Adv. Micro Theory, ECON 6202-090

Assignment 6, Fall 2010

Due: Monday November 29^{th}

Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Consider the firm's short run cost minimization problem when there are two inputs, x_1 and x_2, but x_2 is fixed at $\overline{x}_2$. The production function is

$$y = x_1^{1/3} x_2^{1/3}$$

and input prices are w_1 and w_2 for inputs 1 and 2 respectively. Find the firm's short run cost function $c(w_1, w_2, y, \overline{x}_2)$ and show that the short run profit function for this problem is

$$: \pi(p, w_1, w_2, \overline{x}_2) = \frac{2\sqrt{3}}{9}\left(\sqrt{\frac{\overline{x}_2 p^3}{w_1}}\right) - w_2\overline{x}_2$$

Note that $\sqrt{27} - \sqrt{3} = 2\sqrt{3}$.

2. Suppose that the technology for producing q is identical for all firms. The cost function for a representative firm is given by

$$c(q) = a + bq + cq^2,$$

where $a > 0, b > 0$, and $c > 0$. Find the long-run equilibrium price, and the quantity of output produced by each firm.

3. Consider a two-period monopoly facing the negatively sloped inverse demand function $p_t = p(q_t)$ in each period $t = 1, 2$. The firm maximizes the present discounted value of profits

$$PDV = \sum_{t=0}^{1} (1+r)^{-t} \pi_t$$

where $r > 0$ is the market interest rate and π_t is period-t profit. In each of the following, assume that costs each period are increasing in that period's output and are strictly convex, and that PDV is strictly concave.

   a If costs are $c_t = c(q_t)$ for $t = 0, 1$, show that the firm will "short-run profit maximize" in each period by choosing output to equate marginal cost and marginal revenue in each period.

   b Now suppose that the firm can "learn by doing". Its first period costs are simply $c_0 = c_0(q_0)$. Its second-period costs, however, depend on first-period output; $c_1 = c_1(q_0, q_1)$, where $\partial c_1/\partial q_0 < 0$. Does the firm still "short run profit maximize" in each period? Why or why not? Interpret your results.

4. The monopolist faces demand:

$$q = p^{-\varepsilon}$$

where $\varepsilon > 1$, and has constant marginal cost of c with no fixed costs so that the cost function is $c(q) = cq$.

a Find the monopolist's profit maximizing price.

b A per-unit tax, $t > 0$, is levied on the output of a monopolist (this means the monopolist pays an amount t for each unit sold). Find the monopolist's new profit maximizing price under this condition.

c Compare the prices in part **a** and part **b** and show that the monopolist raises its price by more than the amount of the tax when the monopolist is taxed.

5. A monopolist faces linear demand $p = \alpha - \beta q$ and has cost $C = cq + F$, where all parameters (α, β, c, and F) are positive, $\alpha > c$, and $(\alpha - c)^2 > 4\beta F$.

   a Solve for the monopolist's output, price, and profits.

   b Calculate the deadweight loss and show that it is positive.

   c If the government requires this firm to set the price that maximizes the sum of consumer and producer surplus, and to serve all buyers at that price, what is the price the firm must charge? Show that the firm's profits are negative under this regulation, so that this form of regulation is not sustainable in the long run.