moves that the white pieces player can use to begin Chess, and the black pieces player must have a plan of action for what he will do for EVERY possible move the white pieces player would make. That's a list of 20 moves that the black pieces player must write out just to make his FIRST move. Thus, a complete strategy of Chess is very, very, very, lengthy (even with the increases that we have seen in computing power no one has been able to program a computer to solve Chess).

2.2 Monopoly as a "game"

It is possible to consider the monopoly market as a 1-player game (some texts will say that a game must have 2 players whereas a "game" with 1 player is not really a game but a decision – we will not concern ourselves with that detail). Look at the features of this game:

1. Player(s): The monopolist

2. Rules: The monopolist must choose a quantity level between 0 and ∞. The price in the market will be determined by $P(Q) = 400 - 5Q$. The monopolist's costs are given by: $TC(Q) = 5Q^2 + 100$, with $MC = 10Q$.

3. Outcomes: The outcome in this game is a set of outcomes that will lead to a profit level.

4. Payoffs: In this case, the payoff is the outcome level, so $\Pi = (400 - 5Q)Q - (5Q^2 + 100)$.

Using the tools we already have, we "know" that the solution to this game can be found by finding the quantity level where $MR = MC$. Since $MR = 400 - 10Q$ and $MC = 10Q$, we have:

$$400 - 10Q = 10Q$$

$$20 = Q$$

We could also set up a table to find the monopolist's optimal strategy (which is the quantity choice that maximizes his profit).

A possible table (with only a few of the strategies listed) is below:

Strategy (qty. choice)	Payoff (profits)
$Q = 1$	\$290
$Q = 10$	\$2900
$Q = 19$	\$3890
$Q = 20$	\$3900
$Q = 21$	\$3890
$Q = 80$	$-\$32,100$

If we wanted to be sure that this was the monopolist's optimal strategy, we would either need to look at all of his possible strategy choices (every quantity from 0 to ∞) and see which gives the highest profit, or solve for the optimal strategy choice mathematically (which is what we did earlier in the course even though we did not call it a strategy).

2.3   Simple duopoly example

Suppose that there are two firms (Firm A and Firm B) engaged in competition. The two firms will choose quantity levels simultaneously. To keep this example simple, assume that the firms' quantity choices are restricted to be either 48 units or 64 units. If both firms choose to produce 64 units, then both firms will receive a payoff of \$4.1. If both firms choose to produce 38 units, then both firms will receive a payoff of \$4.6. If one firm chooses to produce 48 units and the other chooses to produce 64 units, the firm that produces 48 units receives a payoff of \$3.8 while the firm that produces 64 units receives a payoff of \$5.1.

When analyzing 2 firm simultaneous games (where there are a small number of strategy choices), we can use a game matrix (or the normal form or strategic form or matrix form – it has many names) as an aid in finding the NE to the game. The game matrix is similar to the table above for the monopoly, only now we have 2 firms. I will write out the matrix below and then explain the pieces as well as some terminology.

		Firm B	
		$Q_B = 48$	$Q_B = 64$
Firm A	$Q_A = 48$	\$4.6, \$4.6	\$3.8, \$5.1
	$Q_A = 64$	\$5.1, \$3.8	\$4.1, \$4.1

One player is listed on the side of the matrix (Firm A in this example) and is called the row player, as that player's strategies ($Q_A = 48$ and $Q_A = 64$ in this example)are

listed along the rows of the matrix. The other player is listed at the top of the matrix (Firm B in this example) and is called the column player, as that player's strategies ($Q_B = 48$ and $Q_B = 64$ in this example)are listed along the columns of the matrix.

Each cell inside the matrix lists the payoffs to the players if they use the strategies that correspond to that cell. So the $4.6, $4.6 are the payoffs that correspond to the row player (Firm A) choosing to produce 48 and the column player (Firm B) also choosing to produce 48. The cell with $5.1, $3.8 corresponds to the row player choosing 64 and the column player choosing 48. Note that the row player's payoff is ALWAYS, ALWAYS, ALWAYS listed first (to the left) in the cell.

Now that the game is set-up, how do we find the Nash Equilibrium (NE) to the game? We could look at each cell and see if any player could make himself better off by changing his strategy.

If $Q_A = 48$ and $Q_B = 48$, then Firm A could make himself better off by choosing $Q_A = 64$ (Firm B could also have made himself better off by choosing $Q_B = 64$, but all we need is one player to want to change his strategy and we do not have a NE). Thus, $Q_A = 48$ and $Q_B = 48$ is NOT a NE.

If $Q_A = 48$ and $Q_B = 64$, then Firm A can make himself better off by choosing $Q_A = 64$, because he would receive $4.1 rather than $3.8. Thus, $Q_A = 48$ and $Q_B = 64$ is NOT a NE.

If $Q_A = 64$ and $Q_B = 48$, then Firm B could make himself better off by choosing $Q_B = 64$. Thus, $Q_A = 64$ and $Q_B = 48$ is NOT a NE.

If $Q_A = 64$ and $Q_B = 64$ then neither firm can make himself better off by changing his strategy (if either one of them changes then the firm that changes will receive $3.8 rather than $4.1). Since neither firm has any incentive to change, $Q_A = 64$ and $Q_B = 64$ is a NE to this game.

Working through each cell is a fairly intuitive, albeit time-consuming process. You can use this technique if you want, but a word of caution. You must check EVERY cell in the game as there may be multiple NE to the game – thus, even if you started by checking $Q_A = 64$ and $Q_B = 64$ and found that it was a NE you would still need to check the remaining cells to ensure that they were not NE. However, there is another method.

Another method that works to find NE of game matrices is called "circling the payoffs" (it doesn't really have a technical name). Here's the idea: hold one player's strategy constant (so suppose Firm B chooses $Q_B = 48$), then see what the other player's highest payoff is against that strategy and circle that payoff. So if Firm B chose $Q_B = 48$, then Firm A would circle the payoff of $5.1 in the lower left-cell (the payoff of $5.1 that corresponds to $Q_A = 64$ and $Q_B = 48$). If Firm B chose $Q_B = 64$, then Firm A would circle the payoff of $4.1 since $4.1 > $3.8. So halfway through the process we have:

		Firm B	
		$Q_B = 48$	$Q_B = 64$
Firm A	$Q_A = 48$	$4.6, $4.6	$3.8, $5.1
	$Q_A = 64$	$5.1, $3.8	$4.1, $4.1

Now, we simply hold Firm A's strategy constant and figure out what Firm B

would do in each situation. Firm B would circle the $5.1 payoff if Firm A chose $Q_A = 48$ and Firm B would circle the $4.1 payoff if Firm A chose $Q_A = 64$. Thus, the result would be:

|	Firm B		
		$Q_B = 48$	$Q_B = 64$
Firm A	$Q_A = 48$	$4.6, $4.6	$3.8, $5.1
	$Q_A = 64$	$5.1, $3.8	$4.1, $4.1

Whichever cell (or cells) have both payoffs circled are NE to the game. Note that this is the same NE we found by going through each cell. Again, it is possible to have more than one NE to a game. Also, it is possible to circle more than one payoff at a time. Suppose Firm A chose $Q_A = 48$ and that Firm B received $5.1 if it chose $Q_B = 48$ or $Q_B = 64$. In this case, since the highest payoff corresponds to two different strategies for Firm B you would need to circle both of the payoffs. The "solved" game (with the $5.1 replacing the $4.6 for Firm B only) would look like below:

	Firm B		
		$Q_B = 48$	$Q_B = 64$
Firm A	$Q_A = 48$	$4.6, $5.1	$3.8, $5.1
	$Q_A = 64$	$5.1, $3.8	$4.1, $4.1

3   Market Games

The primary difference between oligopoly markets and either monopolies or perfectly competitive markets is that oligopoly markets are characterized by mutual interdependence among firms. This means that what one firm does affects another firm's decision. In a monopoly there are no other firms to affect the monopolist's quantity (or price) choice, and in the perfectly competitive market no firm has enough market power to affect the market price so firms do not have to worry about each other's production level. Thus, while we had fairly robust results for the monopoly and the perfectly competitive markets, we will see that the results for the oligopoly market may vary greatly depending on the choice of strategic variable. Although there is a vast array of variables that firms may choose as their strategic variable (level of advertising, product quality, when to release a product, product type, etc.), the two standard choice variables are quantity and price. We will examine these two market games using a simultaneous game between 2 firms that produce identical products, face a linear inverse demand function, and have constant marginal costs.

Before we begin the discussion it may be useful to consider the extremes of oligopoly behavior. At one extreme, the oligopolists could collude and act like a monopolist, choosing to produce a quantity that maximizes INDUSTRY profits. At the other extreme, the oligopolists could act like perfect competitors, driving price down to MC. The picture below shows the extreme forms of behavior.
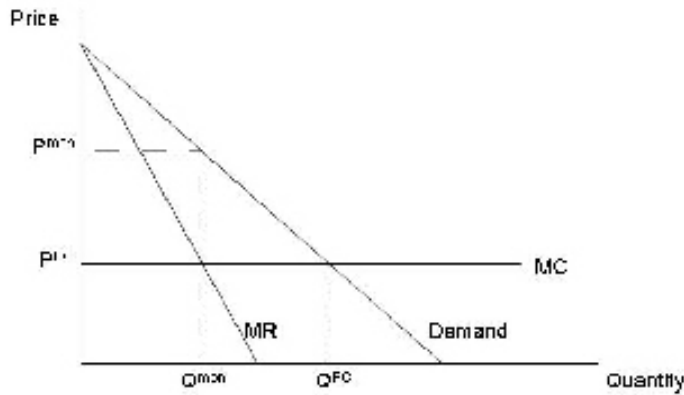
The most likely outcome is that price and quantity will lie somewhere between the two extreme forms of behavior.

3.1 Quantity games

Quantity games are also called Cournot games, after the author who is credited with first formalizing them in 1838. Cournot believed that firms competed by choosing quantities, with the inverse demand function determining the price in the market. Assume that there are 2 identical firms, Firm 1 and Firm 2, each of whom will simultaneously choose a quantity level (q_1 and q_2 respectively). The inverse demand function for this product is $P(Q) = a - bQ$, where Q is the total market quantity, which means $Q = q_1 + q_2$ for this example. Each firm's total cost is as follows: $TC_1 = c * q_1$ and $TC_2 = c * q_2$. Thus, each firm's marginal cost is: $MC_1 = MC_2 = c$. We will first show that the monopoly (or cartel) and perfectly competitive solutions are NOT Nash Equilibria to this game, and then we will find the NE and compare it to the monopoly and perfectly competitive solutions.

3.1.1 Monopoly is NOT a NE to the quantity game

Suppose that the two firms collude to form a cartel. The cartel's goal is to choose the quantity that will maximize industry profits. Each firm will produce $\frac{1}{2}$ of the monopoly quantity and receive the profits from producing that quantity. The monopolist will set $MR = MC$, where $MR = a - 2bQ$ and $MC = c$, so:

$$a - 2bQ = c$$

$$Q = \frac{a-c}{2b}$$

Thus, the total market quantity is $\frac{a-c}{2b}$, so each firm produces $\frac{a-c}{4b}$ (which is $\frac{1}{2} * \frac{a-c}{2b}$). Rather than work in the abstract, we can use some parameters to show that both firms would like to deviate from producing $\frac{a-c}{4b}$. Let $a = 120$, $b = 1$, and $c = 12$. There is nothing particular about these parameters, and these results hold for any parameter specification provided a, b, and c are all positive, and $a > c$. We need $a > c$ because otherwise the marginal cost will be above the highest point on the demand curve, which means a quantity of zero would be sold in the market since marginal cost would be greater than price for any units sold.

Using the parameters we find that: $Q = 54$ and $q_1 = q_2 = 27$. The price in the market is: $P(54) = 120 - (1) * 54 = 66$. The profit to each firm is: $\Pi_1 = \Pi_2 = 66 * 27 - 12 * 27 = 1458$.

Now, suppose that Firm 1 decides to cheat on the agreement and produces more than 27 units (so 28 units). If Firm 1 produces 28 units, then $Q = 55$ and $P(55) = 65$. Firm 1's profits are now: $\Pi_1 = 65 * 28 - 12 * 28 = 1484$, which is greater than the 1458 it was earning when it produced 27 units (to be complete, Firm 2's profits are: $\Pi_2 = 65 * 27 - 12 * 27 = 1431$). Since Firm 1 can earn a higher profit if it changes its strategy (chooses a quantity level greater than 27), the monopoly (or cartel) outcome is NOT a NE. (Note: It may seem as if we've "solved" the game using the cartel quantities as strategies – after all, we do get "answers" for market quantity, individual firm quantity, price and profits. However, this is like saying that you have solved a maze because you have written down a complete strategy, even though that strategy runs you into a wall instead of to the end of the maze.)

3.1.2 Perfect competition is NOT a NE to the game

Suppose that firms act as perfect competitors. In this case, the firms will produce the total market quantity that corresponds to the point where MC crosses the demand curve. Since the two firms are identical, we will assume that each firm produces $\frac{1}{2}$ of this total market quantity. To find the total market quantity, set $MC = demand$ or $c = a - bQ$. Then $Q = \frac{a-c}{b}$, and $q_1 = q_2 = \frac{a-c}{2b}$. Using our parameters, we find that: $Q = 108$, and $q_1 = q_2 = 54$. Now, $P(108) = 120 - (1) * 108 = 12$. The profits to each firm are: $\Pi_1 = \Pi_2 = 12 * 54 - 12 * 54 = 0$. Notice that $P = MC$ and $\Pi_1 = \Pi_2 = 0$, both of which correspond to the theoretical predictions of a perfectly competitive market.

Now, suppose that Firm 1 decides to relax his stance on being competitive, and it produces 53 units rather than 54 units. If Firm 1 produces 53 units, then $Q = 107$ and $P(107) = 13$. Firm 1's profits are now: $\Pi_1 = 13 * 53 - 12 * 53 = 53$, which is greater than the 0 profit it was earning by acting competitively (to

be complete, Firm 2's profits are: $\Pi_2 = 13 * 54 - 12 * 54 = 54$). Since Firm 1 can earn a higher profit if it changes its strategy (chooses a quantity level less than 54), the perfectly competitive outcome is NOT a NE. The intuitive difference between this game and the perfectly competitive market is that each firm in this game has some impact on the price. If this were a true perfectly competitive market, then Firm 1 could NOT have caused the price to increase by reducing its quantity – however, in this game, Firm 1 can cause the price to increase by reducing its quantity.

3.1.3   The Cournot-Nash solution

We have seen that the 2 firms behaving like either extreme (cartel or perfect competition) is NOT a NE. We could set up a game matrix to find the NE, but that would be an extremely large matrix. Instead, we will use the concept of a best-response function to find the NE. A best response function is a function that tells a firm the quantity level it should produce (or, more generally the strategy it should use) given the quantity level that the other firm produces. Thus, a firm's best response function will be a function of the other firm's quantity as well as the parameters of the problem. We will first derive the best response functions using economic intuition and then I will derive them using calculus – either way gives the same answer.

Intuitively, we know that firms maximize their profit by setting $MR = MC$. Now, take Firm 1. We know that $MC = c$, so half of the equation is done for us. Finding MR is a little bit more difficult. We know that $P(Q) = a - bQ$, and that $Q = q_2 + q_1$, so $P(Q) = a - bq_2 - bq_1$. What we are trying to find is a function that tells us how much Firm 1 should produce for a GIVEN (or constant) level of q_2. Since Firm 1 is holding Firm 2's quantity choice (q_2) constant, the entire term $a - bq_2$ can be rewritten as a constant, which I will call A (so $A = a - bq_2$). Now, for Firm 1, $P(Q) = A - bq_1$. This looks very familiar, and we derived a nice result earlier in the course for finding the MR. If $P(Q) = a - bQ$, then $MR(Q) = a - 2bQ$. We can use that same result here, so that $MR = A - 2bq_1$. Now, plugging $a - bq_2$ back in for A gives us: $MR = a - bq_2 - 2bq_1$. Setting this equal to MC we get:

$$a - bq_2 - 2bq_1 = c$$

Solving for q_1 we get:

$$q_1 = \frac{a - c - bq_2}{2b}$$

Thus, for a given quantity choice of q_2 by Firm 2 and a given set of parameters a, b, and c we know the quantity level that Firm 1 should produce to maximize its profits.[1] We can derive Firm 2's best response function in a

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[1] Technically, Firm 1's best response function is the function above for any quantity choice of q_2 between 0 and the entire perfectly competitive quantity, which was 108 using the parameters above. If firm 2 produces more than 108, Firm 1's best response would be to produce 0, since it would then earn 0 profit rather than a negative profit. But this is just a technical note.

similar manner, so that:

$$q_2 = \frac{a - c - bq_1}{2b}$$

Before continuing on to find the actual quantity levels that each firm would produce I would like to point out one thing. Notice that if Firm 2 decides to produce $q_2 = 0$, then Firm 1's best response is to produce the entire monopoly quantity, which would be $q_1 = \frac{a-c}{2b}$. This is consistent with the results that we have already seen.

As for finding the NE quantities, recall that a NE is a set of strategies that are best responses to one another. To find the NE, we want to find the q_1 and q_2 that are best responses to one another. We can do this by plugging in the best response function for q_2 into the best response function for q_1 (essentially we have 2 equations and 2 unknowns, q_1 and q_2, and we want to find the 2 unknowns). Substituting in we get:

$$q_1 = \frac{a - c - b\left(\frac{a-c-bq_1}{2b}\right)}{2b}$$

Simplifying:

$$2bq_1 = a - c - b\left(\frac{a - c - bq_1}{2b}\right)$$

Simplifying:

$$2bq_1 = a - c - \left(\frac{a - c - bq_1}{2}\right)$$

Simplifying:

$$4bq_1 = 2a - 2c - (a - c - bq_1)$$

Distributing the negative:

$$4bq_1 = 2a - 2c - a + c + bq_1$$

Solving for q_1:

$$q_1 = \frac{a - c}{3b}$$

Thus, Firm 1 should produce $q_1 = \frac{a-c}{3b}$. We can solve for q_2 using a similar method to find that $q_2 = \frac{a-c}{3b}$. Plugging in our numbers shows us that $q_1 = q_2 = \frac{120-12}{3} = 36$, so $Q = 72$ and $P(Q) = 120 - (1) * 72 = 48$. Thus, since both firms are identical and producing the same amount, $\Pi_1 = \Pi_2 = 48 * 36 - 12 * 36 = 1296$. If Firm 1 decides to deviate by producing a larger quantity (say 37), then $Q = 73$ and $P(73) = 47$. Firm 1's profits are: $\Pi_1 = 47 * 37 - 12 * 37 = 1295$, which is less than the 1296 Firm 1 would earn if it produced 36 units. So producing a quantity greater than 36 is not more

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profitable than producing a quantity of 36.[2] Suppose Firm 1 decided to deviate by producing a lower quantity than 36 (say 35). Then $Q = 71$ and $P(71) = 49$. Firm 1's profits are: $\Pi_1 = 49 * 35 - 12 * 35 = 1295$, which is less than the 1296 Firm 1 would earn if it produced 36 units. So producing a quantity less than 36 is not more profitable than producing a quantity of 36. Thus, if Firm 2 produces 36 units then Firm 1's best response is to produce 36 units. If Firm 1 produces 36 units, then Firm 2's best response is to produce 36 units. Since each firm is using a strategy that is a best response to the other firm's strategy, we have a NE.

Calculus method We could also use calculus to find the firm's best response function, and I will use calculus right now for those of you who have had the course. To find Firm 1's best response function, we want to find a function that maximizes Firm 1's profits for any given choice of q_2 by Firm 2. So:

$$\Pi_1 = (a - bq_1 - bq_2) q_1 - cq_1$$

Now, take the derivative of profit with respect to q_1 (technically it is the partial derivative of profit with respect to q_1). We find:

$$\frac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c$$

Set this equal to zero to find the maximum (we know it's a maximum because the 2nd derivative is $(-2b)$, which is always negative for positive b). We get:

$$a - 2bq_1 - bq_2 - c = 0$$

Solving for q_1:

$$q_1 = \frac{a - c - bq_2}{2b}$$

We can use a similar process to find that $q_2 = \frac{a-c-bq_1}{2b}$. Note that these best response functions are the same as the ones derived in the previous section. Now just follow the steps in the previous section to find that the NE quantities are $q_1 = q_2 = \frac{a-c}{3b}$.

Graphical representations of the Cournot-Nash solution Another way to find the Cournot-Nash solution is to plot the best response functions. We can rewrite $q_1 = \frac{a-c-bq_2}{2b}$ and $q_2 = \frac{a-c-bq_1}{2b}$ as $q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2$ and $q_2 = \frac{a-c}{2b} - \frac{1}{2}q_1$. If we plot these on a graph we will get:

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[2]The intuition is that selling one more unit generates additional revenue of $47 (since we sell one more unit), but the additional cost is the direct cost of selling one more unit (the $12 MC) plus the decrease in revenue that occurs from selling the first 36 units at one dollar less than they were being sold before. Thus, the total additional cost is $12 + 36 = 48$, so the firm loses $48 while only gaining $47, which means it is less profitable to increase production.
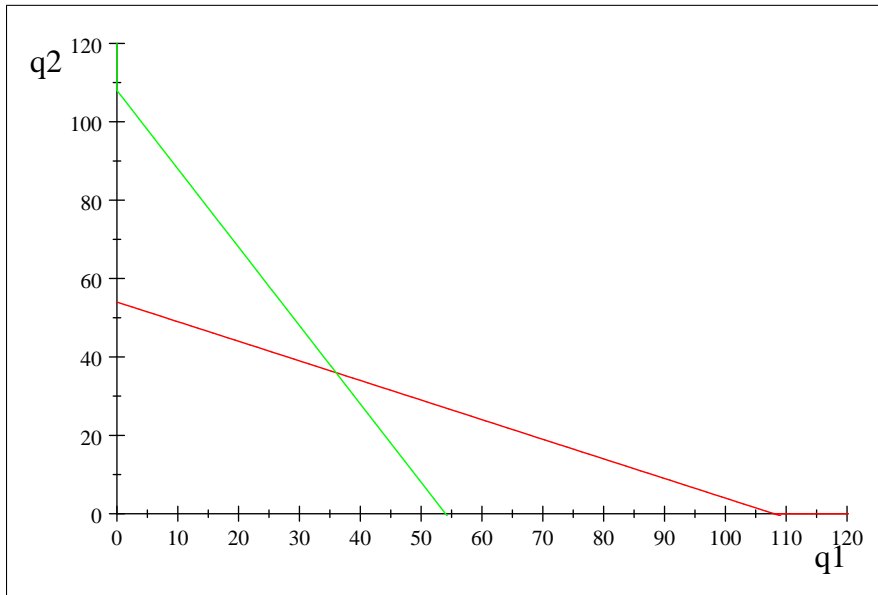
The red line (flatter line) is Firm 2's best response function and the green line (steeper line) is Firm 1's best response function. The point of intersection is the Nash equilibrium point – it is where both players are choosing their best responses to each other. Note that the lines intersect when $q_1 = 36$ and $q_2 = 36$.

Finally, we can look at Firm 1's profit when Firm 2 chooses 36 and Firm 2's profit when Firm 1 chooses 36.
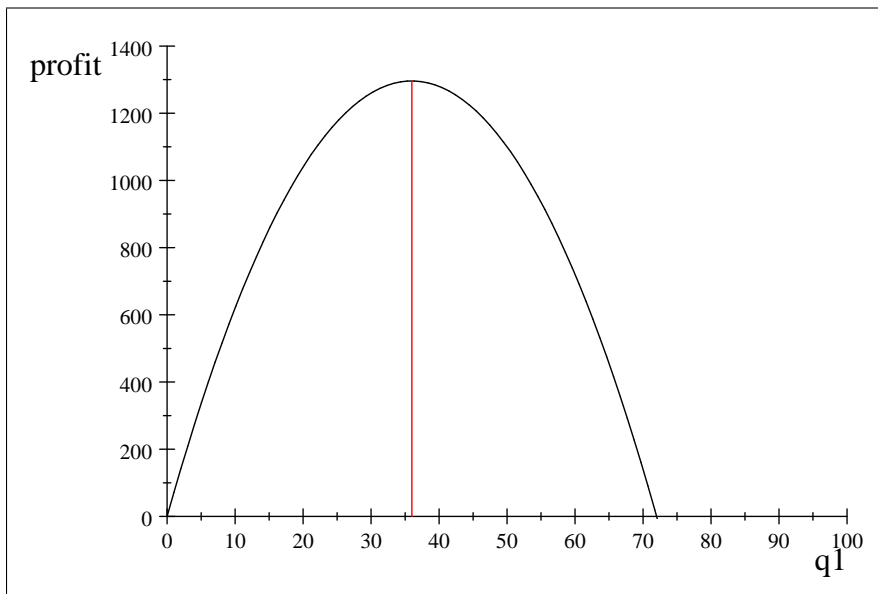
The parabola is Firm 1's profit when $q_2 = 36$. The vertical red line corresponds to when $q_1 = 36$, which is the maximum of the profit function. Thus, when $q_2 = 36$, Firm 1 is maximizing its profit when $q_1 = 36$. Since the firms are identical, the same picture will result for Firm 2 (holding $q_1 = 36$).

3.1.4 Comparing the cartel, perfect competition, and Cournot outcomes

We began the discussion of oligopoly behavior by looking at the two extreme forms of behavior (cartel and perfect competition) and asserting that the real-world outcome was likely between those two. The table below compares the cartel, perfect competition, and Cournot outcomes using the parameters $a = 120$, $b = 1$, and $c = 12$.

	Q	q_1	q_2	$Price$	Π_1	Π_2
Cartel	54	27	27	66	1458	1458
Cournot	72	36	36	48	1296	1296
Perfect Competition	108	54	54	12	0	0

We can see that the price and quantity that result from Cournot competition falls between the extreme forms of behavior of the firms, which corresponds nicely to our assertion.

3.1.5 Cournot behavior and k firms

One other aspect of Cournot behavior that conforms with intuition is that as the number of firms increases the profit per firm decreases, and when there is an infinite number of firms profits become zero. Thus, if there is a very large number of firms then Cournot behavior approaches perfectly competitive behavior. We can show this by analyzing the profit a particular firm earns.

In the two-firm case the Cournot quantities are $\frac{a-c}{3b}$ for both firms, which leads to a total market quantity of $\frac{2a-2c}{3b}$. The price in the market is then:

$$P(Q) = a - b\left(\frac{2a - 2c}{3b}\right)$$

Simplifying this expression gives:

$$P(Q) = \frac{a + 2c}{3}$$

Firm profits are then:

$$\Pi_1 = \Pi_2 = \left(\frac{a + 2c}{3}\right) * \left(\frac{a - c}{3b}\right) - c * \left(\frac{a - c}{3b}\right)$$

Factoring out the $\left(\frac{a-c}{3b}\right)$ term gives:

$$\Pi_1 = \Pi_2 = \left(\left(\frac{a + 2c}{3}\right) - c\right) * \left(\frac{a - c}{3b}\right)$$

Simplifying the first bracketed term, $\left(\left(\frac{a+2c}{3}\right) - c\right)$ gives:

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$$\Pi_1 = \Pi_2 = \left(\frac{a-c}{3}\right) * \left(\frac{a-c}{3b}\right)$$

Or:

$$\Pi_1 = \Pi_2 = \frac{(a-c)^2}{9b}$$

Note that this is the profit for each firm in a duopoly. The general profit function for an oligopoly with k firms is:

$$\Pi_1 = \Pi_2 = ... = \Pi_k = \frac{(a-c)^2}{(k+1)^2 b}$$

Notice that if we plug in $k = 2$ we get the previous result, with $9b$ in the denominator. As k becomes very large, the profits to the firms fall, since we are divided the same number, $(a-c)^2$ in this case, by an even larger number as k becomes bigger. Again, this result conforms with our previously held belief that if we have a large number of firms in the industry and the firms are in equilibrium then we should see zero economic profits.

3.2   Pricing games

About 50-60 years after Cournot, another economist (Bertrand) found fault with Cournot's work. Bertrand believed that firms competed by choosing prices, and then letting the market determine the quantity sold. Recall that if a monopolist wishes to maximize profit it can choose either price or quantity while allowing the market to determine the variable that the monopolist did not choose. The resulting price and quantity in the market is unaffected by the monopolist's decision of which variable to use as its strategic variable. We will see that this is not the case for a duopoly market.

The general structure of the game is as follows. There are identical 2 firms competing in the market – the firms produce identical products, have the same cost structure ($TC = c*q$ and $MC = c$), and face the same downward sloping inverse demand function, $P(Q) = a - bQ$. However, in this game it is more useful to structure the inverse demand function as an actual demand function (because the firms are choosing prices and allowing the market to determine the quantity sold), so we can rewrite the inverse demand function as a demand function, $Q(P) = \frac{a}{b} - \frac{1}{b}P$. Consumers have no brand or firm loyalty, and it is assumed that all consumers know the prices of both firms in the market. Consumers will purchase from the lowest priced producer according to the demand function. This last assumption means that each firm's quantity is determined by the table below (p_1 is Firm 1's price choice and p_2 is Firm 2's price choice):

	q_1	q_2
if $p_1 > p_2$	0	$\frac{a-p_2}{b}$
if $p_1 = p_2$	$\frac{1}{2} * \frac{a-p_1}{b}$	$\frac{1}{2} * \frac{a-p_2}{b}$
if $p_1 < p_2$	$\frac{a-p_1}{b}$	0

Thus, the firm with the lowest price will sell the entire market quantity at that price. If the firms have equal prices then they will each sell $\frac{1}{2}$ the total market quantity at that price. Now we will see what happens if the firms choose the monopoly, the Cournot, or the perfectly competitive price. These prices correspond to the ones derived in the section on the quantity games, using the parameter $a = 120$, $b = 1$, and $c = 12$.

3.2.1 Choosing the monopoly price

Suppose that the 2 firms both choose the monopoly price, which was $66. Each then sells $\frac{1}{2}$ of the monopoly quantity, which means that $q_1 = q_2 = 27$. Firm profits are then $\Pi_1 = \Pi_2 = 1458$. Suppose that Firm 1 decides to cheat and chooses a lower price of $65. Since $p_1 < p_2$, Firm 1 then produces $\frac{120-65}{1} = 55$ units. Firm 1's profits are: $\Pi_1 = 65*55 - 12*55 = 2915$, which is greater than 1458. So Firm 1 has the incentive to lower its price (as does Firm 2), which means choosing the monopoly price is NOT a NE to the Bertrand game.

3.2.2 Choosing the Cournot price

Suppose that the 2 firms both choose the Cournot price, which was $48. Each firm then sells $\frac{1}{2}$ of the total Cournot quantity, which means that $q_1 = q_2 = 36$. Firm profits are then $\Pi_1 = \Pi_2 = 1296$. Suppose that Firm 1 decides to cheat (just as a reminder, the firms are NOT jointly deciding to produce the Cournot quantity when playing the Cournot game – each is acting in its own self-interest) and chooses a lower price of $47. Since $p_1 < p_2$, Firm 1 then produces $\frac{120-47}{1} = 73$ units. Firm 1's profits are now: $\Pi_1 = 47*73 - 12*73 = 2555$, which is greater than 1296. So Firm 1 has the incentive to lower its price (as does Firm 2), which means choosing the Cournot price is NOT a NE to the Bertrand game.

3.2.3 Choosing the perfectly competitive price

Suppose that the 2 firms both choose the perfectly competitive price, which was $12. Each firm then sells $\frac{1}{2}$ of the total perfect competition quantity, which means that $q_1 = q_2 = 54$. Firm profits are then $\Pi_1 = \Pi_2 = 0$. Suppose that Firm 1 wishes to change its strategy by lowering its price to $11. It captures the entire market, and sells $\frac{120-11}{1} = 109$. Firm 1's profits are now: $\Pi_1 = 11*109 - 12*109 = (-109)$. Clearly, lowering the price makes Firm 1 worse off. If Firm 1 attempts to raise the price above $12, then $p_2 < p_1$, and Firm 2 captures the entire market. This means that Firm 1's profit (if it raises the price to $13) is still 0, so it did not make itself better off. Thus, the perfectly competitive outcome is the NE to this Bertrand game.

3.2.4   Comparing Cournot and Bertrand

Under Cournot competition each firm made a positive economic profit, and the perfectly competitive outcome is only achieved when the number of firms becomes large. Under Bertrand competition the perfectly competitive outcome is achieved with only two firms. Thus, we tend to assume that the Cournot outcome is more applicable in the framework we have been discussing – however, there are other applications of the Bertrand outcome.