These notes correspond to chapter 4 of Jehle and Reny.

In these notes we combine firms and consumers into one market economy. This chapter concerns the notion of partial equilibrium, while the next focuses on general equilibrium. Partial equilibrium typically focuses on the market for one good, and in particular on a good in which income effects and feedback effects to other markets are minimal. The market for Beanie Babies would be the type of good for which we might use partial equilibrium analysis (at least today – in the late 1990s, when they were an "investment" collectable, it may not have been appropriate to use partial equilibrium analysis to study the market). Partial equilibrium analysis is likely to be inadequate for the market for gasoline, where there are likely non-negligible income effects and substantial feedback effects.

1 A market economy

Before discussing any notions of behavior in a market economy we need to define all the relevant terms. To begin, we have a set of potential buyers for the good in question q. Each buyer has his or her own preferences, consumption set, and income. We assume there are I buyers, with $I = 1, ..., I$. Each buyer has a nonnegative demand for good q represented by $q^i\left(p_q, p_{-q}, y^i\right)$, where p_q is the price of good q, p_{-q} is the vector of prices of all other goods except q, and y^i is consumer i's income. Market demand at price p_q is the sum of all buyers' individual demands at that price:

$$q^d\left(p_q\right) = \sum_{i=1}^{I} q^i\left(p_q, p_{-q}, y^i\right) \tag{1}$$

Note that market demand depends not only on the price of the good but also on the prices of other goods as well as the aggregate income in the economy and the distribution of that income. We know that individual demands are homogeneous of degree zero in prices and income, so that market demand will be homogeneous of degree zero in prices and the vector of incomes (so if we double all prices and every individual's income demand will stay the same). This is the only restriction that follows from utility maximization for market demand.

There are J potential suppliers of the good which are already in operation in the market. We consider the number J to be the number of sellers in the short-run, so that there is some fixed factor (such as plant size). The short-run market supply function is the sum of the individual firm market supply functions $q^j\left(p_q, w\right)$:

$$q^s\left(p_q\right) = \sum_{j=1}^{J} q^j\left(p_q, w\right) \tag{2}$$

1.1 Perfect competition

In a perfectly competitive market all buyers and sellers simply "take" the market price as given and no individual buyer or seller exerts any effect on the market price. Market supply and demand determine the equilibrium price of the good $\left(p_q^*\right)$ and the equilibrium quantity traded $\left(q^*\right)$. This occurs where $q^d\left(p_q^*\right) = q^s\left(p_q^*\right)$. In equilibrium each buyer is buying the optimal amount of the good at p_q^* and each seller is maximizing profits at p_q^*. Since no agent (consumer or producer) wishes to change his or her decision, we have an equilibrium in that everything in the system is at rest.

Suppose that there are J identical firms in a competitive market. The short-run profit function in the market when x_2 is fixed is:

$$\pi\left(p, w_1, w_2, \overline{x}_2\right) = \frac{2\sqrt{3}}{9}\left(\sqrt{\frac{\overline{x}_2 p^3}{w_1}}\right) - w_2\overline{x}_2 \tag{3}$$

and output supply is:

$$\frac{\partial \pi\left(p, w_1, w_2, \overline{x}_2\right)}{\partial p} = \frac{\sqrt{3}}{3}\left(\sqrt{\frac{\overline{x}_2 p}{w_1}}\right) \tag{4}$$

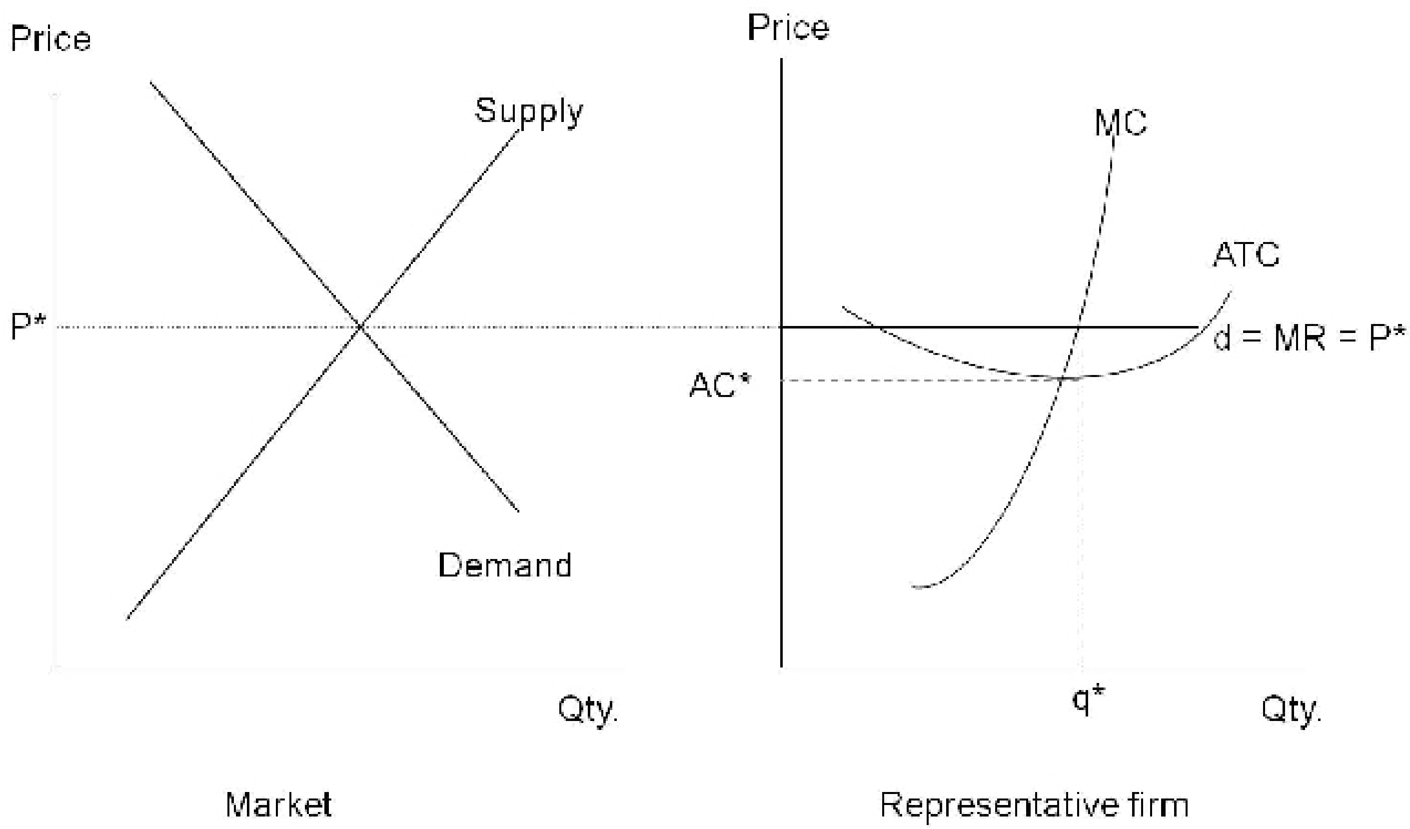

Figure 1: A profit maximizing firm earning positive economic profit in a perfectly competitive market.

Let $w_1 = 3$, $w_2 = 5$, and $\overline{x}_2 = 1$. Then output supply for each firm becomes:

$$q^j = \frac{\sqrt{p}}{3} \tag{5}$$

If there are $J = 48$ firms then we have that market supply is equal to:

$$q^s = 16\sqrt{p} \tag{6}$$

If we assume that market demand is:

$$q^d = \frac{256}{\sqrt{p}} \tag{7}$$

then we have that $p^* = 16$. So that:

$$\begin{gathered} p^* = 16 \\ q^* = 64 \\ q^j = \tfrac{4}{3} \\ \pi^j = 9.\overline{2} \end{gathered} \tag{8}$$

A picture of a firm in short run equilibrium is in Figure 1.

Note that the example leads to firms in the industry making positive economic profits, which may occur in a short-run competitive equilibrium. However, in a long-run competitive equilibrium if there were positive economic profits this would entice other firms to enter the market since they could do better in this market than in their next best alternative. These firms can enter the market because there are no barriers to entry and if they choose to enter then they can choose the optimal plant size. Also, profits in a long-run competitive equilibrium cannot be negative because then firms would be free to exit the market. Thus, we

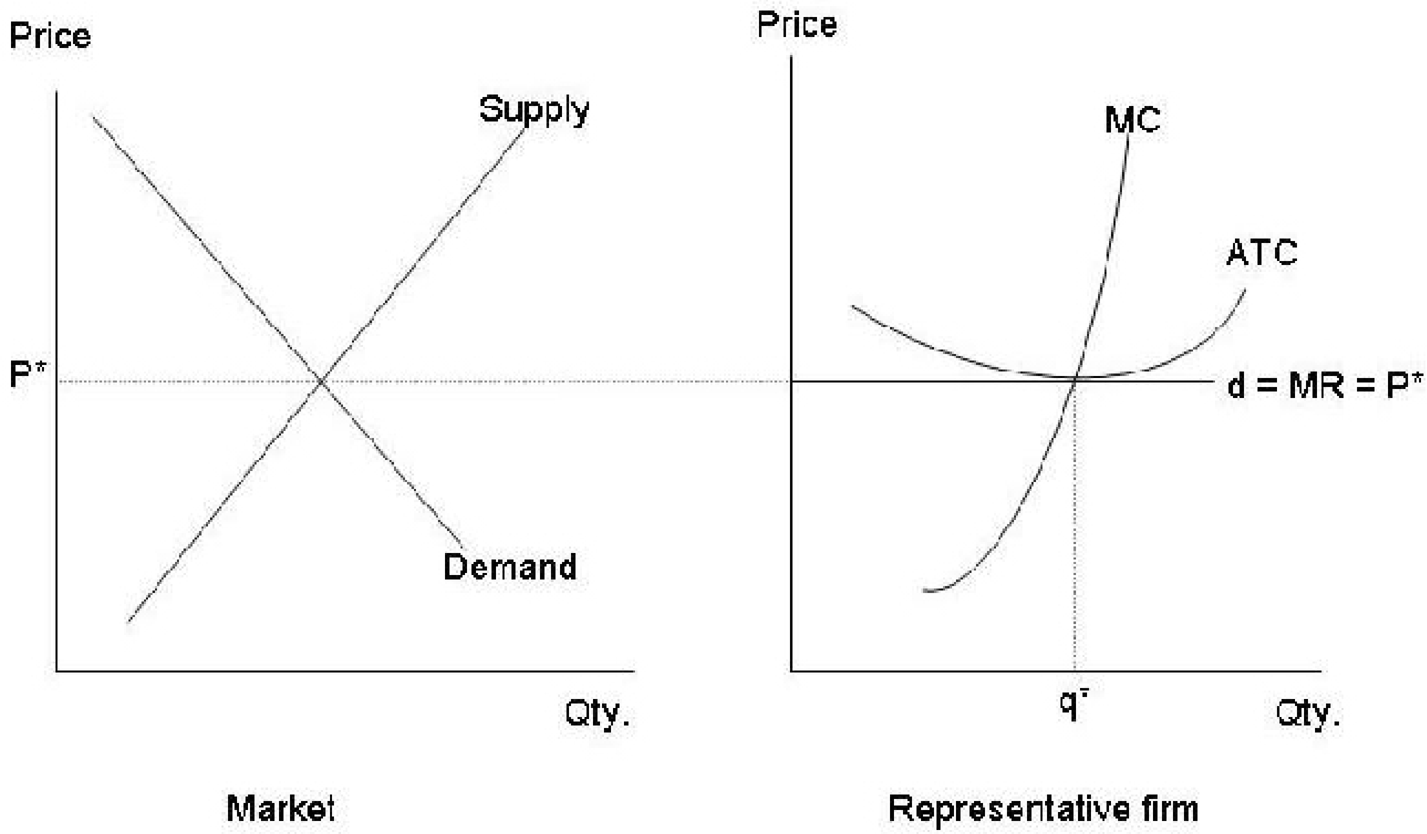

Figure 2: Long-run equilibrium in a perfectly competitive market

come to the familiar conclusion that in a long-run equilibrium of a perfectly competitive market there are zero economic profits.[1] The conditions that need to hold in long-run equilibrium for a perfectly competitive market are now:

$$q^d(\widehat{p}) = \sum_{j=1}^{\widehat{J}} q^j(\widehat{p}) \tag{9}$$

$$\pi^j(\widehat{p}) = 0,\ j = 1, ..., \widehat{J} \tag{10}$$

A picture of the firm and the market in long-run equilibrium is in Figure 2.

1.2 Imperfect competition

In many markets either buyers or sellers have some control over the market price. These markets range from more to less competitive. On one end of the spectrum is perfect competition in which no buyer or seller has any impact on the market price. At the other end of the spectrum is monopoly, in which there is a single seller which can set the price of the good.[2] In between we call those markets with many sellers

[1] Recall that when we discuss "economic" profits we assume that opportunity costs are included as costs. This is different from accounting profit which does not include opportunity costs (if you have zero accounting profit it is generally a good sign that you should exit the industry).

[2] It is a common misconception that a monopolist can "charge whatever it likes". While this is technically true, if monopolists charge too much then they will not have any sales. Thus, while a monopolist can set the price, the market determines what the quantity traded is at that price.

who have some impact on the price "monopolistic competition" while markets with few sellers who have a large impact on the market price of the good are called "oligopoly".

1.2.1 Monopoly

In monopoly there is a single seller of a well-defined good with no close substitutes. There is also some entry barrier blocking potential entrants from entering the market, which could lead to a firm earning positive economic profits even in long-run equilibrium. While the monopolist can choose the price (or quantity) in the market, the market demand determines the other variable. The convention is to assume the firm chooses quantity but the results we will see hold up even if the monopolist chooses price.

The monopolist's profit function is:

$$\Pi(q) = r(q) - c(q) \tag{11}$$

where $r(q)$ is the monopolist's revenue function and $c(q)$ is its cost function. If we find the q^* that maximizes profit we have:

$$\begin{aligned} \frac{d\Pi(q)}{dq} &= r'(q^*) - c'(q^*) && (12)\\ 0 &= r'(q^*) - c'(q^*) && (13)\\ r'(q^*) &= c'(q^*) && (14) \end{aligned}$$

Once again this is the familiar result that marginal revenue equals marginal cost when $q^* > 0$. If we let market demand be $q(p_q)$, then inverse market demand is $p_q(q)$, so that total revenue is $r(q) = p_q(q) * q$. We now have:

$$\begin{aligned} \Pi(q) &= p_q(q) * q - c(q) && (15)\\ \frac{d\Pi(q)}{dq} &= p_q(q) + p_q'(q)\, q - c'(q) && (16) \end{aligned}$$

If $q^* > 0$ we have:

$$p_q(q^*) + p_q'(q^*)\, q^* = c'(q^*) \tag{17}$$

This is still the same marginal revenue equals marginal cost result but if we rearrange some terms we have:

$$p_q(q^*) = c'(q^*) - p_q'(q^*)\, q^* \tag{18}$$

We now know that the monopolist will make positive economic profit in equilibrium if it produces a positive quantity since:

$$p_q(q^*) > c'(q^*) \tag{19}$$

We know this to be true because:

$$-p'(q^*)\, q^* > 0 \tag{20}$$

since $q^* > 0$ and $p'(q^*) < 0$ so that $-p'(q^*) > 0$. A picture of the monopolist's profit-maximizing choice of price and/or output is in Figure 3. Note that it is assumed that inverse demand is linear in this picture, but the same basic results still hold.

We can also relate the monopolist's profit to the elasticity of demand for the good. The monopolist's marginal revenue is:

$$\begin{aligned} mr(q) &= p(q) + qp'(q) && (21)\\ mr(q) &= p(q)\left[1 + \frac{q}{p(q)} p'(q)\right] && (22)\\ mr(q) &= p(q)\left[1 + \frac{1}{\varepsilon(q)}\right] && (23) \end{aligned}$$

Note that $\varepsilon(q)$ is the price elasticity of demand for good q and so $\varepsilon(q) < 0$. Rewriting we have:

$$mr(q) = p(q)\left[1 - \frac{1}{|\varepsilon(q)|}\right] \tag{24}$$

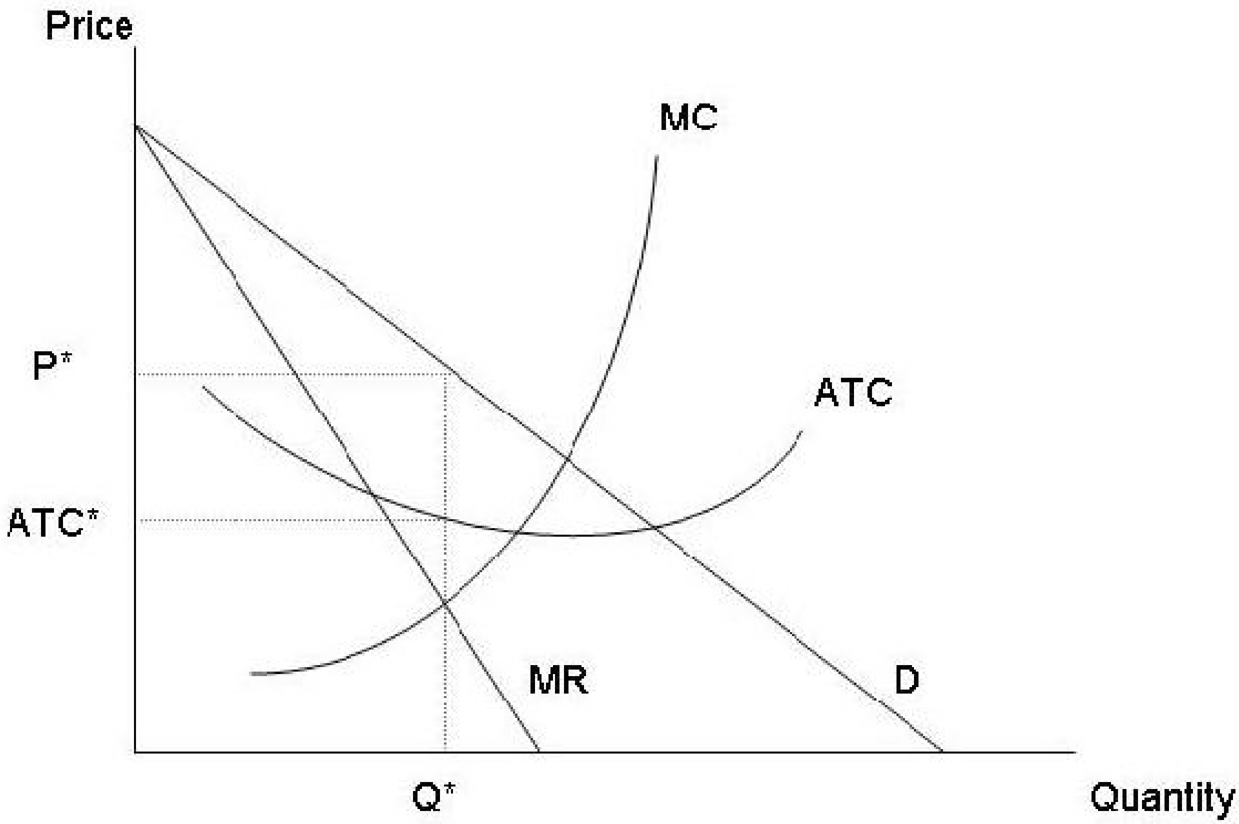

Figure 3: Optimal price and quantity for a profit maximizing monopolist.

where we write $|\varepsilon(q)|$ to be clear that this term is positive. At q^* we have:

$$p(q^*)\left[1-\frac{1}{|\varepsilon(q^*)|}\right]=c'(q^*)\geq 0 \tag{25}$$

Since $c'(q)\geq 0$, we know that $mr(q^*)\geq 0$. We know $p(q^*)>0$, so we also need:

$$\begin{aligned} 1-\frac{1}{|\varepsilon(q^*)|} &\geq 0 && (26)\\ |\varepsilon(q^*)| &> 1 && (27)\end{aligned}$$

What this means is that the monopolist's profit maximizing price is never on the inelastic portion of the demand curve. We can also rewrite:

$$\begin{aligned} p(q^*)\left[1-\frac{1}{|\varepsilon(q^*)|}\right] &= c'(q^*) && (28)\\ p(q^*)-\frac{p(q^*)}{|\varepsilon(q^*)|} &= c'(q^*) && (29)\\ \frac{p(q^*)-c'(q^*)}{p(q^*)} &= \frac{1}{|\varepsilon(q^*)|} && (30)\end{aligned}$$

This shows that the price-cost markup is related to the elasticity of demand for the good. As long as $|\varepsilon(q^*)|$ is not infinity this number will be positive. The more inelastic demand is, the more the monopolist is able to increase price above marginal cost in equilibrium.

1.2.2 Monopolistic competition

In monopolistic competition there are a large number of firms who all produce different variations of a product (or differentiated products). These products are close, but not perfect, substitutes. Because the goods are close substitutes each firm has some degree of monopoly power in the market. There are no barriers to enter in this market, so that entry occurs when a firm introduces a new product.

Let $j=1,...,\infty$ be the number of potential product variations. For simplicity, assume each firm only produces one product variant, so that $j=1,...,J$ represents the number of firms in the market. The demand for each firm's product is a function of its own price as well as the price of other variants. We have $q_j(p_j,p_{-j})$ where $\partial q_j/\partial p_j<0$ and $\partial q_j/\partial p_k>0$ for all $k\neq j$. This means that own-price effects are negative while all other goods are substitutes for good j. Assume there is a price $\overline{p}_j>0$ such that $q_j(p_j)=0$ for all $p_j\geq\overline{p}_j$. The firm's profit is:

$$\Pi_j(p_j,p_{-j})=q_j(p_j,p_{-j})\,p_j-c_j(q_j(p_j,p_{-j})) \tag{31}$$

We can now distinguish between long-run and short-run equilibrium. In the short-run there are a fixed number of firms which maximize profit by choosing price. Suppose there are J firms in the market. If firm j produces a positive amount of output, so that $q_j(p_j,p_{-j})>0$. We then have:

$$\begin{aligned} \frac{\partial \Pi_j(p_j,p_{-j})}{\partial p_j} &= \frac{\partial q_j(p_j,p_{-j})}{\partial p_j}p_j+q_j(p_j,p_{-j})-c'(q_j(p_j,p_{-j}))\frac{\partial q_j(p_j,p_{-j})}{\partial p_j} && (32)\\ 0 &= \frac{\partial q_j(p_j,p_{-j})}{\partial p_j}p_j+q_j(p_j,p_{-j})-c'(q_j(p_j,p_{-j}))\frac{\partial q_j(p_j,p_{-j})}{\partial p_j} && (33)\\ 0 &= \frac{\partial q_j(p_j,p_{-j})}{\partial p_j}\left[p_j+q_j(p_j,p_{-j})\frac{\partial p_j}{\partial q_j}-c'(q_j(p_j,p_{-j}))\right] && (34)\\ 0 &= \frac{\partial q_j(p_j,p_{-j})}{\partial p_j}\left[mr(q_j)-mc(q_j)\right] && (35)\end{aligned}$$

We know that $\frac{\partial q_j(p_j,p_{-j})}{\partial p_j}<0$ by assumption, so that once again this firm is setting $mr(q_j)=mc(q_j)$. It is possible that the monopolistically competitive firm may have positive, negative, or zero profits in the short-run.

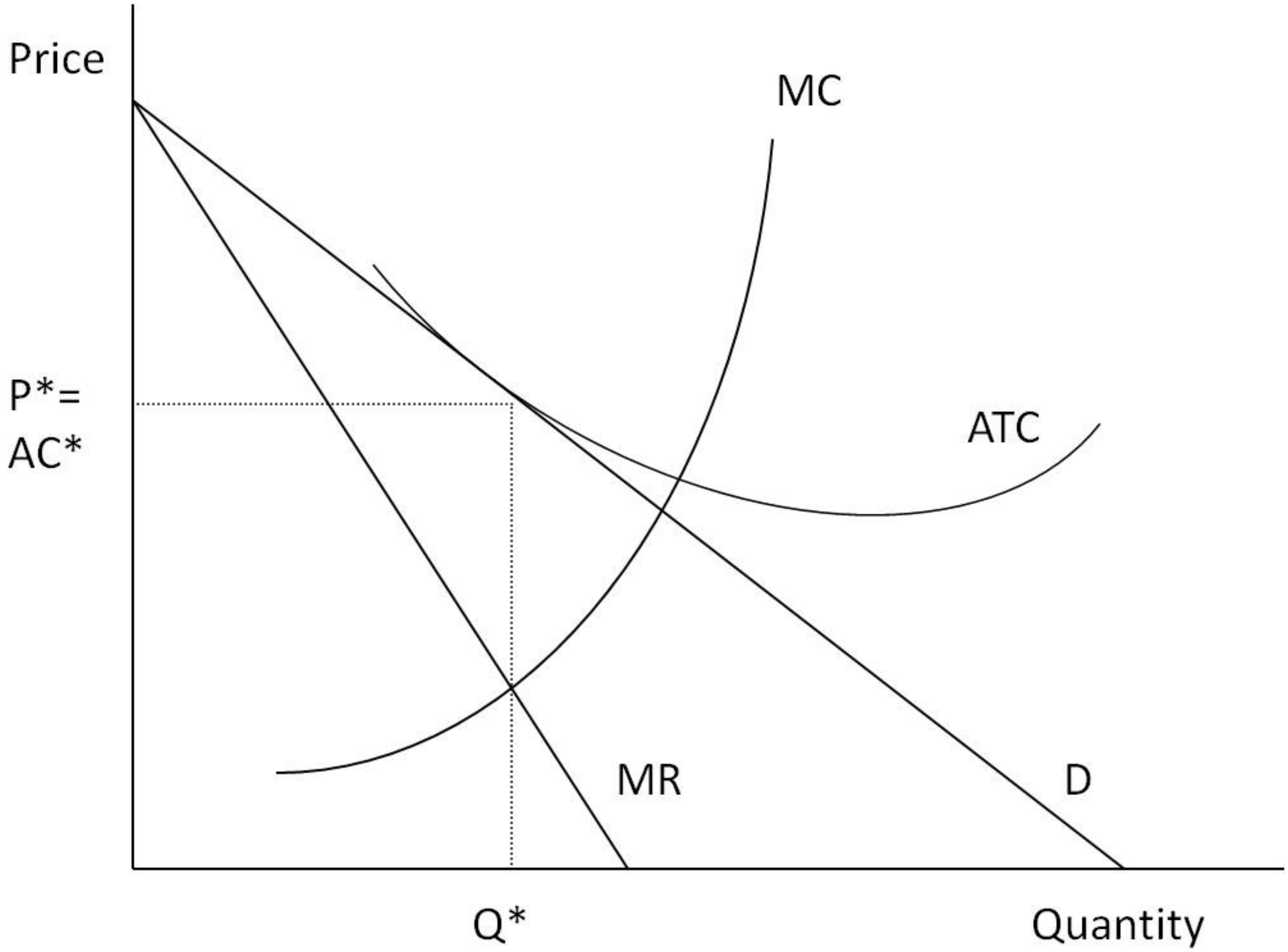

Figure 4: Long run equilibrium for a profit maximizing firm in monopolistic competition.

In the long-run, a firm will exit the market if its profit is negative and positive economic profits will attract new firms. Thus, as in perfect competition, in long-run equilibrium economic profits will be zero. We define a long-run equilibrium by the following two conditions for all firms active in the market:

$$\frac{\partial q_j\left(p_j^*, p_{-j}^*\right)}{\partial p_j}\left[mr\left(q_j\left(p_j^*, p_{-j}^*\right)\right) - mc\left(q_j\left(p_j^*, p_{-j}^*\right)\right)\right] = 0 \quad (36)$$

$$\Pi_j\left(q_j\left(p_j^*, p_{-j}^*\right)\right) = 0 \quad (37)$$

In the short-run, a monopolistically competitive firm choosing the profit maximizing price and quantity will have a picture that looks like the monopolist's picture in Figure 3. However, in the short run a firm in a monopolistically competitive market will earn zero economic profits, and so a picture of a representative firm would look like (again, assuming a linear inverse demand function) the one if Figure 4.

1.2.3 Oligopoly

An oligopoly market is one with a few sellers where there are substantial barriers to entry. Oligopoly theory has undergone radical changes since game theory became a popular tool in economics.[3] Depending upon

[3] If you have seen the movie "A Beautiful Mind" you may remember at the end of the movie when Nash is being told about being considered for the Nobel Prize the man informing him says that his equilibrium has been used in antitrust cases.

whether firms use price or quantity the equilibrium predictions of the market vary dramatically. In a very simple model where firm's choose prices if there are two or more firms in the market then profits are driven down to zero as in the perfectly competitive market. In a model where firms choose quantities the firms can make positive economic profits in equilibrium.

A proper discussion of these models requires some basic game theory which we will not discuss in this class.

2 Equilibrium and welfare

We have discussed how prices and quantity are determined in various market settings. However, we can also evaluate these market structures to determine if one is "better" than the other. To do so we will need a notion of "better", which we will define shortly. We can then consider how we might wish to intervene in a market which is "worse" than others. When intervening there are two questions to ask. The first is how the welfare of each individual is affected. The second is how much weight to give the welfare of each individual.

2.1 Individual welfare

Policies that might be implemented in markets are taxes, subsidies, and price controls. We want to examine how these policies impact welfare. In partial equilibrium analysis we want to focus on the impact of a price change for a single good, holding all other prices constant.

Since the prices of all other goods are held constant, we can write the consumer's indirect utility function as $v(p_j, p_{-j}, y)$ as $v(p_j, y)$ so that we only focus on changes of utility when the price of good j changes. We can then create a composite commodity out of all the other goods that are not good j. This composite commodity, m, is the amount of income spent on all other goods. If $x(p_j, p_{-j}, y)$ is the demand for all other goods then $m(p_j, p_{-j}, y) \equiv p_{-j} * x(p_j, p_{-j}, y)$. We will call this $m(p, y)$. If $u(q, x)$ satisfies the standard assumptions, then $\overline{u}(q, m) = \max_x u(q, x)$ subject to $px \leq m$ also satisfies those assumptions. We then use $\overline{u}(q, m)$ to analyze the consumer's problem considering only to goods m and q. The demand for goods m and q solve:

$$\max_{q,m} \; \overline{u}(q, m) \text{ s.t. } pq + m \leq y \tag{38}$$

where the maximized value of $\overline{u}$ is $v(p, y)$.

Suppose now that the government is going to take steps to lower the cost of some product it provides. However, to fund this one time cost the government will need to impose a tax. The question is whether or not the consumers are willing to bear the burden of the tax in order to receive the good at a lower cost.

Suppose there is a consumer with income y^0. The initial price of the good is p^0 and it will fall to p^1. Let $v(p^0, y^0)$ denote his utility before the price decrease and $v(p^1, y^0)$ the utility afterwards. The only thing that we can change is the consumer's income, so the question is how much income would the consumer be willing to give up (since $v(p^1, y^0) \geq v(p^0, y^0)$). Call the amount of the income change the compensating variation, or CV, and it is the amount of income such that:

$$v(p^0, y^0) = v(p^1, y^0 + CV) \tag{39}$$

In this instance $CV \leq 0$.

We can also look at this problem from the expenditure function. We have:

$$\begin{aligned} e(p^1, v(p^0, y^0)) &= e(p^1, v(p^1, y^0 + CV)) && (40) \\ e(p^1, v(p^0, y^0)) &= y^0 + CV && (41) \end{aligned}$$

Also, $y^0 = e(p^0, v(p^0, y^0))$, so we have:

$$\begin{aligned} e(p^1, v(p^0, y^0)) &= e(p^0, v(p^0, y^0)) + CV && (42) \\ CV &= e(p^1, v(p^0, y^0)) - e(p^0, v(p^0, y^0)) && (43) \end{aligned}$$

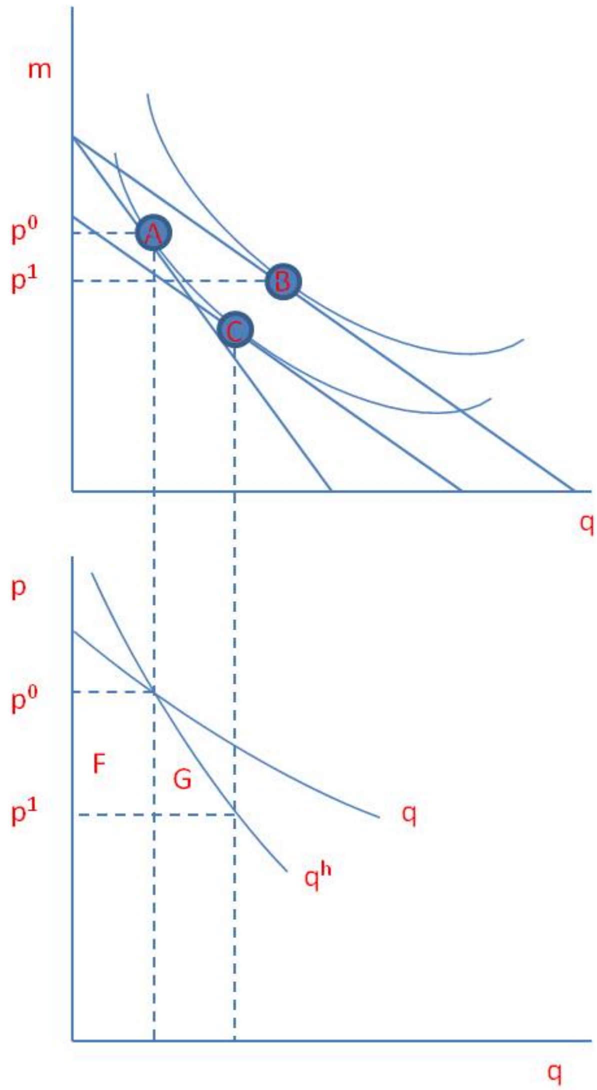

Figure 5: Finding the compensating variation for a consumer after a price decrease in good q.

Since Hicksian demand is the partial of the expenditure function we have:

$$CV = e\left(p^1, v\left(p^0, y^0\right)\right) - e\left(p^0, v\left(p^0, y^0\right)\right) \tag{44}$$

$$CV = \int_0^{p^1} \frac{\partial e\left(p, v\left(p^0, y^0\right)\right)}{\partial p} dp - \int_0^{p^0} \frac{\partial e\left(p, v\left(p^0, y^0\right)\right)}{\partial p} dp \tag{45}$$

$$CV = \int_{p^0}^{p^1} \frac{\partial e\left(p, v\left(p^0, y^0\right)\right)}{\partial p} dp \tag{46}$$

$$CV = \int_{p^0}^{p^1} q^h\left(p, v\left(p^0, y^0\right)\right) dp \tag{47}$$

When $p^1 < p^0$ then CV is the negative of the area to the left of the Hicksian demand curve for base utility level $v\left(p^0, y^0\right)$ and if $p^1 > p^0$ then it is the positive. Figure 5 shows this area as it is labelled by the two letters F and G. It looks a little odd to be taking the area to the left of the curve, but our range of integration is on the y-axis (you can rotate the figure so that p is on the x-axis if that helps).

The problem with using the compensating variation is that it is some area to the left of some Hicksian demand curve which is unobservable. We can however use the relationship between Hicksian and Marshallian demand via the Slutsky equation to get an estimate of the CV. Whenever Marshallian demand depends on income the two demand curves will diverge, which is why this is only an estimate.

Consumer surplus, or CS, is based on Marshallian demand. By definition:

$$CS\left(p^0, y^0\right) = \int_{p^0}^{\infty} q\left(p, y^0\right) dp \tag{48}$$

as consumer surplus is just the area under the demand curve but above the price in the market. If we want to find the change in consumer surplus, ΔCS we simply find the difference between the two consumer surplus measures so that:

$$\begin{aligned}
\Delta CS &\equiv CS\left(p^1, y^0\right) - CS\left(p^0, y^0\right) && (49)\\
\Delta CS &= \int_{p^1}^{\infty} q\left(p, y^0\right) dp - \int_{p^0}^{\infty} q\left(p, y^0\right) dp && (50)\\
\Delta CS &= \int_{p^1}^{p^0} q\left(p, y^0\right) dp && (51)
\end{aligned}$$

So if there is a price decrease then $\Delta CS \geq 0$ whereas if there is a price increase then $\Delta CS \leq 0$.

2.2 Efficiency

In the previous section the cost of the project was less than the benefits received from the project so it was possible to make everyone better off by taking a little income from them through a tax and then implementing the project. Whenever we can make at least one individual better off while leaving the utility of all other individuals unchanged we call this a Pareto improvement. If there is no possible Pareto improvement then the current outcome is Pareto efficient. The primary difference between the market structures is the price and quantity traded they yield in the market. The question is which of these market structures generates a Pareto efficient outcome (or which comes the closest if none do).

2.2.1 Competitive outcome

Consider a market with one consumer and one producer for simplicity. Figure 6 shows the firm's marginal cost curve, $mc(q)$, the consumer's Marshallian demand curve, $q\left(p, y^0\right)$, and the consumer's Hicksian demand curve $q^h\left(p, y^0\right)$. Assume that the firm charges p^0 and the resulting market quantity is q^0. The question now becomes: Is this a Pareto efficient outcome? The answer is no. Consider dropping the price to p^1. The consumer now consumes q^1 and is willing to pay the compensating variation, $A + B$, to obtain this new quantity of q^1. So we can take the amount $A + B$ away from the consumer and the consumer is still as well off as before. The firm however has to produce more units so this changes profit. Let $c(q)$ be the cost of producing q units. We then have that:

$$\begin{aligned}
\Delta\Pi &= \left[p^1 q^1 - c\left(q^1\right)\right] - \left[p^0 q^0 - c\left(q^0\right)\right] && (52)\\
\Delta\Pi &= \left[p^1 q^1 - p^0 q^0\right] - \left[c\left(q^1\right) - c\left(q^0\right)\right] && (53)\\
\Delta\Pi &= \left[p^1 q^1 - p^0 q^0\right] - \int_{q^0}^{q^1} mc(q)\, dq && (54)
\end{aligned}$$

Looking at the figure, we have that

$$\begin{aligned}
\Delta\Pi &= C + D + E + F - [A + E + F] - D && (55)\\
\Delta\Pi &= C - A && (56)
\end{aligned}$$

So we can take A from the consumer and give it to the producer, and the producer will have a positive change in profit to C, while giving back B to the consumer, so that the consumer has a positive change in payoff of B. Since we can find an outcome where both are better off, the original outcome was NOT Pareto efficient. The only point at which we achieve Pareto efficiency is at the competitive equilibrium outcome where market supply intersects market demand.

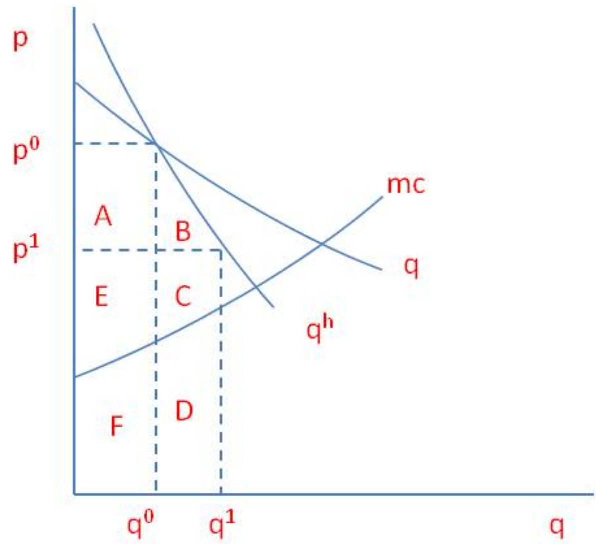

Figure 6: Why the monopoly outcome is inefficient

2.2.2 Total surplus maximization

Consumer surplus is close to being a dollar measure of gains to the consumer as a result of purchasing the good. If there are no income effects then consumer surplus is an exact measure of those gains. For the producer we have the concept of producer surplus, which is simply the firm's revenue over and above its variable cost (if fixed costs are zero then producer surplus is profit). If wealth effects were nonexistent then we would know that to obtain an efficient outcome we would simply maximize the total surplus. However, even if wealth effects are present as long as demand is downward sloping and marginal costs are increasing then we will only achieve efficiency if the sum of consumer and producer surplus is maximized.

Consider the total surplus $CS(q) + PS(q)$:

$$CS(q) + PS(q) = \left[\int_0^q p(z)\,dz - p(q)\,q\right] + p(q)\,q - c(q) + F \tag{57}$$

$$CS(q) + PS(q) = \int_0^q p(z)\,dz - c(q) + F \tag{58}$$

where $p(q)$ is the inverse demand curve, q is the quantity, $c(q)$ is the firm's total cost, and F is the amount of fixed costs. Well, $c(q) = vc(q) + F$, so that $-c(q) + F = -vc(q) - F + F = -vc(q)$. In a sense fixed costs are "lost" and so we are not concerned with how they enter into surplus. Rewriting we have:

$$CS(q) + PS(q) = \int_0^q p(z)\,dz - \int_0^q mc(z)\,dz \tag{59}$$

$$CS(q) + PS(q) = \int_0^q (p(z) - mc(z))\,dz \tag{60}$$

Maximizing that expression simply leads to choosing the price at which quantity supplied equals quantity demanded, or the competitive outcome.