

Assignment 4

ECON3122, Intermediate Microeconomics

Due: Tuesday, March 18th, by end of class

Directions: Answer all questions completely. Note the due date of the assignment. Late assignments will be accepted at the cost of 15 points per day, up until 12:30pm (beginning of class) on Thursday, March 20th. At that time I will return the graded assignments and post the answers on the web. You may turn in assignments to me after that time so that I can check your work for you, but please realize that you will not receive a grade for the assignment. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1 Production (50 points)

Consider the following production functions, where q is the quantity produced of the good, K is the quantity of capital used, and L is the quantity of labor used:

Production function 1

$$q(K, L) = K^\alpha L^\beta$$

Production function 2

$$q(K, L) = K^\alpha + L^\beta$$

- (5 points) Suppose that $\alpha = 1$ and $\beta = 1$. Does Production function 1 have decreasing, constant, or increasing returns to scale? Explain.
- (5 points) Suppose that $\alpha = 1$ and $\beta = 1$. Does Production function 2 have decreasing, constant, or increasing returns to scale? Explain.
- (5 points) Suppose that $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$. Does Production function 1 have decreasing, constant, or increasing returns to scale? Explain.
- (5 points) Suppose that $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$. Does Production function 2 have decreasing, constant, or increasing returns to scale? Explain.
- (5 points) Below is a Total Product table for production function 1 when $\alpha = 1$ and $\beta = 1$. Draw the isoquant curves for 6 units of Total Product and for 12 units of Total Product. Be sure to label the relevant points as determined by the Total Product table below.

K\L	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

6. (5 points) Use the production table above. Suppose that the level of capital is fixed at 5 units. What is the marginal product of labor (MP_L) for each unit of labor employed? Does the MP_L vary with number of units of labor used?
7. (5 points) Does Production function 1 have diminishing marginal returns to labor if $\alpha = 1$ and $\beta = 1$? If so, at what level of employment of labor does diminishing marginal returns begin? If not, explain why not.
8. (5 points) Now suppose that $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$. Fill in the Total Product chart for Production function 1 below, based on the input combinations provided. Since there are square roots involved, round your answers to 2 decimal places.

K \ L	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

9. (5 points) Suppose that the level of capital is fixed at 5 units. What is the marginal product of labor (MP_L) for each unit of labor employed? Does the MP_L vary with number of units of labor used?
10. (5 points) Does Production function 1 have diminishing marginal returns to labor if $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$? If so, at what level of employment of labor does diminishing marginal returns begin? If not, explain why not.

2 Cost minimization (40 points)

The firm's production function is $q(K, L) = L^\beta K^\alpha$, so that the $MP_L = \beta L^{\beta-1} K^\alpha$ and the $MP_K = \alpha L^\beta K^{\alpha-1}$. Let $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. Let the slope of the isocost line be $-\frac{w}{r}$, and let $w = \$4$ and $r = \$27$.

1. (10 points) Find the marginal rate of technical substitution.
Suppose that the firm wishes to produce 1080 units of the good.
2. (10 points) What is the lowest cost at which it can produce 1080 units?
3. (5 points) What is the amount of capital used at the cost minimization bundle?
4. (5 points) What is the amount of labor used at the cost minimization bundle?
Suppose that the price of capital increases, so that now $r = \$64$. The firm still wishes to produce 1080 units.
5. (10 points) Find the new cost minimizing bundle of inputs for this firm, and total cost, for this firm.

3 Entering a Market (10 points)

Suppose that a firm has two potential plants from which to choose, each of which produces identical outputs. The two total cost functions are:

$$\begin{aligned} \text{Plant 1} & : TC_1 = 10q - 4q^2 + q^3 \\ \text{Plant 2} & : TC_2 = 10q - 3q^2 + \frac{99}{100}q^3 \end{aligned}$$

For which range of total output should this firm use Plant 1 and for which range of total output should this firm use Plant 2?

4 Bonus (5 points each)

1. Hal goes to the XYZ grocery store. He spends his fixed budget of $\$Y$. Given the prices at the XYZ grocery store, he makes his optimal purchase of goods. A reporter stops him outside grocery store XYZ and says, "Let's go to EFG grocery store – I hear they have better prices." Hal and the reporter go to EFG grocery store, and they find that the exact same bundle of goods that Hal bought at XYZ grocery store is $\$5$ cheaper at EFG grocery store. The reporter says, "You are definitely better off shopping at EFG grocery store." Abstracting away from any non-market effects (distance of the grocery stores to his house, the line lengths at the grocery stores, the quality of customer service, etc.) and assuming all goods that Hal purchases are normal goods, must Hal be better off shopping at EFG grocery store? Explain. (Hint: You may want to use indifference curve analysis to answer your question.)
2. Hal goes to the XYZ grocery store. He spends his fixed budget of $\$Y$. Given the prices at the XYZ grocery store, he makes his optimal purchase of goods. A reporter stops him outside grocery store XYZ and says, "I believe the prices at XYZ grocery store are the best in town. To prove it, I'll show you that the exact bundle of goods that you bought costs more at EFG grocery store." Hal and the reporter go to EFG grocery store, and they find that the exact same bundle of goods that Hal bought at XYZ grocery store is $\$5$ more expensive at EFG grocery store. The reporter says, "I told you – XYZ grocery store has the best prices in town. You are definitely better off shopping there." Abstracting away from any non-market effects (distance of the grocery stores to his house, the line lengths at the grocery stores, the quality of customer service, etc.) and assuming all goods that Hal purchases are normal goods, must Hal be better off shopping at XYZ grocery store? Explain. (Hint: You may want to use indifference curve analysis to answer your question.)