Adv. Micro Theory, ECON 6202-090

Assignment 2, Fall 2010

Due: Monday, September 27^{th}

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Consider the following utility functions: $u(x_1, x_2) = \sqrt{x_1 x_2}$ and $v(x_1, x_2) = \ln(x_1) + \ln(x_2)$. Verify that u and v have the same indifference curves and the same marginal rate of substitution. Explain why.

2. Graph an indifference curve, and compute the marginal rate of substitution and the Marshallian demand functions for the following utility functions:

   (a) Perfect substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$, where $\alpha > 0$ and $\beta > 0$;

   (b) Perfect complements: $u(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$, where $\alpha > 0$ and $\beta > 0$.

3. We have noted that $u(x)$ is invariant to positive monotonic transformations. One common transformation is the logarithmic transform, $\ln(x)$. Take the logarithmic transform of the Cobb-Douglas utility function; then using that as the utility function, derive the Marshallian demand functions and verify that they are identical to those derived in class.

4. A consumer of two goods faces positive prices and has a positive income. Her utility function is

$$u(x_1, x_2) = \max\{ax_1, ax_2\} + \min\{x_1, x_2\}, \text{ for } 0 < a < 1$$

   Derive the Marshallian demand functions.

5. Bob consumes ice cream cones (x_1) and hamburgers (x_2). His utility function is

$$u(x_1, x_2) = (x_1)^{\frac{1}{2}} (x_2)^{\frac{1}{2}}$$

   Bob's income is \$100. The price of each hamburger is \$2. The price of ice cream depends on the quantity that Bob consumes. Specifically, he can buy the first ten ice cream cones at the price of \$2 each. For each additional ice cream cone there is a discount, and Bob has to pay only \$1 each.

   Derive Bob's budget constraint and compute his optimal consumption plan.