These notes essentially correspond to chapter 14 of the text.

1 Dynamic (or sequential) games

We had been studying simultaneous games, where each firm makes its quantity choice or price choice without observing the other firm's choice. Now, we want to extend the analysis to include sequential games, where one firm moves first, the second firm observes this decision, and then the second firm makes its decision. To analyze sequential games, a structure, called a game tree, that is slightly different than the game matrix should be used. The game tree provides a picture of who decides when, what decisions each player makes, what decisions each player has seen made prior to his decision, and which players see his decision when it is made. We can start by translating the simple quantity choice game from chapter 13 (when the firms could each only choose to produce a quantity of 64 or 48) into a sequential games framework.

Suppose that there are two firms (Firm A and Firm B) engaged in competition. Firm A will choose its quantity level first, and then Firm B will choose its quantity level after observing Firm A's choice. To keep this example simple, assume that the firms' quantity choices are restricted to be either 48 units or 64 units. If both firms choose to produce 64 units, then both firms will receive a payoff of \$4.1. If both firms choose to produce 48 units, then both firms will receive a payoff of \$4.6. If one firm chooses to produce 48 units and the other chooses to produce 64 units, the firm that produces 48 units receives a payoff of \$3.8 while the firm that produces 64 units receives a payoff of \$5.1. This game is sequential since Firm A chooses first and Firm B observes Firm A's decision.[1]

While we could use the matrix (or box or normal) form of the game for the sequential game, there is another method for sequential games that makes the sequential nature of the decisions explicit. The method that should be used is the game tree. A game tree consists of:

1. Nodes – places where the branches of the game tree extend from

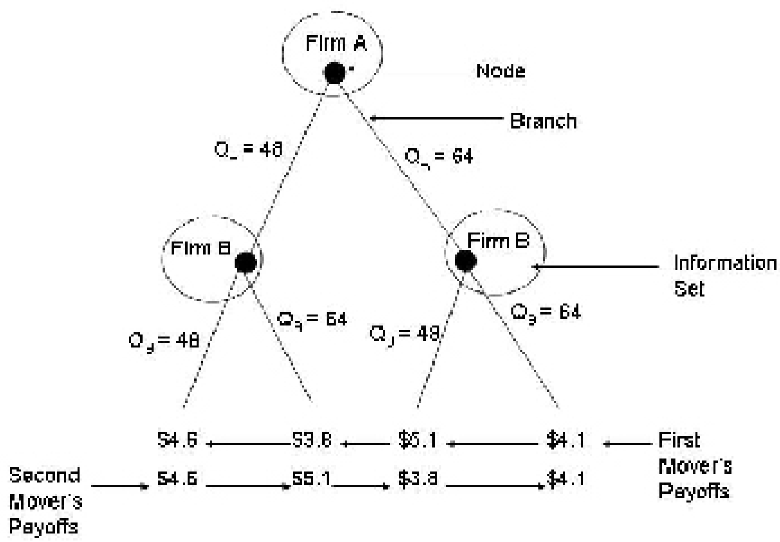
2. Branches – correspond to the strategies a player can use at each node

3. Information sets – depict how much information the player has when he moves (if the second player knows that he follows the first player but cannot observe the first player's decision then his information set is really no different than in the simultaneous move game; however, if the second player can observe the first player's decision, then his information set has changed)

A game tree corresponding to the quantity choice game previously described is depicted below. The individual pieces of a game tree are also labelled. The

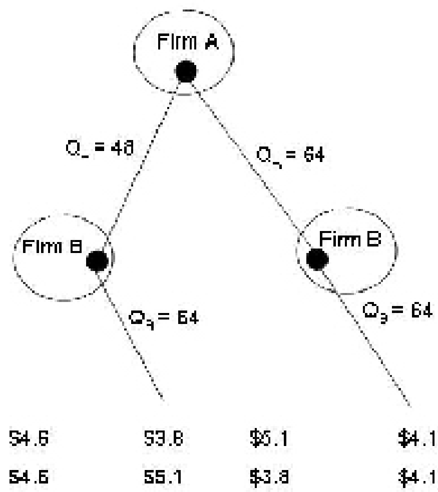
[1] In the real-world Firm A may actually choose a quantity before Firm B, but if Firm B gains no additional information from Firm A's decision (such as a change in the market price), then the game is essentially one where Firm A and Firm B choose simultaneously.

label for information set is pointing to the open circle that encircles the term "Firm B". Thus, Firm B can see how much Firm A has decided to produce. If Firm B could not determine if Firm A decided to produce 48 or 64 units, then Firm B would have one information set, and there would be one open circle encircling both of Firm B's decision nodes.



To solve sequential games we start from the end of the game and work our way back towards the beginning. This is called backward induction. To find the Nash Equilibrium (NE), we first determine what Firm B would do given a quantity choice by Firm A. In this example, Firm B would choose $Q_B = 64$ as its strategy if Firm A chose $Q_A = 48$ because \$5.1 > \$4.6. Also, Firm B would choose $Q_B = 64$ if Firm A chose $Q_A = 64$ because \$4.1 > \$3.8. Thus, Firm B's strategy is: {Choose $Q_B = 64$ if Firm A chooses $Q_A = 48$; choose $Q_B = 64$ if Firm A chooses $Q_A = 48$}. We now know what Firm B will do for any given choice by Firm A, which means that we have an entire strategy for Firm B.

Firm A, knowing that Firm B will choose $Q_B = 64$ regardless of its quantity choice, can now "lop off the branches" that correspond to $Q_B = 48$. The reason that Firm A can lop off these branches is that it knows that it will never see the payoffs associated with following those branches because Firm B will never follow them. Thus, to Firm A, the game tree looks like:



I have left the payoffs there but removed the branches. Firm A has one decision to make, produce a quantity of 48 or a quantity of 64. If it produces a quantity of 48, Firm B will produce 64, and Firm A will receive a payoff of \$3.8. If it produces a quantity of 64, Firm B will produce 64, and Firm A will receive a payoff of \$4.1. Since \$4.1 > \$3.8, Firm A will choose $Q_A = 64$. Thus, the complete NE for this game is:

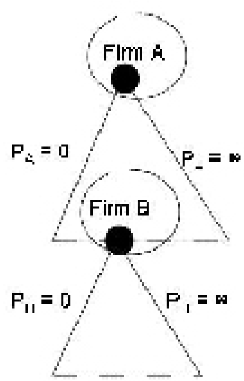
Firm A: Choose $Q_A = 64$

Firm B: Choose $Q_B = 64$ if Firm A chooses $Q_A = 48$; choose $Q_B = 64$ if Firm A chooses $Q_A = 48$

Now, when the game is played only one payoff is received. To find this payoff just follow the path outlined by the NE strategy. Firm A chooses $Q_A = 64$, and if Firm A chooses $Q_A = 64$ then Firm B chooses $Q_B = 64$, which leads to a payoff of \$4.1 for Firm A and \$4.1 for Firm B. Notice that we didn't use the fact that Firm B chooses $Q_B = 64$ if Firm A chooses $Q_A = 48$ because Firm A did not choose $Q_A = 48$. We still need to include that piece as part of our NE strategy even though we don't use it when we find the path that the game actually follows.

2 Sequential Bertrand Game

Recall that in a Bertrand game the competing firms choose the price that they want to sell at in the market. The firm with the lowest price sells the quantity that corresponds to the entire market quantity at that price, while the firm with the higher price sells nothing. If the two firms choose the same price, then each firm sells $\frac{1}{2}$ the market quantity at that price. Assume that the firm's are identical, and that each firm has constant MC equal to c. To make this a sequential Bertrand game, assume that Firm A chooses its price first, and then Firm B observes Firm A's choice and sets its own price. The game tree is depicted below, with a slight modification. Since firms can choose any price greater than 0 they have an infinite amount of strategies ($P_A = 0, P_A = 1, P_A = 1.5, ...$). Since it is impossible to write down an infinite amount of branches that correspond to the infinite amount of strategies we simplify the game tree by drawing two branches corresponding to the lowest possible price ($P_A = 0$) and the highest possible price ($P_A = \infty$) and then connect those two branches with a dotted line to represent the fact that there are an infinite amount of possibilities there.[2] Also note that the payoffs have been removed as listing an infinite amount of payoffs to correspond to the infinite amount of strategies is unrealistic.



Again, to find the solution of this game use backward induction. We want to find out what Firm B would do in response to any price choice that Firm A could make. Suppose that Firm A sets a really high price, above the MC of c. Firm B's best response would be to charge a slightly lower price and capture the entire market. Suppose that Firm A sets a really low price, less than the MC of c. Firm B's best response in this case is NOT to undercut Firm A. If it undercuts Firm A then it captures the entire market, but it captures the entire market at a price below cost which means it is making a loss, which it could

[2] Technically no firm would choose a price above a (the intercept of the inverse demand function) as any price above this level implies that the firm sells 0 units and thus earns 0 profits.

avoid by not producing at all, which means that if Firm A chooses a price less than c that Firm B should choose a price greater than Firm A. We can assume that if Firm A chooses a price less than c that Firm B will choose to set its price equal to c to ensure that it does not make any losses. Suppose that Firm A chooses a price equal to the MC of c. If Firm B chooses a price below c then it captures the entire market, but at a price less than cost, which means that it is making a loss. Clearly, Firm B could do better if it decided to stay out of the market. If Firm B charges a price above c then it will not earn any profits as it allows Firm A to capture the entire market. If Firm B charges a price exactly equal to c, then it will still earn zero economic profit but at least it will then produce half of the market quantity. Formalizing this thought process into a strategy we can write down:

$$P_B = \begin{cases} P_A - \varepsilon & \text{if } P_A > c \\ c & \text{if } P_A = c \\ c & \text{if } P_A < c \end{cases}$$

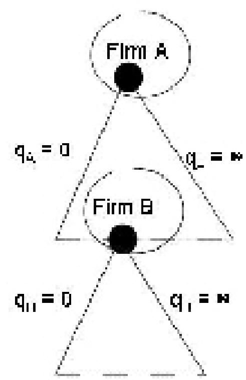
The term ε means the smallest possible amount by which Firm B can undercut Firm A's price (perhaps a penny). Firm A now knows that Firm B will use this strategy.[3] Firm A then has to decide what it will do. If it prices below MC it will capture the entire market but will make a loss. If it prices above MC then Firm B will undercut its price and Firm A will sell nothing. If Firm A chooses to price at MC then it splits the market quantity with Firm B. Thus, Firm A chooses to set $P_A = c$, which means that Firm B will set $P_B = c$, which means that in the sequential Bertrand game the result is the same as in the simultaneous Bertrand game.[4]

3 Sequential Quantity Game

The sequential quantity game is called a Stackelberg game, after its "creator". In this game one firm chooses its quantity first and then the other firm observes this quantity decision and chooses its quantity. We will assume the linear inverse demand function, $P(Q) = a - bQ$, where $Q = q_A + q_B$ and where firms costs are such that $TC = c * q_A$ and $MC = c$. The game tree for this example is:

[3] It's not that Firm B tells Firm A the strategy it will use, it's that Firm A knows the game that will be played and can also see what Firm B's best responses will be given Firm A's choice of price. Also, Firm B's strategy could have one more tier to it. If Firm A chose any price above the monopoly price, Firm B's best response would be to choose the monopoly price, not to undercut Firm A by a tiny amount. Then, for any price between the monopoly price and MC, Firm B's best strategy would be to undercut Firm A by the smallest possible amount. This, however, does not effect the result of the game.

[4] Technically, if the price space is discrete then there is a NE where both firms choose a price at the lowest possible increment above MC. If $c = 12$, and firms must price in increments of pennies, then the NE result is that both firms charge \$12.01 and make very, very small economic profits. This is true of the simultaneous game as well.



Notice that this is the same picture as the sequential Bertrand game, only now the firms are making quantity choices. Again, begin with finding Firm B's strategy. When we worked the simultaneous Cournot game we found the best response functions for each firm. Firm B's best response function, for a given choice of q_A, was:

$$q_B = \frac{a - c - bq_A}{2b}$$

Since this problem has the same basic structure, Firm B's best response function is the same as it was in the Cournot game. Thus, for any choice of q_A we know the exact quantity amount that Firm B would choose. There is one slight caveat to this. If Firm A were to choose an amount of $q_A \geq \frac{a-c}{b}$, then Firm B would choose to produce 0. The reason why is that if Firm A chooses $q_A = \frac{a-c}{b}$, then it is choosing to produce the competitive quantity, where the price in the market equals marginal cost. If Firm A for some reason decides to produce a quantity $q_A > \frac{a-c}{b}$, then Firm A is producing a quantity such that the price in the market is LESS than MC. In this case, Firm B would opt out of the market and produce 0, as producing 0 ensures Firm B of receiving 0 profits, while producing any positive quantity will only force the price lower and ensure that Firm B earns a loss. To summarize, Firm B's strategy is:

$$q_B = \begin{cases} \frac{a-c-bq_A}{2b} & \text{if } 0 \leq q_A \leq \frac{a-c}{b} \\ 0 & \text{if } q_A > \frac{a-c}{b} \end{cases}$$

Firm A then takes Firm B's strategy as given. Firm A is like any other profit maximizing firm, and will set $MR = MC$. We know what Firm A's MC

is as it is given. We need to find Firm A's MR. Let's look at the inverse demand function, $P(Q) = a - bQ$. We know that:

$$P(Q) = a - bq_B - bq_A$$

However, as long as Firm A does not produce more than the perfectly competitive quantity then Firm B will produce $q_B = \frac{a-c-2bq_A}{2b}$. We can plug this into the inverse demand function to find:

$$P(q_A) = a - b\left(\frac{a-c-bq_A}{2b}\right) - bq_A$$

Simplifying:

$$P(q_A) = a - \left(\frac{a-c-bq_A}{2}\right) - bq_A$$

Simplifying:

$$P(q_A) = a - \frac{a}{2} + \frac{c}{2} + \frac{bq_A}{2} - bq_A$$

Combining terms:

$$P(q_A) = \frac{a}{2} + \frac{c}{2} - \frac{bq_A}{2}$$

Now, we know that this is almost in the form of $P(Q) = a - bQ$, which we know has $MR = a - 2bQ$. If we let $\frac{a+c}{2} = A$, then $P(q_A) = A - \frac{bq_A}{2}$, which means that $MR = A - 2 * \frac{bq_A}{2} = A - bq_A$. Thus,

$$MR = \frac{a+c}{2} - bq_A$$

Now, setting $MR = MC$ we have:

$$\frac{a+c}{2} - bq_A = c$$

Or:

$$\frac{a+c}{2} - c = bq_A$$

Or:

$$\frac{a-c}{2} = bq_A$$

Or:

$$\frac{a-c}{2b} = q_A$$

Thus, if Firm B uses the strategy that we found, Firm A will produce $q_A = \frac{a-c}{2b}$. So the NE to the Stackelberg game is:

$$q_A = \frac{a-c}{2b}$$

$$q_B = \begin{cases} \frac{a-c-bq_A}{2b} & \text{if } 0 \leq q_A \leq \frac{a-c}{b} \\ 0 & \text{if } q_A > \frac{a-c}{b} \end{cases}$$

We can find the payoffs to the firms of using these strategies by plugging $q_A = \frac{a-c}{2b}$ into Firm B's best response function to determine how much Firm B will produce.

$$q_B = \frac{a-c-b\left(\frac{a-c}{2b}\right)}{2b}$$

Or:

$$q_B = \frac{a-c-\left(\frac{a-c}{2}\right)}{2b}$$

Or:

$$q_B = \frac{a-c-\frac{a}{2}+\frac{c}{2}}{2b}$$

Or:

$$q_B = \frac{\frac{a}{2}-\frac{c}{2}}{2b}$$

Or:

$$q_B = \frac{a-c}{4b}$$

Thus, if Firm A produces $q_A = \frac{a-c}{2b}$, Firm B will produce $q_B = \frac{a-c}{4b}$. Note that this is NOT the NE strategy for Firm B, just what the result is of Firm B using its NE strategy. Total market quantity is then $q_A + q_B = \frac{a-c}{2b} + \frac{a-c}{4b} = \frac{3}{4} * \frac{a-c}{b}$, or $\frac{3}{4}$ of the perfectly competitive quantity.

3.1 Comparing the results

It's important to compare the results of the different market models. In the table below, I have used the values that we have been using in class, $a = 120$, $b = 1$, and $c = 12$ to compare the monopoly (or cartel), Cournot, Stackelberg, and perfectly competitive (or Bertrand, both simultaneous and sequential) outcomes. The column for CS stands for consumer surplus and the column for TS stands for total surplus, where total surplus is defined as the sum of the firm's profits and the consumer surplus.

	Q	q_A	q_B	$P(Q)$	Π_A	Π_B	CS	TS
Monopoly	54	27	27	66	1458	1458	1458	4374
Cournot	72	36	36	48	1296	1296	2592	5184
Stackelberg	81	54	27	39	1458	729	3280.5	5467.5
Bertrand	108	54	54	12	0	0	5832	5832

As should be clear from the total, consumer's are made better off at the expense of the firms as we move down the table. It is interesting to note that the Cournot case, with two identical firms, is slightly less efficient than the Stackelberg case, with one large firm and one small firm (in terms of relative quantities produced). This raises the question of why antitrust policy may focus on the industry with one large firm and one small firm, rather than the one with two equal-sized firms. The reason has to do with the dynamic aspects of the markets, which we will now discuss in the form of entry prevention by a monopolist.

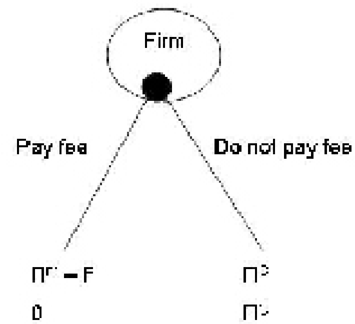
3.2 Entry prevention strategies

It is possible to model entry decisions by new firms as well as entry prevention strategies by incumbent firms as sequential games. Consider the case of an incumbent monopolist who wishes to deter a possible entrant from entering the market. There are three possible scenarios that could occur:

1. Blockaded entry: The entrant finds it unprofitable to enter or cannot enter due to legal restrictions. It is possible that no strategic actions have been made by the monopolist in this case.
2. Deterred entry: The monopolist undertakes strategic actions that make market conditions such that the entrant will not enter the market.
3. Accommodated entry: The monopolist allows the entrant to enter the market as it is too costly to deter the entrant.

3.2.1 Blockaded entry

We could actually model blockaded entry as a 1-player decision by the monopolist. Suppose that the government grants a firm monopoly power in a market provided that the firm pays the government a fee. If the firm pays the fee, F, then it will become a monopoly and will receive Π^m when it is in the market. However, if the firm refuses to pay the fee, then a second firm will also produce in the market. Both firms will then receive the Cournot profit, Π^C. As a simple game, this would look like (Note that I have also included the 2^{nd} firm's payoff in the game tree):



We can actually solve for the amount of the fee that the government could charge the firm. The firm will pay the fee if the profit from paying the fee and receiving monopoly power is greater than the profit from not paying the fee and receiving the 2-firm Cournot payoff, or:

$$\Pi^m - F \geq \Pi^C$$

Using some earlier results, we know that, for a linear inverse demand function and constant marginal costs:

$$\begin{aligned} \Pi^m &= \frac{(a-c)^2}{4b} \\ \Pi^C &= \frac{(a-c)^2}{9b} \end{aligned}$$

We can plug those in to find:

$$\frac{(a-c)^2}{4b} - F \geq \frac{(a-c)^2}{9b}$$

Or:

$$\frac{(a-c)^2}{4b} - \frac{(a-c)^2}{9b} \geq F$$

Getting a common denominator:

$$\frac{9\left(a-c\right)^2}{36b} - \frac{4\left(a-c\right)^2}{36b} \geq F$$

Combining terms:

$$\frac{5\left(a-c\right)^2}{36b} \geq F$$

Now, one little trick. We know that $\Pi^m = \frac{(a-c)^2}{4b}$. The term on the left-hand side of the equation is just $\frac{5}{9} * \frac{(a-c)^2}{4b}$. This means that:

$$\frac{5}{9} * \frac{\left(a-c\right)^2}{4b} \geq F$$

As long as the fee is less than $\frac{5}{9}$ of the monopoly profit the firm would be willing to pay the fee. Think about that – the government can essentially receive 55% of the monopoly profit if it grants a firm the right to be a monopoly producer. This could be a decent method of generating revenue for the government.

3.2.2 Deterred entry

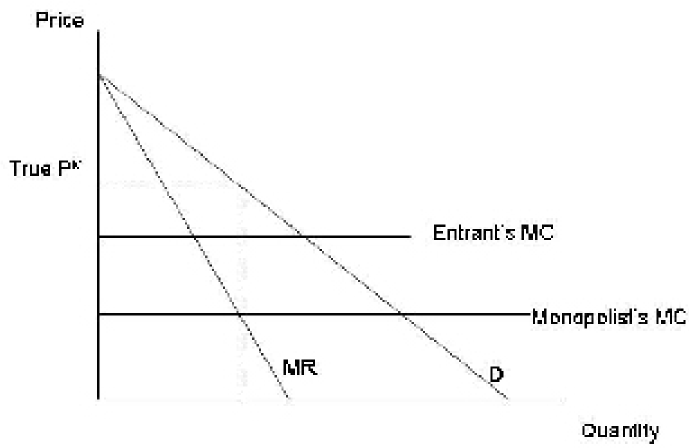
There are many strategic actions an incumbent monopolist might take to deter entry. In all cases it needs to be asked whether or not these actions are credible. If they are credible then the monopolist will achieve its goal and keep the competitor out. If they are not credible then the monopolist has just wasted money pursuing a strategy that the entrant will ignore. Some of those are listed below:

1. Monopolist could make a large fixed cost (or sunk cost) investment to appear to credibly commit to producing a large amount if a potential competitor decides to enter.

2. Over time, the monopolist could develop a reputation as a fierce competitor when someone threatens his market. This reputation could act as a credible commitment.

3. In multi-product markets, monopolists may attempt to "fill in all the niches". Consider the market for breakfast cereals. There are healthy cereals, frosted cereals, fruit-flavored cereals, chocolate cereals etc. If the monopolist can fill in all the niches before his competitors get there, then he can capture the market.

4. The monopolist, if it has a cost advantage over the potential entrant, can set a price below the true profit-maximizing price in order to keep the potential entrant out. This will work if the monopolist can charge a price lower than the entrant's cost but still above his own cost.



In this picture the true profit-maximizing price for the monopolist is above the potential entrant's MC. Thus, the potential entrant may decide to enter the industry. However, if the monopolist wishes to deter entry then it could charge a price slightly below the entrant's MC. The monopolist will still receive positive profits since the price is above cost, but the entrant will not enter because the price is below its cost, preserving the monopoly for the monopolist.