1 Introduction[1]

Why study theory? The most important reason is that theory provides a way for a researcher to determine the assumptions he or she views as important, reduce the model of behavior to those assumptions, and then make predictions about behavior *that follow logically* from those assumptions. The key is that predictions follow logically from the assumptions (hence why it is emphasized) – anyone can make statements about what might occur, and just about all of those things may actually come true at some point in time, but the point is to understand what behavior drives us to those predictions. It is completely possible that the researcher neglects one or two important aspects, but the theory can always be revised to incorporate new assumptions.[2] A second reason to study theory is because the language of economics is theory, so in order to communicate with other economists it is important to understand that in which they believe. Also, in order to effectively criticize a theoretical model you need to be able to understand it. Simply making a statement like "Well, that's just theory" is not going to win you an argument with a theorist, nor should it.

Now, why study game theory? Most of standard economic theory assumes that individual actors are non-strategic in the sense that their actions have little to no effect on others actions. For example, consider the classic case of a consumer in a market. The consumer has a budget constraint and a utility function, and sees a set of prices, and then chooses the affordable bundle of goods that provides the highest utility. For large numbers of consumers or firms this non-strategic view may be true, but what about when there are only small numbers of either firms or consumers? The classic example is oligopoly theory, which was basically the theory of the kinked demand curve prior to the introduction of game theory (recall in the movie "A Beautiful Mind" that when John Nash is being informed about his receiving the Nobel Prize that the informer tells him his equilibrium concept has been used in antitrust cases). The idea is that when agents are allowed to make strategic decisions, and other agents know that the other agents will be strategic, predictions may change.

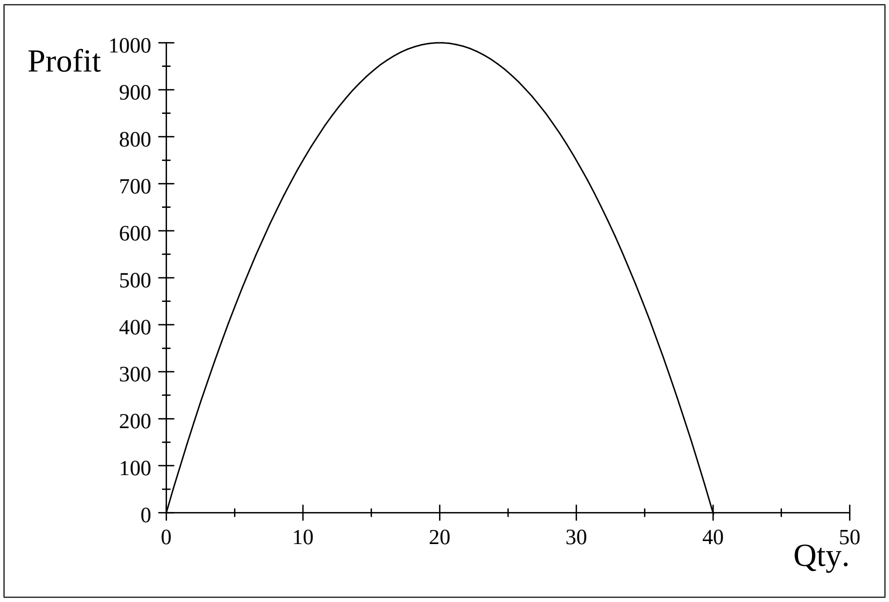
2 Decision-making

In the English language it is easy to see how "decision-making" and "strategy" might be confused – both words imply that some action be undertaken (we will discuss the difference between an action and a strategy later – they are different to a game theorist). However, decision-making is typically used in situations where the actor is non-strategic, as in the example of consumer behavior above. Thus, a consumer choosing the optimal bundle, a monopolist choosing the quantity to maximize profit, and a firm in a perfectly competitive market choosing its profit-maximizing quantity are all decisions, as their actions do not depend on the actions of others (at least in the standard undergraduate setting with exogenous prices or exogenous demand curves – we can allow consumers to act strategically, which may alter the predictions of some of these models; in particular, if you consider a dynamic analysis of the monopolist's problem then the predictions may change dramatically from the standard prediction given in an undergraduate text). Decision-making choices are fairly easy to determine – simply choose the decision which yields the highest profit or utility or whatever the objective measure happens to be. We usually use graphs to represent these decisions as it is easier to use a graph to represent the many choices that a consumer/firm has but we could just as well use a list. Consider the following monopolist who wishes to maximize profit, with inverse demand function $P(Q) = 100 - 2Q$, so that marginal revenue is $MR(Q) = 100 - 4Q$ and let the monopolist have marginal cost of $MC(Q) = Q$ with no fixed costs (so that total cost is just $TC(Q) = \frac{1}{2}Q^2$). We know the monopolist's profit-maximizing quantity is $100 - 4Q = Q$, or $Q = 20$. We can find this by (1) knowing that we need to set $MR = MC$ (we know this through using calculus) (2) plotting out the profit function and finding the quantity at which

[1] These are some introductory notes that somewhat correspond to chapter 1 of the Harrington text.

[2] You should also attempt to understand *why* the theory was developed. For instance, a lot of people criticize general equilibrium models without understanding why general equilibrium was developed in the first place. Walras and later Arrow and Debreu and some others I am forgetting set out with showing that markets can be efficient, and provided a nice set of axioms to show the conditions under which this is true. I do not believe any of them ever argued that all the conditions were true in the real world, just that if these conditions held then markets would be efficient. If you think about this in the context of the time, this is extremely important given that the prevailing thought was that some central planner was needed to coordinate activity (check out Samuelson's prediction about how the Soviet Union was supposed to surpass the US in some of the early editions of his text).

profit attains a maximum or (3) making a list of all quantities and seeing which yields the maximum profit. These are all the same things. In essence, the profit function is a "list"



but we could make a traditional list as well:

Qty.	Profit
1	97.5
5	437.5
10	750.0
20	1000.0
25	937.5
30	750.0
40	0.0

Of course, this list does not contain all of the possible quantities, which is why we use the mathematics. The reason that I bring up list-making is that is pretty much what decision-making is – choosing the optimal (fill in the objective variable – bundle, quantity) from a set list of options. For some games we will use a depiction of the game that looks like a list, although we will call it a matrix.

3 Games

A game consists of four components, listed as follows. They can be actual games – Chess, Baseball, Candy Land, Monopoly. They can be economic games – auctions, oligopoly markets. They can even span across other disciplines – voting.

1. Players – All games must have players, who undertake the actions in the game. Generally speaking, a game should have two or more players, otherwise it turns into a decision. However, what should we do about "games" like Solitaire or Minesweeper? In one sense they are decisions, as the player simply makes (hopefully optimal) choices. In another sense, we could model "Nature" as the second player, which acts somewhat randomly.

2. Rules – Who makes what decisions or moves? When do they make the moves? What are they allowed to do at each move? What information do they know? In a standard Chess game, White moves first and there are 20 moves that White can make (8 pawns that can move either one or two spaces ahead, and 2 knights that can move to one of 2 different spots on the board). Players alternate turns, so that Black also has 20 moves that can be made on his first turn. Furthermore, there are restrictions on how the pieces can move, how pieces are removed and returned to the board, how a winner is determined, how long a player has to make a move – in short, there are a lot of rules to Chess.

3. Outcomes – What occurs as a result of the rules and the decisions players make? At the end of a Chess match one of three things occurs – White wins, Black wins, or there is a draw. Those are the end results of the game. Much simpler than the rules.

4. Payoffs – What utility is assigned to each of the outcomes? Essentially each player has a utility function over outcomes and acts in a manner to best maximize utility, taking into consideration that the other player is doing the same. It does not have to be the case that "winning" has a higher utility than "losing". It may be that one's payoff is tied to who the other players are. If the Chess match is a professional or amateur match and you can win money (or fame) by winning the match, then typically winning will have a higher payoff than losing. However, if you are playing a game with your child or sibling and you are attempting to build their self-esteem then perhaps losing has a higher payoff. Basically, there is a utility function that is a function of all the relevant variables and this utility function determines the players payoffs. In most cases we will simply assume the payoffs are interchangeable with the outcomes, so that specifying a payoff specifies an outcome.

3.1 Solving games

The ultimate goal will be to discuss the standard solution methods to games. The basis for most of the solutions is the Nash equilibrium, named after John Nash. However, for different types of games we will use different refinements of the Nash equilibrium concept. One of the issues with game theory is that in some games there are many, many (possibly infinite) Nash equilibria. The refinements add some additional assumptions that must be met in order to narrow the set of Nash equilibria. The common refinements are subgame perfection and perfect Bayesian – but those are not our concern now.

A standard assumption we will make is *common knowledge* – all players know what all other players know (not that everyone knows the same information, just which information is known), and all players know that all other players know what all players know, etc.

There are also two different definitions of information that will be useful. With perfect information all players know exactly which decisions have been made by the players who move before them – thus, a simultaneous game is one of imperfect information. With incomplete information, there is some uncertainty over the other player's payoff function. A classic example is an auction. In a fairly standard auction setting each bidder knows his own private value but not the value of the other bidders. All bidders have the same payoff function, which is their value minus the price they pay for the object (this is simply a calculation of consumer surplus), but since bidders only know their own value they have incomplete information about the other bidder's actual payoff. There is still common knowledge, in that all bidders know what other bidders know (and don't know).