Assignment 6

Intermediate Micro, Spring 2008

Due: Thursday, April 24^{th}, by end of class

Directions: Answer all questions completely. Note the due date of the assignment. Late assignments will be accepted at the cost of 15 points per day, up until the beginning of class on Tuesday, April 29^{th}. At that time I will return the graded assignments and post the answers on the web. You may turn in assignments to me after that time so that I can check your work for you, but please realize that you will not receive a grade for the assignment. Also, you may work in groups of up to 3 people on this assignment if you wish. Be sure to put each person's name on the paper – all group members will receive the same grade.

1 Chicken (25 points)

Ren and Chuck are engaged in a game of chicken using tractors (chicken is the game where people drive at one another and the first one to go off the road is the chicken). The players each have two strategies – stay on the road (*stay*) or veer off the road (*veer*). If both players choose *stay* then they are both not chickens, but they crash into each other. Thus, each player's payoff is (-10). If both players choose *veer* then they are both chickens but they do not crash. Each player's payoff is (0). However, if one player chooses *stay* and the other player chooses *veer*, then the player who chooses *stay* gets (10) and the player who chooses *veer* gets (-1). To be complete, chicken is a simultaneous move game.

1. (10 points) Draw the matrix for this game. Be sure to include the players, the strategies and the payoffs.
2. (10 points) There are 2 Nash Equilibria (NE) for this game. Find them both. (For those of you who have had some exposure to game theory there are actually 3 NE, but I do NOT want you to find the mixed strategy Nash Equilibrium).
3. (5 points) Suppose now that Ren can credibly commit to staying on the road and that Chuck can see this commitment. Which of the 2 NE would occur? Explain.

2 Different MC (25 points)

Assume the following: there are two firms competing in a Cournot (quantity) game. The firms face the following inverse demand function: $P(Q) = a - bQ = 15000 - 50Q$. Firm 1 has a cost structure such that $TC_1 = c_1 * q_1$, so that Firm 1's marginal cost is $MC_1 = c_1$. Firm 2 has a cost structure such that $TC_2 = c_2 * q_2$, so that Firm 2's marginal cost is $MC_2 = c_2$. Let $c_1 = 50$ and $c_2 = 100$, so that we have $c_1 < c_2$.

The best response functions for the firms are:

$$q_1 = \frac{a - c_1 - bq_2}{2b}$$

$$q_2 = \frac{a - c_2 - bq_1}{2b}$$

1. (5 points) In equilibrium, do you expect that Firm 1's quantity will be larger than Firm 2's, that Firm 2's quantity will be larger than Firm 1's, or that they will be equal? Explain why you think this.

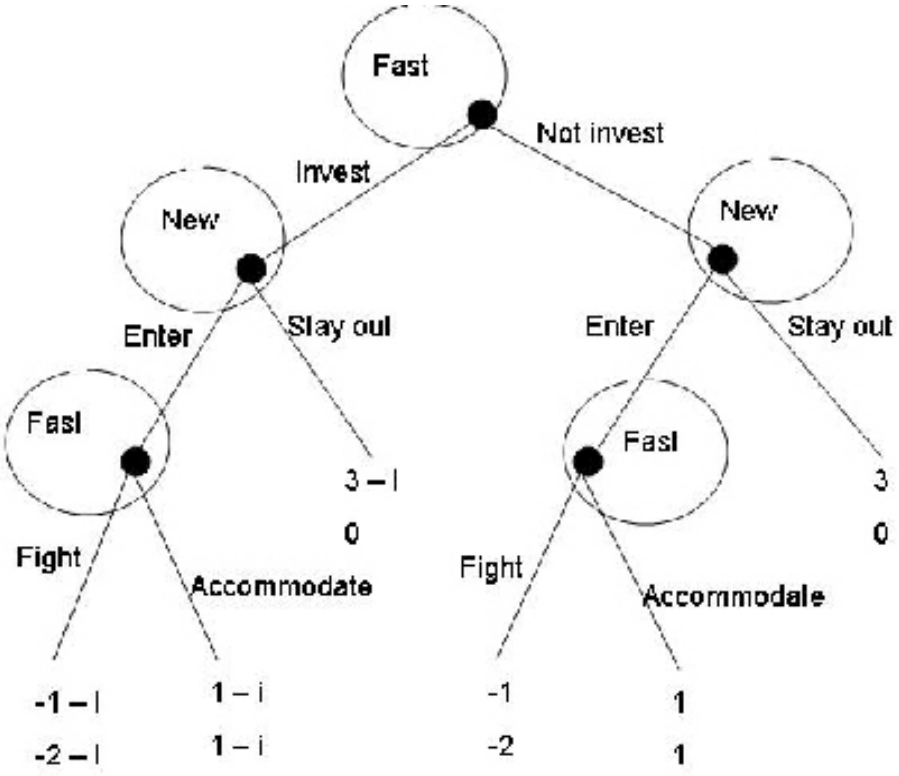
2. (10 points) Find the NE to this Cournot game.
3. (5 points) What is the market price that results when firms play the NE?
4. (5 points) What is each firm's profit when firms play the NE?

3 Preventing entry (50 points)

Consider the following game played between two competitors (called Fastcleaners and Newcleaners). Fastcleaners is an incumbent monopolist and Newcleaners is thinking of entering the market as a competitor. Fastcleaners has the option of initiating a price war with Newcleaners or accommodating entry.

1. (10 points) Suppose that Newcleaners moves first and can decide to either *enter* or *stay out* of the market. If Newcleaners chooses to *enter*, then Fastcleaners gets to decide whether to *fight* or *accommodate entry*. If Newcleaners chooses *enter* and Fastcleaners chooses *fight*, then Newcleaners receives (-2) and Fastcleaners receives (-1). If Newcleaners chooses *enter* and Fastcleaners chooses *accommodate*, then Newcleaners receives 1 and Fastcleaners receives 1. If Newcleaners chooses to *stay out* of the market, then the game ends (without Fastcleaners making a decision). In this case Newcleaners receives a 0 payoff while Fastcleaners receives a payoff of 3. Draw the game tree for this scenario.
2. (5 points) Using backward induction, what is the NE of the game in part 1?

   Now suppose that Fastcleaners can undertake a costly investment in capacity (labelled i in the tree below) to keep Newcleaners out of the market. Formally, the game is as follows: Fastcleaners can either *invest* or *not invest*, which is completely observable to Newcleaners. Upon seeing Fastcleaners' decision, Newcleaners can either *enter* or *stay out* of the market. Finally, if Newcleaners enters then Fastcleaners can either *fight* or *accommodate entry*. This gives way to the following game tree (the names Fast and New refer to Fastcleaners and Newcleaners respectively):



3. (10 points) How much must Fastcleaners invest (in other words, how much should i be) in order to ensure that Newcleaners will not enter? Explain how you arrived at this amount.

4. (10 points) What is the NE, using backward induction, to the game given your answer for the investment level in part 3?

   Now suppose that if Fastcleaners chooses to invest that each unit of investment causes a 2 unit decrease in its own payoffs (so instead of seeing $(3-i)$, $(-1-i)$, and $(1-i)$ Fastcleaners sees $(3-2i)$, $(-1-2i)$, and $(1-2i)$ as its payoffs).

5. (5 points) What is the NE, using backward induction, if Fastcleaners uses the investment level you found in part 3?

6. (10 points) Is there any investment level that Fastcleaners could make so that it would be better off investing than not investing? Explain.

4 Bonus (13 points)

1. (3 points) What movie is the scene in question 1 (Chicken) from (1 point) and how does Ren credibly commit to staying on the tractor (2 points)?

2. (10 points) The citizens of Circleburg live in a city that is laid out in a perfect circle. The circumference of the circle is 12 miles. Residents live in houses which are evenly spaced (uniformly distributed) uniformly over the 12 miles. There are three competing gas stations, Chi Station, Delta Station, and Tau Station. They are attempting to determine where to locate their respective stations. They know that residents of Circleburg will go to the gas station closest to their home. Assume that gas stations are concerned with maximizing the number of customers who visit their station. You may want to use a diagram to aid you when answering the questions. A Nash Equilibrium for this game is a set of locations for the gas stations. Note that gas stations may locate at the same point on the circle. **Note: Customers and gas stations can only locate on the perimeter of the circle. Assume that the interior of the circle is a huge chasm or a steep mountain. Someone always tries to locate stations/customers in the middle of the circle - do NOT do that.** *Hint: It may help to think of the circle as the face of a clock.*

   a (3 points) Find a NE to this game where all firms receive the same number of customers. Explain why this is a NE.

   b (3 points) Find a NE to this game where all firms do NOT receive the same number of customers. Explain why this is a NE.

   c (4 points – not easy) Describe the set of NE to this game.