Adv. Micro Theory, ECON 6202-090

Assignment 1, Fall 2010

Due: Monday, September 13^{th}

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Find all first-order partial derivatives for each of the following:

   a $f(x, y) = 2x - x^2 - y^2$

   b $f(x, y, z) = \ln\left(x^2 + yz - z^2\right)$

   c $f(x, y) = x^{\alpha} y^{1-\alpha}$, where $0 < \alpha < 1$

   d $f(x, y) = \alpha \ln x + (1 - \alpha) \ln y$, where $0 < \alpha < 1$

2. Suppose $f(x, y) = (xy)^2$ and $g(x, y) = \left(x^2 y\right)^3$.

   a What is the degree of homogeneity for $f(x, y)$? For $g(x, y)$?

   b What is the degree of homogeneity for $h(x) = f(x, y) * g(x, y)$

3. Solve the following problems and state the optimized value of the objective function at the solution:

   a $\min_{x_1, x_2} x_1^2 + x_2^2$ subject to $x_1 x_2 = 1$

   b $\max_{x_1, x_2, x_3} x_1 x_2^2 x_3^3$ subject to $x_1 + x_2 + x_3 = 1$

4. Consider the set $\mathbb{R}^2$. Now consider a subset X of $\mathbb{R}^2$, where $X = \mathbb{R}^2_+$, which is the positive quadrant of the Cartesian plane including the x and y axes. Argue that the set X is closed and convex.

5. Answer each of the following as "yes" or "no" and justify your answer.

   a Suppose $f(x)$ is an increasing function of one variable. Is $f(x)$ quasiconcave?

   b Suppose $f(x)$ is a decreasing function of one variable. Is $f(x)$ quasiconcave?

   c Suppose $f(x)$ is a function of one variable and there is a real number b such that $f(x)$ is decreasing on the interval $(-\infty, b]$ and increasing on $[b, \infty)$. Is $f(x)$ quasiconcave?

   d Suppose $f(x)$ is a function of one variable and there is a real number b such that $f(x)$ is increasing on the interval $(-\infty, b]$ and decreasing on $[b, \infty)$. Is $f(x)$ quasiconcave?