1 Voting as a strategic game

Although some may believe that voting should be done without strategic concerns, one could argue that voting is a game played between players where the outcome depends on the actions (votes) of the players. Thus we will analyze voting from a strategic perspective. We will compare the outcomes of various voting mechanisms when people vote strategically.

1.1 2-candidate elections

When there are only two candidates (or choices) and no later rounds of voting, there can be no strategic behavior on the part of the voters. Sincere voting is voting for your preferred outcome without consideration of other voters' preferences or how you could impact the outcome of the election by changing your vote. In contrast, strategic voting considers the preferences of other voters as well as the individual's preferences over the remaining options and the impact the individual vote can have upon the outcome of the election. Note that you may still find it in your best interest to vote for your preferred choice when voting strategically, it is just that you went through the process of determining that voting for your preferred choice would leave you best off after the election has taken place. In the case of the 2 candidate election, we will study the decisions of the candidates running in an election and assume that voters simply choose their preferred candidate.

We know that candidates represent many different issues in an election – however, voters are only able to cast one vote, so they must take all the available information as well as their own personal feelings (utility) about how strong a stance a candidate takes on particular issues and map it into a ranking for the candidates. We will assume that voters rank candidates along the "political spectrum". The political spectrum runs on the unit interval (from 0 to 1), with 0 representing the extreme left and 1 representing the extreme right. Candidates will choose a location, L_i, on the spectrum. A location of 0.5 means the candidate is directly in the center of the political spectrum; a location of 0.6 means he is right of center and a location of 0.4 means he is left of center. The 2 candidates will choose their locations simultaneously. Voters will vote for the candidate who is nearest to their own preferences. We will assume that voters are distributed uniformly along the unit interval – essentially there is one voter located at every spot along the unit interval. If you want to think about it mathematically, let p_j denote the position of the j^{th} voter. The voter will choose the candidate whose location maximizes $-|p_j - L_i|$, where 0 is the highest number that could occur (a 0 occurs if $p_j = L_i$, which means the candidate locates exactly at that voter's preference). We will use majority rule voting. If both candidates are the same distance away from a voter then the voter flips a coin to choose between the 2 candidates. Where should the candidates locate?

Suppose they begin by locating at the extreme points along the political spectrum. Both receive 50% of the vote. However, each candidate has an incentive to change his location. Consider the candidate who located at 0 (call him candidate A). He now has the incentive to move just to the left of candidate B (location around 0.999) to take all but one vote in the election. However, if candidate A does this then candidate B has the incentive to locate just to A's left, at 0.9998. Both candidates continue this move leftward, until one of them reaches the location at 0.5. Now, if the other candidate moves a little to the left then he still loses. His only hope is to locate at the exact same spot as the other candidate, at 0.5. Now both candidates are at 0.5. Does either have an incentive to move? No, neither can obtain a larger vote share if he moves. Thus we have found a (the only given our set-up) pure strategy Nash equilibrium to the game: both candidates locate at 0.5. We could have set this up as a giant normal form game and done the circle-square method to find the NE. However, the method of analyzing the game just used is a sufficient shortcut.

You should note that the candidates locating at an identical location that is not 0.5 is NOT a NE to this game. For example, if both candidates locate at 0.75 then they both have an incentive to move to 0.74 because they will gain more votes at that position.

When we have two candidates we get a very nice result called the Median Voter Theorem. Suppose we have 2 candidates and voters have single-peaked preferences (meaning they only have one peak along the political spectrum, or, if you exclude the boundary points of 0 and 1, their preferences exhibit no local minima along the political spectrum) along the political spectrum. If we let m^* be the location of the median voter along the political spectrum, then the NE location choices by the 2 candidates are (m^*, m^*).

1.2 3-candidate election

Now, consider a 3 candidate election. The candidates are to choose their location along the political spectrum described above. We can show that there is no PSNE to the location choices of the three candidates if their goal is to maximize vote share. Although the proof is a little involved, the basic idea is if they all locate at the same spot then someone has an incentive to shift to the side with the most mass. If they all locate at different spots then the 2 candidates on the outside have an incentive to move inward, and at some point as the candidates move inward it will be in the best interest of the candidate in the middle to move from being between the other two candidates to being just on the outside of the candidate that has the most mass on his side. All that being said, our goal is not to use NE to determine why candidates locate where they do in a 3 candidate election but to understand how having 3 candidates in an election allows for strategic voting.

1.2.1 1992 US Presidential Race

In the 1992 US Presidential race the 3 major candidates were Clinton, Bush, and Perot. Given their final vote shares (modified very slightly to fit the example), if we were to locate our candidates along the political spectrum they would locate at 0.25, 0.62, and 0.99, for Clinton, Bush, and Perot respectively. This would give Clinton a 43% share, Bush a 37% share, and Perot a 20% share of the popular vote if voters vote sincerely. We will assume that the plurality winner is elected, abstracting away from the real-world complicating factor of the electoral college.

Given these locations we can back out what the preferences of the voters must be. The preference rankings, along with the percentage of the population that have those preferences, are included in the table below. Note that these preferences and rankings pertain to how the example is structured, and is not based on any survey of actual individuals.

Percentage	43% (CBP)	20% (PBC)	19% (BCP)	18% (BPC)
1^{st}	Clinton	Perot	Bush	Bush
2^{nd}	Bush	Bush	Clinton	Perot
3^{rd}	Perot	Clinton	Perot	Clinton

I found these preferences and percentages as follows:

Anyone who sincerely voted for a candidate must rank that candidate first.

Anyone who ranked Clinton first must prefer Bush to Perot because Bush is more left than Perot.

Anyone who ranked Perot first must prefer Bush to Clinton because Bush is more right than Clinton.

People who voted for Bush could rank Clinton or Perot 2^{nd}, depending on which candidate is closer to them. If we remove Bush from the election we find that Clinton beats Perot, and the vote share for the candidates is 62% to 38%. Thus, Clinton picks up 19% and Perot picks up 18%, which gives us the rankings above.

Now, what would happen if these blocks of people voted strategically? It all really comes down to those who favor Perot – they favor Clinton the least, and by voting for Perot they allow Clinton to win. Thus, if they voted strategically they could alter the election and end up with their second-best choice if they vote for Bush. This would give Bush a 57% vote share to Clinton' 43% vote share. You should notice that no other group has enough power to swing the election. If the 18% BPC people attempted to vote strategically, then Perot would have 38% of the vote to Clinton's 43% and Bush's 19%. However, neither the CBP people nor the BCP people would wish to change their vote to Perot. Thus, if people voted strategically we may have had a different President throughout the 1990s.

1.2.2 A Condorcet election

A Condorcet election is an election in which candidates are paired in a round-robin type of voting tournament. For instance, in the three candidate election above we would have three separate elections: one for Bush vs. Perot, one for Bush vs. Clinton, and one for Clinton vs. Perot. A candidate who goes undefeated in this type of election is called the Condorcet winner. This is different than an agenda-based voting mechanism (described below), as there is no progression of a winner to face the remaining candidate. If we hold a Condorcet election using the preferences above, and people vote sincerely, we find that Bush beats both Perot and Clinton. This means that Bush is the Condorcet winner and is elected. Already we can see the tension building between the different voting systems – plurality gives us Clinton as a winner and Condorcet gives us Bush.

Now suppose that we have a committee of three attempting to choose a flavor of ice cream for their next organizational meeting. The committee members' preferences are as follows:

Person	Greg	Peter	Michael
1^{st}	Chocolate	Vanilla	Strawberry
2^{nd}	Vanilla	Strawberry	Chocolate
3^{rd}	Strawberry	Chocolate	Vanilla

They have decided to use a plurality rule and have not chosen a tie-breaking mechanism. What they find is that all 3 flavors tie. So they decide to use a Condorcet election. With chocolate vs. vanilla, chocolate wins. With vanilla vs. strawberry, vanilla wins. One would think that if chocolate beats vanilla, and vanilla beats strawberry, that chocolate would beat strawberry. However, strawberry beats chocolate in the actual vote, and the committee members are left as they were before, with each flavor ending up 1-1 in the won-loss column. Thus there does not necessarily need to be a Condorcet winner (one option that beats all other options in a head-to-head election) in a Condorcet election.

1.2.3 Agenda-based voting

Suppose that Greg devises a new voting procedure that will ensure a winner. Instead of a round-robin tournament, there will be a single-elimination playoff (for lack of a better term). They will take 2 of the flavors and pit them against each other in an election, with the winner being pitted against the remaining flavor. Technically this is called agenda-based voting. Since Greg devised the voting procedure the committee members agree to allow chocolate to have the bye. In the first round, if the members vote sincerely, vanilla beats strawberry. Then, when vanilla is pitted against chocolate the winner is chocolate. Finally, they have a flavor of ice cream.

A few things to note about agenda based voting. First, chocolate is Peter's least favorite ice cream. He knows that if vanilla (his favorite) is going to go up against chocolate it will lose. So if Peter acts strategically he can change his vote to strawberry in the first round, which is his second favorite flavor, since we know that strawberry will beat chocolate when matched against each other. Thus, strategic voting can run rampant in all rounds of an agenda-based voting system except for the last (in the last round it is essentially a 2-candidate election with no future rounds, so there is no need for strategic voting).

Second, the order of the match-ups is extremely important. If Greg believes that everyone votes sincerely then he wants chocolate to "have the bye" into the last round because chocolate will beat vanilla. However, if Greg believes that the members will vote strategically, then he would suggest that vanilla has the bye into the last round and that chocolate should be pitted against strawberry. If this was the case then Greg would vote for chocolate in the first round, and Michael, knowing that his preferred outcome of strawberry will lose to vanilla in the second round, will vote for chocolate in the first round. Thus Greg can appear to be "nice" by allowing another's favorite to receive the bye into the last round, while in reality he is acting strategically so that his favorite flavor will be chosen.

Finally, you should note that a strategy for an agenda-based voting mechanism must include not only what the voter will choose in the match-ups that are actually reached but also what the voter will do in the match-up(s) that is not reached. In the initial setting we had vanilla vs. strawberry with the winner vs. chocolate. So a strategy for a player would consist of a vote in the case of vanilla vs. strawberry as well as what he would vote for if it ended up to be vanilla vs. chocolate or strawberry vs. chocolate.

1.2.4 Borda Counts

The last voting mechanism we will discuss is a Borda count. The idea behind a Borda count is that in many election systems only the top choice of the voters receives any consideration. It could be the case that in a plurality election 43% of the people have one candidate as their favorite but the other 57% despise that candidate and rank him last. However, since there are 2 more candidates who divide the remaining 57% of the vote the candidate that is despised by 57% of the population ends up winning. Borda counts were devised to give weight to more than one choice among voters preference rankings. The way a Borda count works is that voters are asked to rank candidates in slots 1 through N, where N is either some predetermined cut-off number or determined by the number of candidates. A weight is assigned to each position in the ranking order, and total points are calculated based on how the candidates are ranked and the assigned weights. The candidate with the most points is declared the winner.

Borda counts are frequently used in sports. The NCAA polls use Borda counts and many awards in the major professional sports are also based on Borda counts. However, as we have seen in all of these voting systems, there are potential problems. The first problem is that strategic voting can run rampant as there is no guarantee that voters rank the candidates honestly. Thus, if all the candidates are to be ranked, then voters can attempt to lower the point totals of candidates they think will compete with their favorite by ranking the candidate lower. Or, if a voter's preferred candidate will not win, then voters have the incentive to put a lower ranked choice in their top spot to give that choice more points. Also, if the number of candidates exceeds the cut-off number then voters may just leave candidates who will compete with their preferred candidate completely off the ballot, awarding him no points.

A second problem is the weights assigned to the rankings. The NCAA polls use a constant decrement of 1 point between their rankings. A first-place vote receives 25 points, a second-place vote receives 24 points, ..., down to a twenty-fifth place vote receiving 1 point. In Major League Baseball Most Valuable Player award voting, a first-place vote receives 15 points, while a second-place vote receives 9 points. The increments then decline at a rate of one less point for one lower spot in the ranking, so that a tenth place vote receives 1 point. The question then becomes which of these point award systems is "correct". The answer is no one really knows. There is a paper that looks at how changing the weights would alter baseball MVP voting. The paper is by Jean-Pierre Benoit, and it is called Scoring Reversals: A Major League Dilemma. You can find it in the journal called Social Choice and Welfare, volume 9, published in 1992, on pages 89-97 if you are interested in reading it. Recall that 24 of the 86 elections that Benoit looked at, or about 28% of them, could have had a different winner if a different point awards system was used. And this is only a system that has between 16 and 28 voters, depending on the year and league of the award, with only a few truly viable candidates in most years. One can only imagine what would happen to the number of the possible changes if more voters were able to participate.

Nash Equilibria of voting games You should note that a NE of the voting mechanisms we have studied must have voters choosing their votes in a manner such that no voter can alter the outcome of the election by changing his vote to make himself better off given what the other voters are doing. Thus, sincere voting is typically not a NE in most of these mechanisms.

2 Is there a voting system that works?

From what we have seen so far there appears to be problems with all of the voting systems we have discussed. In many cases we have strategic voting, in some cases two different systems declare two different winners, and in other cases a system may fail to declare a winner. In particular, we would like to find a voting system that is able to aggregate individual preferences and create a logically consistent social ordering of the characteristics. It would also be nice if this voting mechanism would obey the following characteristics:

1. Unrestricted domain – people are allowed to have any logically consistent preference ordering

2. Weak Pareto principle – if all members of a society prefer X to Y, then we should get that X is preferred to Y with our social ordering

3. Independence of Irrelevant Alternatives – if we have 2 options X and Y, such that X is preferred to Y in the social ordering, the introduction of a third option Z should be irrelevant in determining the social ordering of X and Y; basically, if X is preferred to Y without Z, then X should still be preferred to Y when we allow people to vote for Z also

4. Nondictatorship – we would like it to not be the case that one individual decides the social rankings regardless of the preferences of the other player's rankings

Unfortunately, there is no voting mechanism that will satisfy all of these criteria when there are 3 or more options to vote for. This was shown by Kenneth Arrow, who won a Nobel Prize in economics, in part because of this work. This theorem has become known as Arrow's Impossibility Theorem.