Adv. Micro Theory, ECON 6202-090

Assignment 4, Fall 2010

Due: Wednesday October 13^{th} by 5pm

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Consider a risk neutral individual. Show that this individual's Arrow-Pratt measure of absolute risk aversion, $R_a(w)$, is equal to zero. (**Hint**: What type of vN-M utility function must the individual have in order to be risk neutral?)

2. An individual has wealth W. Her von Neumann-Morgenstern utility function over non-negative levels of wealth is $u(w) = w^{\rho}$, where $0 < \rho < 1$. The individual is offered the following bet. If she pays x, with probability 1/2 she receives nothing and with probability 1/2 she receives $x(1+s)$, where $s > 1$. How much will she bet (as a function of s)?

3. (**Harder**) Consider an investor who has initial wealth w and has to decide how to invest it. There is a riskless asset with rate of return r. The risky asset has return x_i with probability $\pi_i, i = 1, \ldots, n$. Denote by α the fraction of wealth that the investor puts into the risky asset, so that $1 - \alpha$ is the fraction he invests in the riskless asset.

   a Write down the investor's optimization problem.

   b Show that if the investor has constant relative risk aversion (CARA), then the fraction of wealth invested in the risky asset α, does not change with w (that is, $\frac{d(\alpha^*/w)}{dw} = 0$, where α^* denotes the solution to the investor's problem). Note that you may assume an interior solution.

4. Consider the quadratic vN-M utility function $u(w) = a + bw + cw^2$.

   a What restrictions, if any, must be placed on paramters a, b, and c for this function to display risk aversion?

   b Over what domain of wealth can a quadratic vN-M utility function be defined?

   c Given the gamble

   $$g = \left(\frac{1}{2} \circ (w+h), \frac{1}{2} \circ (w-h)\right)$$

   show that $CE < E(g)$ and that $P > 0$.

   d Show that this function, satisfying the restrictions in part **a**, cannot represent preferences that display decreasing absolute risk aversion.