

These notes essentially correspond to chapter 2 of the text.

1 Supply and Demand

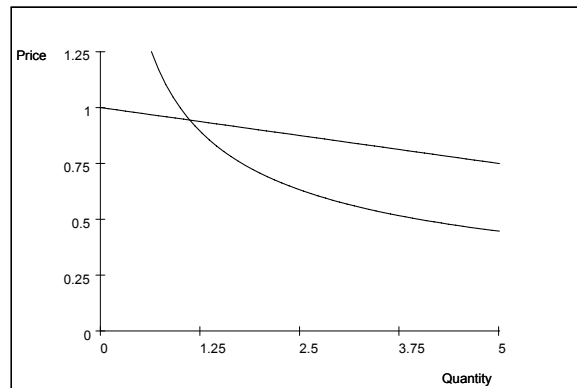
The first model we will discuss is supply and demand. It is the most fundamental model used in economics, and is generally used to predict how equilibrium prices and quantities will change given a change in the underlying determinants of supply and demand.

1.1 Demand

Recall the Law of Demand from your principles of economics courses:

Law of Demand: *There exists an inverse relationship between the price of a good and the quantity demanded of the same good*

This law has been verified using "real-world" data for many goods. If the law holds, then we can draw a demand curve if we place price on the y-axis and quantity on the x-axis. Demand curves for which the law of demand holds will be downward-sloping. They may be linear or non-linear, although we will generally work with linear demand curves for simplicity. Graphed below are portions of a linear and non-linear demand curve:



1.1.1 A simplification of reality

The demand curve is really a simplification of reality. There are many factors that go into determining the demand for a specific product (how many consumers in a market, the prices of related goods, the amount of income consumers have, etc.), but when we graph the demand curve we only consider the price of the good and the quantity demanded. In a sense, we are saying that the quantity demanded of a good is only a function of the price of the good (or the own-price of the good as I have called it). Mathematically, we say that $Q_D = f(P_{own})$. For the more complex case we could write,

$Q_D = f(P_{own}, P_{sub}, P_{comp}, Income, \# \text{ of consumers})$. However, to show this in a picture we would need more and more dimensions (2 dimensions for Q_D and P_{own} , 3 dimensions for Q_D , P_{own} and the price of one substitute, 4 dimensions for Q_D , P_{own} , the price of one substitute and the price of one complement, etc.). Since it is difficult to draw and picture such higher dimension objects we only consider the graph of Q_D and P_{own} .

1.1.2 Demand functions and inverse demand functions

As you can see above, we will be working with demand equations in the course. When Q_D is isolated, so that $Q_D = f(P_{own})$, this is called a demand function. If P_{own} is isolated, so that $P_{own} = f(Q_D)$, then this is called the inverse demand function. (Note: If you are going to graph a demand curve you need to use the inverse demand function, since price is on the y-axis and quantity is on the x-axis and we typically think of graphing equations of the form $y = f(x)$.)

To find the inverse demand function when given the demand function you simply have to solve for P_{own} . Suppose that you have the linear demand function $Q_D = 12 - 6P_{own}$. Then the inverse demand function would be: $P_{own} = 2 - \frac{1}{6}Q_D$. In general, our linear demand functions will take the form of $Q_D = a - bP_{own}$. (Note: Be careful here. The inverse demand functions may also be generally written as $P_{own} = a - bQ_D$. However, the a 's and b 's will not be the same. If a demand function is written as $Q_D = a - bP_{own}$, then the inverse demand function is actually $P_{own} = \frac{a}{b} - \frac{1}{b}Q_D$. If an inverse demand function is written as $P_{own} = a - bQ_D$, then the demand function is actually $Q_D = \frac{a}{b} - \frac{1}{b}P_{own}$. The main point: KNOW WHICH FUNCTION YOU ARE WORKING WITH!!!)

Examples A simple demand function example is one where Q_D is only a function of P_{own} . Thus, $Q_D = 286 - 20P_{own}$ is a simple demand function. If we rewrite this as the inverse demand function we get: $P_{own} = 14.3 - 0.05Q_D$. We can now graph the inverse demand function on our plane using routine methods. The number 14.3 is the price (or y) intercept. The slope of the line is (-0.05). Note that the slope of the demand curve will always be negative if the law of demand holds.

A more complex demand function takes the form of $Q_D = f(P_{own}, P_{sub}, P_{comp}, Y)$. You should note that Y is income. Writing this out we get: $Q_D = 171 - 20P_{own} + 20P_{sub\#1} + 3P_{sub\#2} + 2Y$. The inverse demand function would be: $P_{own} = 8.55 - 0.05Q_D + 1P_{sub\#1} + 0.15P_{sub\#2} + 0.1Y$. Attempting to graph this would be difficult, so we hold the values of the variables other than P_{own} and Q_D at their constant (or average or ceteris paribus) levels. Suppose $P_{sub\#1} = 4$, $P_{sub\#2} = \frac{10}{3}$, and $Y = 12.5$. We then plug these constant values in to the complex demand (or inverse demand) function to find the simple demand (or inverse demand) function. Plugging them in gives:

$$P_{own} = 8.55 - 0.05Q_D + 1(4) + 0.15\left(\frac{10}{3}\right) + 0.1(12.5)$$

Simplifying gives:

$$P_{own} = 8.55 - 0.05Q_D + 4 + 0.5 + 1.25$$

Or

$$P_{own} = 14.3 - 0.05Q_D$$

Thus a simple demand function assumes the values of other variables are held at their constant level.

1.1.3 Changes in demand

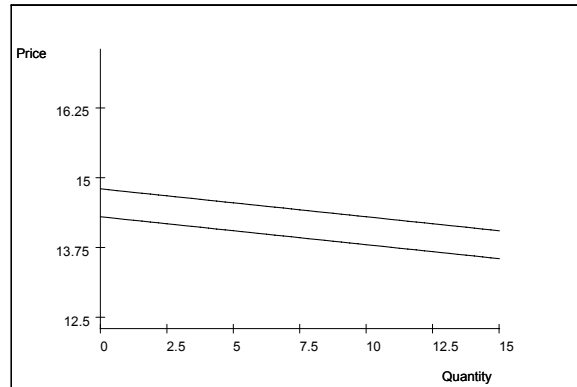
What happens when one of the values of a variable held at its constant level changes? Suppose that $P_{sub\#1}$ increases from \$4 to \$4.5. Now we need to recalculate the simple demand curve. We do this by plugging in the new constant value for $P_{sub\#1}$. We get:

$$P_{own} = 8.55 - 0.05Q_D + 1(4.5) + 0.15\left(\frac{10}{3}\right) + 0.1(12.5)$$

$$P_{own} = 8.55 - 0.05Q_D + 4.5 + 0.5 + 1.25$$

$$P_{own} = 14.8 - 0.05Q_D$$

If we graph the new and old demand curves we will have:



The new demand curve (after $P_{sub\#1}$ increases) is the demand curve to the right with the higher intercept (the unfortunate part of using the actual functions is that I cannot place labels inside the box when I graph them). Recall that a shift to the right is an increase in demand. This should make sense because as the price of a substitute good increases, the demand for our good also increases. Thus the factors other than P_{own} determine where the demand curve is placed on the graph.

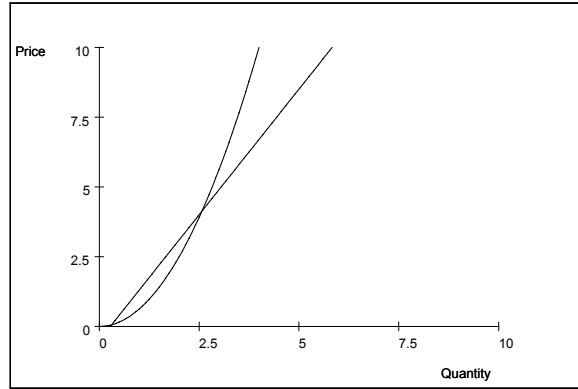
1.2 Supply

Recall the Law of Supply from your principles of economics courses:

Law of Supply: *There exists a direct relationship between the price of a good and its quantity supplied.*

While not as strong as the law of demand (we will see why later in the course), we will assume that the law of supply holds for now. Supply curves for which the law holds will be upward sloping. They may be linear or non-

linear, although we will generally work with linear supply curves for simplicity. Graphed below are portions of a linear and non-linear supply curve:



1.2.1 Supply functions and inverse supply functions

There are both supply functions and inverse supply functions. We denote the supply function as $Q_S = f(P_{own})$ and the inverse supply function as $P_{own} = f(Q_S)$. Note that these are the simple supply and inverse supply functions – the complex functions can include many other factors, such as resource prices, the number of sellers in a market, etc.

Examples A simple supply function example is one where Q_S is only a function of P_{own} . Thus, $Q_S = 88 + 40P_{own}$ is a simple demand function. If we rewrite this as the inverse demand function we get: $P_{own} = -2.2 + 0.025Q_S$. We can now graph the inverse supply function on our plane using routine methods. The number (-2.2) is the price (or y) intercept. The slope of the line is 0.025. Note that the slope of the supply curve will always be positive if the law of supply holds.

A more complex demand function takes the form of $Q_S = f(P_{own}, P_{resource})$. Writing this out we get: $Q_S = 178 + 40P_{own} - 60P_{resource}$. The inverse supply function would be: $P_{own} = -4.45 + 0.025Q_S + 1.5P_{resource}$. Attempting to graph this would be difficult, so we hold the value of $P_{resource}$ at its constant (or average or *ceteris paribus*) level. Suppose $P_{resource} = 1.5$. We then plug this constant value in to the complex supply (or inverse supply) function to find the simple supply (or inverse supply) function. Plugging it in gives:

$$P_{own} = -4.45 + 0.025Q_S + 1.5(1.5)$$

Simplifying gives:

$$P_{own} = -4.45 + 0.025Q_S + 2.25$$

Or

$$P_{own} = -2.2 + 0.025Q_S$$

Thus a simple supply function assumes the values of other variables are held at their constant level.

1.2.2 Changes in supply

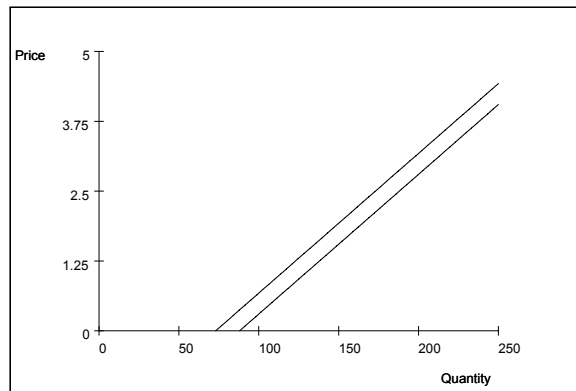
What happens when the value of a variable held at its constant level changes? Suppose that $P_{resource}$ increases from \$1.5 to \$1.75. Now we need to recalculate the simple supply curve. We do this by plugging in the new constant value for $P_{resource}$. We get:

$$P_{own} = -4.45 + 0.025Q_S + 1.5 (1.75)$$

$$P_{own} = -4.45 + 0.025Q_S + 2.625$$

$$P_{own} = -1.825 + 0.025Q_S$$

If we graph the new and old supply curves we will have:



In this case the new supply curve, after $P_{resource}$ changed from 1.5 to 1.75, is the one to the left. Note that the supply curve has decreased (shifts to the left are decreases). Again, this should coincide with what you were taught in principles – when the price of a resource increases, the supply of a good decreases.

2 Equilibrium Determination

Alfred Marshall, whose principles of economics text was most likely read by every economics student from 1900 – 1950, compared supply and demand to the blades of a pair of scissors. In order for the pair of scissors to function properly, both blades are needed. The same is true with supply and demand – in order to properly understand how prices bring about equilibrium, we need to use both supply and demand.

I have no doubt that you all are capable of finding the equilibrium price and quantity if given a graph. Simply find the coordinates of the point where supply and demand intersect and you have your equilibrium price and quantity. However, accurately graphing the supply and demand functions and determining their price and quantity at the intersection point from a graph is a daunting task. It is much easier (especially if you don't have the graph given to you) to determine the equilibrium price and quantity by simply solving a system

of equations. Using our supply and demand functions from above, can we determine an equilibrium price and quantity? We have:

$$\text{Demand function: } Q_D = 286 - 20P_{own}$$

$$\text{Supply function: } Q_S = 88 + 40P_{own}$$

With only these 2 equations we CANNOT solve for a unique price and quantity pair. Notice that we have 3 variables: Q_D , Q_S , and P_{own} but only 2 equations. However, we do know that a 3rd equation holds at the equilibrium point: $Q_D = Q_S$, which must be true if a market is in equilibrium. We now have 3 equations and 3 unknowns (although this does not guarantee that a solution exists).

2.1 Steps to solve for equilibrium prices and quantities

Begin with your 3 equations:

$$\begin{aligned} Q_D &= 286 - 20P_{own} \\ Q_S &= 88 + 40P_{own} \\ Q_D &= Q_S \end{aligned}$$

You can use whatever method you want to solve for the unknowns. Given the current set-up, I would say:

1. Substitute Q_S in for Q_D in the demand function.
2. Next, the left-hand side of the supply and demand functions are now both equal to Q_S . Set the two functions equal to each other.
3. Solve for P_{own} .
4. Plug P_{own} back into the demand function to find Q_D .
5. Plug P_{own} back into the supply function to find Q_S . You should make sure that $Q_D = Q_S$. (The purpose of this 5th step is to check your algebra.)

Now, to do the work:

$$\begin{aligned} Q_S &= 286 - 20P_{own} \\ Q_S &= 88 + 40P_{own} \end{aligned}$$

Next,

$$286 - 20P_{own} = 88 + 40P_{own}$$

Next,

$$P_{own} = \frac{198}{60} = 3.3$$

Next,

$$Q_D = 286 - 20 * (3.3) = 220$$

Finally,

$$Q_S = 88 + 40 * (3.3) = 220$$

Since $Q_D = Q_S$ at P_{own} , we have solved for the equilibrium price and quantity. Either that or we made so many mistakes along the way that things just worked out. I'll assume it's done correctly...

You should also be able to recalculate the equilibrium price and quantity given that one or more of the underlying factors of the supply or demand functions has changed. In those cases, you would need to plug the new constant value into either the new supply or demand function, recalculate the simple supply or demand function, and then work through the steps to solve for the equilibrium price and quantities.

3 Times when $Q_D \neq Q_S$

There are some cases when determining the equilibrium price and quantity cannot be done as described above. Typically, these cases involve some restriction imposed on either price or quantity in the market. We will work through an example of a price floor.

3.1 Price Floor example

Recall that a price control is a government mandated price. A price floor is a price set by the government which the market price cannot fall below. A price ceiling is a price set by the government which the market price cannot rise above.

Suppose we have the following supply and demand functions:

$$\begin{aligned} Q_D &= 286 - 20P_{own} \\ Q_S &= 88 + 40P_{own} \end{aligned}$$

These are the same supply and demand functions from above, so the equilibrium price is \$3.30 and the equilibrium quantity is 220. Suppose the government imposes a price floor of \$4. We now know that the price cannot fall below \$4. How would we go about solving for the price and quantity traded (to be honest, it is really NOT an equilibrium quantity because $Q_D \neq Q_S$ which is why I call it the quantity traded) in the market? I propose the following steps:

1. First, calculate the equilibrium price and quantity without imposing the price floor. I say this because if the price floor is BELOW the equilibrium price, then the price floor does not bind because the market price is greater than the price floor. Thus, the equilibrium price and quantity would be the price and quantity traded in the market. So, if the government had

decided to set a price floor of \$3 in this market, the outcome would just be the equilibrium price and quantity because $\$3 < \3.30 . (And yes, I know I mentioned the possibility that a price floor that is set below the true equilibrium price may end up being a focal point in the market, but that is more food for thought than anything else. If you want to look at some economic journal articles of non-binding price controls, a few references are: Smith and Williams (1981), Isaac and Plott (1981), and Cottle and Wallace (1983). The “conventional wisdom” is that non-binding price controls have no effect on the market.)

2. If the price control does bind then you need to calculate Q_D and Q_S by substituting the value of the price control (\$4 in the example) into the demand and supply functions. We find that $Q_D = 206$ when $P_{own} = \$4$ and that $Q_S = 248$. Note that this is NOT an equilibrium solution because $Q_D \neq Q_S$.
3. Finally, choose the quantity level that is lower: in this case, $Q_D < Q_S$. The reason that we choose the lesser amount is that even if you have 248 units for sale, if people only want to buy 206 units at the price you are charging then only 206 units will be traded.

References

- Cottle, R. and M. Wallace (1983). Economic Effects of Non-Binding Price Constraints. *Journal of Industrial Economics* 31:4, 469–474.
- Isaac, R. M. and C. Plott (1981). Price Controls and the Behavior of Auction Markets: An Experimental Examination. *American Economic Review* 71:3, 448–459.
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