These notes essentially correspond to chapter 3 of the text.

1 Elasticities

An important concept in economics is elasticity. Recall that when we are measuring elasticity we want to see how responsive a quantity measure (demanded or supplied) is to a change in a dollar measure (a price or income). In particular, elasticities measure the percentage change in the quantity measure to a percentage change in a dollar measure. Some of the more common elasticities and their uses are discussed below.[1]

1.1 Own-price elasticity of demand

The own-price elasticity of demand is commonly called the price elasticity of demand (PED). When we calculate this elasticity we are measuring the percentage change in quantity demanded ($\%\Delta Q_D$) to a given percentage change in the own price of the good ($\%\Delta P_{own}$). Mathematically, we have:

$$PED = \frac{\%\Delta Q_D}{\%\Delta P_{own}}$$

The basic formula to find the percentage change in anything is to take the new amount, subtract off the old amount, and then divide the difference by the old amount. So we can write:[2]

$$\%\Delta Q_D = \frac{Q_D^{new} - Q_D^{old}}{Q_D^{old}} \text{ and } \%\Delta P_{own} = \frac{P_{own}^{new} - P_{own}^{old}}{P_{own}^{old}}$$

So our elasticity can then be written as:

$$PED = \frac{\frac{Q_D^{new} - Q_D^{old}}{Q_D^{old}}}{\frac{P_{own}^{new} - P_{own}^{old}}{P_{own}^{old}}}$$

Rearranging some terms,

$$PED = \frac{Q_D^{new} - Q_D^{old}}{P_{own}^{new} - P_{own}^{old}} * \frac{P_{own}^{old}}{Q_D^{old}}$$

Letting $Q_D^{new} - Q_D^{old} = \Delta Q_D$ and $P_{own}^{new} - P_{own}^{old} = \Delta P_{own}$ (note that these are NOT percentage changes but simply the net change in quantity and price), we have:

[1] For a quick refresher on PED, you can see my principles of micro notes. The link to them is: http://www.belkcollege.uncc.edu/azillant/prmicroch7out.pdf

[2] In your principles of micro class you may have calculated the arc price elasticity of demand where you had $\%\Delta Q_D = \frac{Q_{new} - Q_{old}}{\frac{Q_{new} + Q_{new}}{2}}$. This formula was used so that you would get the same measure of elasticity regardless of whether you started from the old quantity or the new quantity.

$$PED = \frac{\Delta Q_D}{\Delta P_{own}} * \frac{P_{own}^{old}}{Q_D^{old}}$$

Now, take a generic simple demand function of the form (why you would want to do this will become clear in a few steps):

$$Q_D = a - bP_{own}$$

We know that:

$$Q_D^{new} = a - bP_{own}^{new}$$
$$Q_D^{old} = a - bP_{own}^{old}$$

Now, subtract the bottom equation from the top equation to get:

$$Q_D^{new} - Q_D^{old} = a - bP_{own}^{new} - a + bP_{own}^{old}$$

Simplify, recalling that $Q_D^{new} - Q_D^{old} = \Delta Q_D$ and $P_{own}^{new} - P_{own}^{old} = \Delta P_{own}$:

$$\Delta Q_D = -bP_{own}^{new} + bP_{own}^{old}$$

$$\Delta Q_D = -b\left(P_{own}^{new} - P_{own}^{old}\right)$$

$$\Delta Q_D = -b\left(\Delta P_{own}\right)$$

$$\frac{\Delta Q_D}{\Delta P_{own}} = -b$$

So we now have a very simple result for our PED. If we have a demand function (NOTE that this is a demand function and NOT an inverse demand function), then:

$$PED = b * \frac{P_{own}^{old}}{Q_D^{old}}$$

A verbal way of stating this is that PED is the ratio of a price to its corresponding quantity (as given by the demand function) times the coefficient on P_{own} (which is $-b$ in the example). For those of you with a calculus background, recall that for infintesimal (very small) changes in P_{own}, the relationship $\frac{\Delta Q_D}{\Delta P_{own}}$ becomes the derivative, $\frac{dQ_D}{dP_{own}}$, or if the demand function is a more complex version with income and prices of substitutes and complements, we get the partial derivative, $\frac{\partial Q_D}{\partial P_{own}}$. Hopefully, this makes some sense as $\frac{\Delta Q_D}{\Delta P_{own}}$ is simply a rate of change, which is exactly what a derivative is.

Now that the gory mathematical details have been laid out, What does it all mean? We say that the demand for a good is elastic when the $|\%\Delta Q_D| > |\%\Delta P_{own}|$ or when $\frac{|\%\Delta Q_D|}{|\%\Delta P_{own}|} > 1$. We say that the demand for a good is inelastic when $|\%\Delta Q_D| < |\%\Delta P_{own}|$ or when $\frac{|\%\Delta Q_D|}{|\%\Delta P_{own}|} < 1$. In the rare case when $|\%\Delta Q_D| = |\%\Delta P_{own}|$ or $\frac{|\%\Delta Q_D|}{|\%\Delta P_{own}|} = 1$, we say that demand is unit elastic. The elasticity for a good is primarily determined by the availability of substitutes for the good. Goods that have many available substitutes are more elastic than goods that have few substitutes. Graphically, the flatter (less steep) a demand curve becomes the more elastic it becomes, until we reach a perfectly horizontal demand curve which we call perfectly elastic. As the demand curve becomes steeper it becomes more inelastic, until we reach a perfectly vertical demand curve which we call perfectly inelastic.

1.2 Income elasticity

Income elasticity (IE) measures how responsive quantity demanded is to a change in a consumer's income (Y). Technically, it is the percentage change in quantity demanded in response to a percentage change in income. Mathematically,

$$IE = \frac{\%\Delta Q_D}{\%\Delta Y}$$

Or, plugging in $\frac{Q_D^{new} - Q_D^{old}}{Q_D^{old}}$ for $\%\Delta Q_D$ and $\frac{Y^{new} - Y^{old}}{Y^{old}}$ for $\%\Delta Y$, we get:

$$IE = \frac{Q_D^{new} - Q_D^{old}}{Y^{new} - Y^{old}} * \frac{Y^{old}}{Q_D^{old}}$$

Again, letting $\Delta Q_D = Q_D^{new} - Q_D^{old}$ and $\Delta Y = Y^{new} - Y^{old}$, we have:

$$IE = \frac{\Delta Q_D}{\Delta Y} * \frac{Y^{old}}{Q_D^{old}}$$

Now, suppose that our demand function is:

$$Q_D = a - bP_{own} + cY$$

What we can show is that $\frac{\Delta Q_D}{\Delta Y} = c$. You can do this by plugging in a new and old Q_D and Y and subtracting the old equation from the new, or you can take the partial derivative of quantity demanded with respect to income. Either way you should find the result. Plugging that result into our initial formula we get:

$$IE = c * \frac{Y^{old}}{Q_D^{old}}$$

Verbally, the income elasticity of a good is equal to the product of the income to quantity demanded ratio and the coefficient on income in the demand function. It is important to note that c can be positive or negative, and that the sign has important implications for the good as discussed below.

1.2.1 Normal and Inferior goods

The sign of the IE determines if the good is a normal good or an inferior good. A normal good is a good with a positive IE. We call it normal because if income increases then the quantity demanded of the good also increases (recall this rule from the principles class). An inferior good is a good with a negative IE. We call them inferior because as consumers earn more income they shift away from these goods, which is contrary to what we usually believe happens when consumers earn more income. Examples of such inferior goods are used clothing and Ramen noodles.

Classifying normal goods – necessities and luxuries If a good is a normal good we can classify it as either a luxury good or a necessity. A necessity is a good with an IE between 0 and 1. Consider electricity. If you receive a moderate increase in your income you are unlikely to change your electricity purchases given this change in income. Perhaps you will consume a slight amount more, perhaps not. Since there is such a small change in quantity demanded when your income rises, we classify goods such as electricity as necessities. However, with even small to moderate increases in income we may see large increases in quantity demanded of other items. Suppose you receive a $15 per week raise and this causes you to increase the number of times that you eat out per week to increase from 1 to 2. You have seen a 100% increase in the amount of times you eat out per week even though you only had a minor increase in income. This would be considered a luxury good.

1.3 Cross-price elasticity of demand

Cross-price elasticity of demand measures how responsive quantity demanded is to a change in the price of a different good $\left(\%\Delta P^B\right)$. Technically, it is the percentage change in quantity demanded in response to a percentage change in the price of another good. Mathematically,

$$X - price = \frac{\%\Delta Q_D^A}{\%\Delta P^B}$$

Using the same steps as above, we can find that:

$$X - price = \frac{\Delta Q_D^A}{\Delta P^B} * \frac{P^{B,old}}{Q_D^{A,old}}$$

Specify the demand function as:

$$Q_D^A = a - bP_{own} + cY + dP^B$$

We can then find that the formula for $X - price$ elasticity is:

$$X - price = d * \frac{P^{B,old}}{Q_D^{A,old}}$$

Verbally, the cross-price elasticity of a good is equal to the ratio of the price of good B to the quantity demanded of good A times the coefficient on the price of good B in the demand function. It is important to note that d can be positive or negative, and that the sign has important implications for the two goods as discussed below.

1.3.1 Substitutes and Complements

The sign of the cross-price elasticity measure is important in determining which goods are substitutes and which goods are complements. If the cross-price elasticity between two goods is positive, then the two goods are substitutes. The logic is that if the quantity demanded of one good increases when the price of another good increases, consumers must be substituting away from the good with the now relatively higher price by replacing purchases of the second good with more purchases of the first good.

If the cross-price elasticity between two goods is negative, then the two goods are complements. The logic is that if the quantity demanded of one good decrease when the price of another good increases, consumers are responding to the price increase of the second good by cutting their purchases of the first good. This implies that the goods are consumed together, which means that they are complements.

1.4 Price elasticity of supply

Price elasticity of supply (PES) measures how responsive quantity supplied (Q_S) is to a change in the own-price of a good (P_{own}). Technically, it is the percentage change in quantity supplied in response to a percentage change in the own-price of the good. Mathematically,

$$PES = \frac{\%\Delta Q_S}{\%\Delta P_{own}}$$

We find that PES is:

$$PES = \frac{\Delta Q_S}{\Delta P_{own}} * \frac{P_{own}^{old}}{Q_S^{old}}$$

Using a SUPPLY function such as:

$$Q_S = z + vP_{own}$$

We can find that the term $\frac{\Delta Q_S}{\Delta P_{own}} = v$. So our formula for the PES is:

$$PES = v * \frac{P_{own}^{old}}{Q_S^{old}}$$

Verbally, the price elasticity of supply for a good is equal to the ratio of the price to the quantity supplied times the coefficient on the price of the good in the supply function. You should note that v will always be positive if the supply curve is upward-sloping.

We can classify supply curves as elastic or inelastic. If $PES > 1$ we say that the supply curve is elastic at the point at which we are evaluating the elasticity. If $PES < 1$ we say that the supply curve is inelastic at the point at which we are evaluating the elasticity. Graphically, the flatter (less steep) a supply curve becomes the more elastic it becomes, until we reach a perfectly horizontal supply curve which we call perfectly elastic. As the supply curve becomes steeper it becomes more inelastic, until we reach a perfectly vertical supply curve which we call perfectly inelastic. If a firm can use its resources to produce many other goods, then the supply curve will be relatively elastic. However, if the firm can produce only the specific good with its resources then the supply curve will tend to be more inelastic.

1.5 Interpreting elasticities

The general form for interpreting elasticities is as follows: A 1% increase in (fill in dollar measure) will cause a X% (increase or decrease depending on the sign of the elasticity) in the (fill in quantity measure).

Suppose the $PED = -0.6$. This means that a 1% increase in the own-price of the good will cause a 0.6% decrease in the quantity demanded of that good.

Suppose the $IE = 1.4$. This means that a 1% increase in the consumer's income will cause a 1.4% increase in the quantity demanded of that good.

Suppose the $X - price = -1.5$. This means that a 1% increase in the price of good B will cause a 1.5% decrease in the quantity demanded of good A.

Suppose the $PES = 0.2$. This means that a 1% increase in the own-price of the good will cause a 0.2% increase in the quantity supplied of that good.

As you can see, they all follow the general form for interpreting elasticities that is given above – you simply need to fill in the correct dollar measure and quantity measure.

2 Taxes

We will analyze the effects that a per-unit tax has on the equilibrium price and quantity in a market. To be specific, we will do a partial equilibrium analysis of the per-unit tax. A partial equilibrium analysis isolates a particular market (or sector of the economy) and considers any effects on other markets to be negligible. A general equilibrium analysis, which is a little more involved, would consider more markets.

Suppose that the government places a per-unit tax on a good. We know that the supply curve will decrease (shift to the left) and that the result will be a higher price and lower equilibrium quantity. What we want to do is see how the consumers, producers, and the government are affected.

2.1 Example

I will use the processed pork market, and we will suppose that the government wants to place a per-unit tax (use τ as the symbol for the tax) on the producers of processed pork. We know that the supply and demand conditions before the tax are:

$$Q_D = 286 - 20P_{own}$$
$$Q_S = 88 + 40P_{own}$$

The initial equilibrium price and quantity in this market are \$3.30 and 220 units (see chapter 2 notes on how to get this result). When the government places this tax on the suppliers, nothing happens to the demand curve, so we still have $Q_D = 286 - 20P_{own}$. However, we know that for every unit sold the producer will have to give back τ to the government. The easiest way to incorporate τ into the supply function is to rewrite it as the inverse supply function:

$$P_{own} = -2.2 + .025Q_S$$

Now, all we need to do is subtract the tax, τ, from P_{own} because the producer will receive only $P_{own} - \tau$ for each unit. The after-tax inverse supply function is then:

$$P_{own} - \tau = -2.2 + .025Q_S$$

Rewriting this as the supply function, we get:

$$Q_S = 88 + 40\left(P_{own} - \tau\right)$$

Or,

$$Q_S = 88 + 40P_{own} - 40\tau$$

Now, if we know the amount of the tax, all we need to do is plug in the amount of the tax and then solve for the equilibrium price and quantity as we did in chapter 2. Suppose the tax is \$1.05 per-unit sold. Then our supply function becomes: $Q_S = 88 + 40P_{own} - 42$. Subtracting 42 from 88 will give us supply and demand functions of:

$$Q_D = 286 - 20P_{own}$$
$$Q_S = 46 + 40P_{own}$$

Now all we need to do is solve for the equilibrium price and quantity. Set $286 - 20P_{own} = 46 + 40P_{own}$ and solve for P_{own}. You should get $P_{own} = \$4$. Now plug \$4 in for the price to find that $Q_S = Q_D = 206$. You should note that the buyers in this market pay \$4, but that the sellers only keep \$2.95 because they must give \$1.05 to the government.[3]

[3] To see this done with a graph, go to my principles of micro chapter 4 lecture. It is not the same example, but the graph works in the same manner. The link is http://www.belkcollege.uncc.edu/azillant/prmicroch4out.pdf.

2.2 Some additional calculations

Now that we have found the new equilibrium price and quantity, we can see how much revenue the government receives, how much society loses from the tax, and who actually bears the burden of the tax – the consumer or the producer.

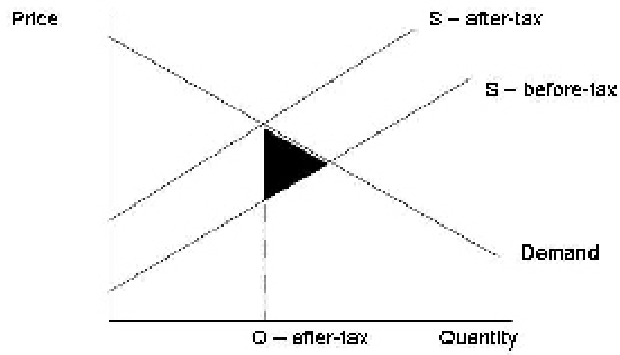
2.2.1 Government revenue

This is a fairly quick calculation to make with a per-unit tax. The government receives the amount of the tax, τ, for every unit sold in the after-tax market. So all we need is the new equilibrium quantity, which we found to be 206, and the tax rate, which was \$1.05 per-unit. Multiply the two of them together to find how much revenue the government generates. In this case, it is \$216.30.

2.2.2 Deadweight loss

If a market is allowed to operate unimpeded then all of the gains from trade should be captured by the buyers and sellers in the market. The gains from trade can be defined as the total amount of consumer and producer surplus that a market can generate. When a market has some restriction imposed on it, such as a price control or a tax, or if the market is not competitive, as in the case of a monopoly, the market is no longer unimpeded. This generates a deadweight loss to society. Deadweight loss is best defined as gains from trade that fail to be captured by the market due to some restriction in the market.

Look at the picture below, with a linear demand curve and a linear supply curve. The triangular area shaded in black as the deadweight loss due to the tax. When the market was untaxed, consumers and producers shared these gains. Now, however, since the tax is placed on the market, no one receives these potential gains from trade because no units beyond Q – after-tax are traded. Thus, this is society's loss.



To calculate the deadweight loss given a linear demand curve and a linear supply curve, use the formula for a triangle, $\frac{1}{2} * base * height$. The best way to do this is to rotate the triangle 90 degrees. Then the base of the triangle will be the amount of the tax, τ, and the height of the triangle will be the change in quantity traded. Just subtract the new equilibrium quantity from the initial (pre-tax) equilibrium quantity. So we can rewrite the formula as $\frac{1}{2} * \tau * \left(Q^{before-tax} - Q^{after-tax}\right)$. Plugging in the numbers from the example we get that the deadweight loss is $\frac{1}{2} * 1.05 * (220 - 206) = 7.35$.

2.2.3 Who bears the burden of the tax

When the government imposes a tax on the seller of a good there is a common misconception that the seller will pass the entire tax along to the consumer by raising the price of the product by the full amount of the tax. While the seller may do this, it will only be an equilibrium if demand is perfectly inelastic. Eventually market forces will bring the price back down, although not to its former level. The amount by which the price rises (and ultimately the percentage of the tax the consumer must actually pay) depends on the elasticity of the supply and demand curves. The simple method (if we already have the before-tax and after-tax equilibrium prices) to calculating the percentage of the tax that the consumer must pay is to see how much the equilibrium price rises by and divide that by the amount of the tax. In our example, the price that the buyers paid rose from \$3.30 to \$4, so \$0.7. The amount of the tax was \$1.05. Divide 0.7 by 1.05 and we get that the consumers must bear $\frac{2}{3}$ of the burden of the tax.

An alternative method of finding the burden of the tax that the consumers must bear is to use the following formula:

$$consumer\ burden = \frac{PES}{PES - PED}$$

You should note that PES is the price elasticity of supply at the before-tax equilibrium price, which was 0.6. Also, PED is the price elasticity of demand at the before-tax equilibrium price, which was -0.3. Plugging these numbers in we get that the consumers must bear a burden of $\frac{.6}{.6-(-.3)} = \frac{2}{3}$, which is exactly what we found up above. We can also use this formula to see what happens to the consumers burden as demand becomes more elastic, holding supply constant. Suppose the price elasticity of demand is now (-0.9). The consumers burden is $\frac{.6}{.6-(-.9)} = \frac{2}{5}$. So, as demand becomes more elastic the seller is not able to pass as much of the burden along to the consumer. Alternatively, if supply were to become more elastic, say to 0.9, then we would have the consumers burden as $\frac{.9}{.9-(-.3)} = \frac{3}{4}$. Thus, if the supply is more elastic the producer can pass more of the burden on to the consumer.

2.2.4 Government revenue and elasticity

The government would prefer to tax goods with both inelastic demand and inelastic supply. The more inelastic the curves, the less the equilibrium quantity traded will fall after the tax is imposed. As an extreme example, suppose that either the supply curve or the demand curve is PERFECTLY inelastic at a quantity level of 220. Suppose the equilibrium price is \$4. If the government places a \$1 tax on the item, the quantity traded will not fall at all since one curve is perfectly inelastic. While all the burden will fall on whoever has the perfectly inelastic curve, the government will gain a great deal of revenue as the quantity will not fall.

To convince yourself of this fact, draw two markets on the same graph, one with a supply and demand curve that are very inelastic and another with a supply and demand curve that are very elastic. Make sure the initial equilibrium price and quantity in both markets is the same. Then place a tax of the same amount on the sellers in each market. You should see the quantity traded in the market with the elastic curves fall by a greater amount than the quantity traded in the market with the inelastic curves. Thus, the government will gain a greater amount of revenue in the market with the inelastic curves.