Adv. Micro Theory, ECON 6202-090

Assignment 3, Fall 2010

Due: Monday, October 4^{th}

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Consider a consumer with wealth $y > 0$ who has the constant elasticity of substitution (CES) utility function:
$$u(x) = \left[x_1^\sigma + x_2^\sigma\right]^{1/\sigma},$$
where $\sigma < 1$. The expenditure function for the CES utility function is:
$$e(p,u) = u\left[p_1^{\sigma/(\sigma-1)} + p_2^{\sigma/(\sigma-1)}\right]^{(\sigma-1)/\sigma}$$
   a Find the indirect utility function, $v(p,y)$

   b Show that the indirect utility function, $v(p,y)$, is homogenous of degree zero in (p,y).

   c Find the Marshallian demand functions, $x_1(p,y)$ and $x_2(p,y)$.

   d Show that the Marshallian demand functions, $x_1(p,y)$ and $x_2(p,y)$ are homogeneous of degree zero in (p,y).

   e Find the Hicksian demand functions, $x_1^h(p,u)$ and $x_2^h(p,u)$.

   f Are goods x_1 and x_2 normal or inferior goods? How do you know?

   g Let $g(z) = \frac{x_1(p,y)}{x_2(p,y)}$ and $z = \frac{p_1}{p_2}$. The elasticity of substitution between two goods is defined as $\zeta_{1,2}(p,y) = -\left(\frac{\partial g(z)}{\partial z}\right) * \frac{z}{g(z)}$. Verify that $\zeta_{1,2}(p,y)$ is a constant for the CES utility function (hence its name).

2. The substitution matrix of a utility maximizing consumer's demand system at prices $(8,p)$ is:
$$\begin{pmatrix} a & b \\ 2 & -\frac{1}{2} \end{pmatrix}$$
Find a, b, and p.

3. John derives utility from wine x_1 and beer x_2. His utility function is
$$u(x_1,x_2) = \sqrt{(x_1)^2 + (x_2)^2},$$
and his income is \$300.

   a Sketch John's indifference curves.

   b In period 1 the price of wine and beer are $p_{wine} = 15$ and $p_{beer} = 5$. In period 2 the price of wine increases by \$5, i.e. $p'_{wine} = 20$ (the price of beer does not change). Find the income which allows John to obtain the same level of utility in period 2 as he did in period 1.

4. Consider the utility function $u\left(x_1,...,x_N\right)=\prod_{i=1}^{N}\left(x_i-\alpha_i\right)^{\beta_i}$, where α_i and β_i are constants and $\alpha_i>0$ and $\beta_i>0$ for all $i=1,...,n$. The consumer faces budget constraint $y-\sum_{i=1}^{n}p_ix_i\geq 0$ with $y>\sum_{i-1}^{N}\left(\alpha_ip_i\right)$, and $p_i>0$ for all $i=1,...,n$. Note that $\prod_{i=1}^{N}$ is the product operator, so that $\prod_{i=1}^{2}x_i=x_1x_2$ and $\prod_{i=1}^{3}x_i=x_1x_2x_3$, etc.

   a. Calculate the marginal utility for a general good x_i.

   b. For a two-good economy, find the Marshallian demand for goods x_1 and x_2.

   c. Find the Hicksian demand function for goods x_1 and x_2.

   d Suppose that $w=800$, $p_1=4$, $p_2=8$, $\alpha_1=7$, $\alpha_2=5$, $\beta_1=\frac{3}{4}$, and $\beta_2=\frac{1}{4}$. Verify that the optimal consumption bundle is the same under both the Hicksian and Walrasian demand functions.