Games with a continuum of strategies*

1 Introduction

The games we have considered thus far have focused on strategy sets that are discrete, and in most cases there have been a finite amount of choices from which a player could choose. The discussion now turns to games where players can choose from a continuum of strategies. Hopefully it is obvious that writing down the normal form version of the game would be impossible, and so even when games are simultaneous we will use the extensive form version to represent them. The games we will focus on in this section will be price-setting games and quantity choice games.

2 Pricing games (Bertrand game)

When firms choose prices simultaneously this is known as a Bertrand game after its "inventor".[1] We will discuss both the simultaneous and the sequential game.

2.1 Simultaneous Price Choices

In this model there are two competing firms. The firms produce homogeneous products and compete by simultaneously choosing prices. Let p_1 be the price of Firm 1 and p_2 be the price of Firm 2. Note that $p_1 \in [0, \infty)$ and $p_2 \in [0, \infty)$ (alternatively we can write $p_1 \in \mathbb{R}_+$ and $p_2 \in \mathbb{R}_+$ if we wish to look more sophisticated). This is our continuum of strategies. It should be obvious that this is not easily representable by a normal form game. We can represent this as an extensive form game as in Figure 1. The continuum of strategies is represented by the dashed line between the two "extreme" strategies. Note that Firm 2 only has one information set in the simultaneous game and we represent this by circling the entire continuum of strategies for Firm 1. Note that we do not list any payoffs because it is quite difficult to list the payoffs.

There is a demand function for the good given by $x(p)$, where $x(\cdot)$ is continuous and strictly decreasing at all p where $x(p) > 0$. There exists $\bar{p} < \infty$ such that $x(p) = 0$ for all $p \geq \bar{p}$ (if price is too high then there is no demand). Note that this does not explicitly rule out the possibility that firms choose $p = \infty$, which is why a choice of $p = \infty$ is still in each firm's available action space. Assume that the 2 firms are identical and face constant marginal cost of c. There is a socially optimal level of production $x(c) \in (0, \infty)$. Sales for firm j are given by:

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{cases}$$

Thus, firm j is the only seller in the market if its price is less than its competitor's, firm j and firm k split the market evenly if they choose equal prices, and firm j sells nothing if its price is greater than its competitors. This is a produce to order market, so costs are only incurred on when units are actually sold. For a given p_j and p_k, firm j's profit is

$$(p_j - c) x_j(p_j, p_k).$$

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*Based on Chapters 1 and 2 of Gibbons (1992).

[1] The citation for those of you who speak French is Bertrand, J. (1883) Theorie mathematique de la richesse sociale. *Journal des Savants* 67: 499-508.
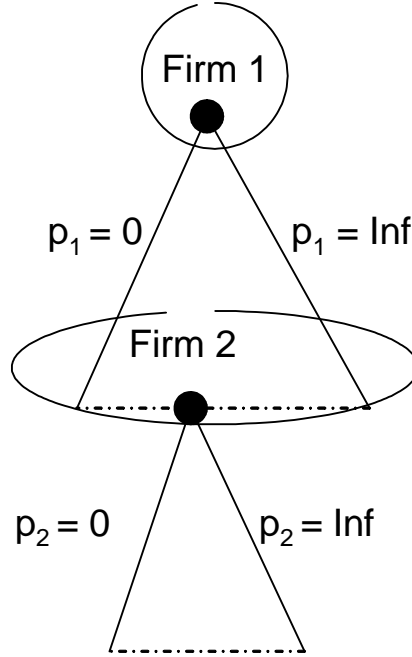
Figure 1: Extensive form representation of the simultaneous Bertrand game

Consider a one-shot game of the Bertrand model. There is a unique NE to this game, where $p_j^* = p_k^* = c$. Note that profits are equal to zero under this proposed strategy because price equals marginal cost. First ask whether any firm would wish to deviate unilaterally from this proposed strategy. If firm j chooses a price less than c while firm k chooses c, then firm j captures the entire market, which is good, but is now charging a price less than marginal cost, which is bad because its profit is now negative. Thus, if one firm chooses a price equal to c, the other has no incentive to decrease price. Firm j also has no incentive to charge a higher price than c if firm k is charging c, because firm j would still earn zero profit, only now firm j would earn zero profit because it sells nothing. Thus, there is no incentive for either firm to deviate from this proposed strategy so it is a NE.

As for uniqueness, we know that neither firm will choose a price below c. This leaves 3 cases:

Case 1: Both firms choose the same price that is greater than c, or $p_j = p_k = \widetilde{p} > c$. This is not a NE. Each firm receives half of the market at price $\widetilde{p}$, but either firm could do better by charging a slightly lower price, $\widetilde{p} - \varepsilon$, and capturing the whole market.

Case 2: Both firms choose a price strictly greater than marginal cost, but one firm chooses a price strictly (but only slightly) greater than the other firm, or $p_j > p_j - \varepsilon = p_k > c$. This is not a NE. Firm j would wish to change its price, as it could capture the entire market by choosing $p_j - \varepsilon - \varepsilon$. It is also possible to argue that firm k would wish to change its price UPWARD if there is some price $p_j - \phi$ such that $p_j > p_j - \phi > p_j - \varepsilon$.

Case 3: One firm chooses a price strictly greater than marginal cost while the other firm chooses a price equal to marginal cost, or $p_j > p_k = c$. We have already seen that firm j can do no better by choosing a different price. However, given that the action space is continuous (prices are chosen from the positive real numbers), there must exist a price $p_j - \varepsilon$ for $\varepsilon \in \mathbb{R}_{++}$ such that $p_j > p_j - \varepsilon > p_k = c$ for some arbitrarily small ε. Thus, firm k would wish to change its price UPWARD to shift from earning zero profits to earning positive profits, which puts us right back at Case 2.

As you can see, with only 2 firms competing in Bertrand competition (at least this version) the competitive outcome is achieved and firms are earning zero economic profit. Intuitively this does not seem logical as we might think that if there are only 2 firms in a market they should earn some positive economic profit. It is possible to modify the Bertrand model in ways that removes this problem – we are assuming here that products are perfect substitutes and that firms can serve the entire market at any price level. If either is
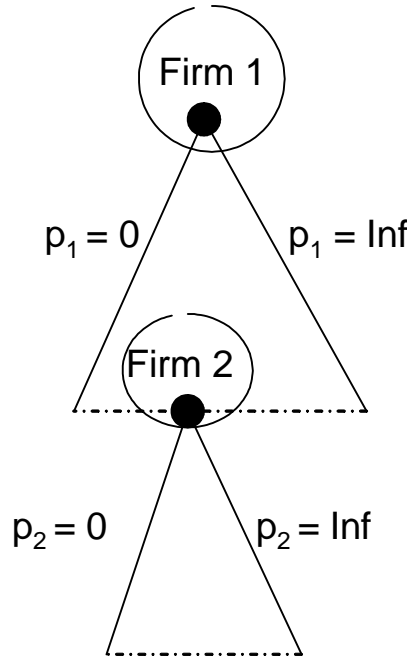
Figure 2: Extensive form representation of the sequential Bertrand game

removed (firms produce differentiated products or are capacity constrained) then the competitive market outcome disappears.

2.2 Sequential Price Choices

We will use the same structure as the simultaneous Bertrand game except that we will assume that Firm 2 observes Firm 1's pricing decision and then will make its own pricing decision. To be clear, I don't think that Bertrand considered the sequential game in his analysis. He certainly didn't contemplate SPNE since this was not "invented" and popularized until much later. The extensive form version of this game is in Figure 2. Note that the only difference between Figure 1 and Figure 2 is in the information set for Firm 2. When there is a continuum of strategies and the game is sequential the convention is to simply place a circle around the second mover's decision node (not including the extreme strategies of the first mover) to denote the sequential nature of the game.

Determining the subgame perfect Nash equilibrium to this game is fairly straightforward even if it is a little more involved than the previous sequential games we have considered. Recall that a strategy for Firm 2 involves specifying an action for every possible decision node Firm 2 can possibly observe. The best way to do this is not to consider individual prices but price ranges. We can look at the following ranges for p_1.

Case 1: Firm 1 chooses a price below marginal cost or $p_1 \in [0, c)$.

Case 2: Firm 1 chooses a price equal to marginal cost or $p_1 = c$.

Case 3: Firm 1 chooses a price between marginal cost and the monopoly price or $p_1 \in (c, p^M]$.

Case 4: Firm 1 chooses a price greater than the monopoly price or $p_1 \in (p^M, \infty)$.

Consider Case 1 with Firm 1 choosing a price below marginal cost. If this is the case then Firm 2 certainly does not want to match or beat Firm 1's price as its profit would be negative. So Firm 2's only option is to choose a price greater than p_1 if $p_1 < c$. Since this leads to a payoff of 0 for Firm 2, ANY price above p_1 is a best response to p_1 if $p_1 < c$. Thus, there is a best response correspondence. For simplicity, let's say that if $p_1 \in [0, c)$ Firm 2 will choose $p_2 = c$. We could have Firm 2 choose $p_2 = p_1 + \varepsilon$ or $p_2 = p^M$. Note that this already leads to multiple SPNE.

Consider Case 2 with Firm 1 choosing price equal marginal cost. Firm 2 will NOT choose $p_2 < c$ as this will yield negative profit. If Firm 2 chooses $p_2 = c$ then it earns 0 and if Firm 2 chooses $p_2 > c$ it still earns

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0. So Firm 2 can choose any price from $[c, \infty)$ and it will earn 0. Again, for simplicity, say that if Firm 1 chooses $p_1 = c$ then Firm 2 chooses $p_2 = c$.

Consider Case 3 with Firm 1 choosing a price above marginal cost but below the monopoly price. For any price choice by Firm 1 above marginal cost but below the monopoly price Firm 2's best response is to choose a price slightly lower than Firm 1's price. We can write this as $p_2 = p_1 - \varepsilon$, where ε is some small positive number. Note that in this portion of Firm 2's strategy Firm 2's action is unique as there is a well-defined best response to any pricing decision by Firm 1.

Consider Case 4 with Firm 1 choosing a price above the monopoly level. At first glance it seems that for any $p_1 > c$ Firm 2 should simply undercut Firm 1 by a small amount but this is incorrect. The monopoly price yields the maximum amount of profit in the market, so if Firm 1 ever chooses a price above the monopoly price Firm 2 should simply choose the monopoly price. This is easily seen if Firm 1 chooses $p_1 > \bar{p}$, where $\bar{p}$ is the maximum price that ANY consumer is willing to pay. If Firm 1 makes this choice, it is obvious that Firm 2 can do much better than choosing $p_1 - \varepsilon$ because choosing $p_1 - \varepsilon$ yields 0 profit for Firm 2 while other price choices would yield positive profit. So Firm 2's strategy as we have defined it is:

$$p_2^* = \begin{cases} c & \text{if } p_1 \in [0, c) \\ c & \text{if } p_1 = c \\ p_1 - \varepsilon & \text{if } p_1 \in \left(c, p^M\right] \\ p^M & \text{if } p_1 \in \left(p^M, \infty\right) \end{cases}$$

We could have condensed the first two portions since Firm 2 is using the same strategy. Now, what should Firm 1 do? Well, it does not matter what Firm 1 does as long as $p_1 \geq c$ because Firm 1 will always earn 0 profit given this strategy by Firm 2. The only time Firm 1 will not earn 0 profit is when $p_1 < c$, and then Firm 1 earns negative profit, so it will not choose $p_1 < c$. We could say that Firm 1 will choose $p_1 = c$ because it wants to participate in the market,[2] but if Firm 1 chooses $p_1 = p^M + \varepsilon$ this yields just as much profit to Firm 1. But we need to choose something for Firm 1, so let's say Firm 1 chooses $p_1 = p^M + \varepsilon$. Thus, a subgame perfect Nash equilibrium to this game is:

$$p_1^* = p^M + \varepsilon$$
$$p_2^* = \begin{cases} c & \text{if } p_1 \in [0, c) \\ c & \text{if } p_1 = c \\ p_1 - \varepsilon & \text{if } p_1 \in \left(c, p^M\right] \\ p^M & \text{if } p_1 \in \left(p^M, \infty\right) \end{cases}.$$

Again, note that this is not the only SPNE to the game. Another potential one is:

$$p_1^* = p^M + \varepsilon$$
$$p_2^* = \begin{cases} p^M & \text{if } p_1 \in [0, c) \\ p^M & \text{if } p_1 = c \\ p_1 - \varepsilon & \text{if } p_1 \in \left(c, p^M\right] \\ p^M & \text{if } p_1 \in \left(p^M, \infty\right) \end{cases}.$$

Note that the difference in this SPNE is that Firm 2 is choosing different actions when $p_1 \in [0, c]$. The typical immediate response is "Well then Firm 1 should choose a price just less than the monopoly price" but then notice that if Firm 1 chooses that strategy then Firm 2 will choose $p_1 - \varepsilon$. So this set of strategies is also a SPNE to the game. It is also possible to obtain the competitive outcome if Firm 1 chooses $p_1 = c$. The point is there are an infinite number of SPNE to the game. However, there are only a few general types of outcomes if Firm 2 uses a strategy of this type: Firm 1 makes negative profit and Firm 2 makes 0 profit (when $p_1 < c$ – which should not happen since a $p_1 < c$ is strictly dominated by $p_1 = c$), both Firms make 0 profit (when $p_1 = c$), and Firm 2 makes a positive profit while Firm 1 makes 0 profit (when $p_1 > c$). As this game shows, it does not always pay to be the first mover in a game.

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[2] It might also be the case that Firm 1 chooses $p_1 = c$ because this will cause Firm 2 to also receive 0 profit. But now we are getting into adding features into the utility function, and if we assume that the payoffs accurately reflect player utility then there is no rationale for Firm 1 to feel this way towards Firm 2.

3 Quantity choice games

An alternative to the price choice game is the quantity choice game or the Cournot game named after its "inventor".[3] Cournot was one of the earliest game theorists, although he was basically neglected for 40-50 years. Bertrand was one of the first to critique Cournot, and his work appeared in 1883. We will see some of the differences between the outcomes of Cournot and Bertrand. But he did influence some of the individuals responsible for the Marginal Revolution (Walras and Jevons). The Marginal Revolution shifted the focus of economics to *marginal* analysis.

3.1 Simultaneous quantity choices

Again, we consider a market with two firms. These 2 firms compete by simultaneously choosing quantity levels. This is also a one-shot game. There are two symmetric firms with constant marginal cost of c. There is an inverse market demand function, $p(Q)$, where $Q = q_1 + q_2$. The function $p(\cdot)$ is differentiable, with $p'(q) < 0$ at all $q \geq 0$. We also have $p(0) > c$ (so that a market exists) and a unique output level $q^0 \in (0, \infty)$ such that $p(q^0) = c$. Firm j's problem is to maximize profit conditional on the output of the other firm.

$$\max_{q_j \geq 0} \ p(q_j + \overline{q_k}) q_j - cq_j$$

This maximization problem has the following first-order condition:

$$p'(q_j + \overline{q_k}) q_j + p(q_j + \overline{q_k}) \leq c, \text{ with equality if } q_j > 0$$

For each $\overline{q_k}$, let $b_j(\overline{q_k})$ denote firm j's choice of quantity. Thus $b_j(\cdot)$ is firm j's best response correspondence. To find b_j simply solve the above equation for q_j. There is one slight modification – it is possible that firm j's best response to firm k is to choose a quantity less than 0. If that is the case, then firm j should choose 0. A pair of quantity choices (q_1^*, q_2^*) is a NE if and only if $q_j^* \in b_j(q_k^*)$ for $k \neq j$ and $j = 1, 2$. Thus, the following need to hold for firm 1 and firm 2:

$$p'(q_1^* + q_2^*) q_1^* + p(q_1^* + q_2^*) \leq c$$
$$p'(q_1^* + q_2^*) q_2^* + p(q_1^* + q_2^*) \leq c$$

We will argue from intuition that $q_1^* > 0$ and $q_2^* > 0$ so that these equations hold with equality. If $q_1^* = 0$, then q_2^* should be the monopoly quantity. But if q_2^* is the monopoly quantity, then firm 1 can produce some small amount and make positive profit. Thus, $q_1^* > 0$ and $q_2^* > 0$, and these equations hold with equality. You should verify the intuitive argument mathematically for practice.

We can show that price is greater than marginal cost by adding the two equations above to get:

$$p'(q_1^* + q_2^*) \left(\frac{q_1^* + q_2^*}{2} \right) + p(q_1^* + q_2^*) = c$$

Since $p'(q_1^* + q_2^*) \left(\frac{q_1^* + q_2^*}{2} \right) < 0$, we must have $p(q_1^* + q_2^*) > c$.

I have not provided diagrams of the extensive form version of this game since it is identical to the extensive form version of the simultaneous price choice game (Figure 1) except that the actions are now labeled with q_1 and q_2 instead of p_1 and p_2.

3.1.1 Linear inverse demand

Now, suppose that $p(Q) = a - bQ$, where $Q = q_j + q_k$. Firms still have constant marginal cost of c, with $a > c \geq 0$ and $b > 0$. We can find firm j's best response function either by solving the maximization problem directly or by using the previous results. We know that firm j's best response function can be found by solving the following equation for q_j:

$$p'(q_j + \overline{q_k}) q_j + p(q_j + \overline{q_k}) = c$$

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[3] Cournot, A. (1838). Recherches sur les Principes Mathematiques de la Theorie des Richesses.

Apparently there is an English version, Researches into the Mathematical Principles of the Theory of Wealth, edited by N. Bacon, London, Macmillan, 1897.

Substituting in for $p'(q_j + \overline{q_k})$ and $p(q_j + \overline{q_k})$ we have:

$$-bq_j + a - b(q_j + \overline{q_k}) = c$$

Solving for q_j we have:

$$q_j = \frac{a - b\overline{q_k} - c}{2b} \text{ or } q_j = \frac{a - c}{2b} - \frac{1}{2}\overline{q_k}$$

Note that if $\overline{q_k} > \frac{a-c}{b}$ then $q_j < 0$. This makes sense on an intuitive level when you realize that $\frac{a-c}{b}$ is the socially optimal quantity where $p(Q) = c$. Thus, if firm k produces more than the socially optimal quantity, firm j would want to produce a negative quantity to "remove" units from the market and bring the price back up to c. But since firm j cannot produce less than 0 units, then firm j will choose to produce 0 units if $\overline{q_k} > \frac{a-c}{b}$. Thus, firm j's best response function is $b_j(q_k) = Max\left[0, \frac{a - bq_k - c}{2b}\right]$. Firm k has a similar best response function, $b_k(q_j) = Max\left[0, \frac{a - bq_j - c}{2b}\right]$. One other useful piece of information is that in this setup the monopoly quantity is $\frac{a-c}{2b}$, so that if $q_k = 0$ then $q_j = \frac{a-c}{2b}$. Again, this conforms with intuition because if one firm chooses not to produce any units the other should choose to produce the amount of units that it would produce if it was a monopolist.

Since these best responses are functions we can depict them graphically. This graph is in Figure 3.1.1. Figure 3.1.1 assumes $a = 4500$, $b = 1$, and $c = 0$.
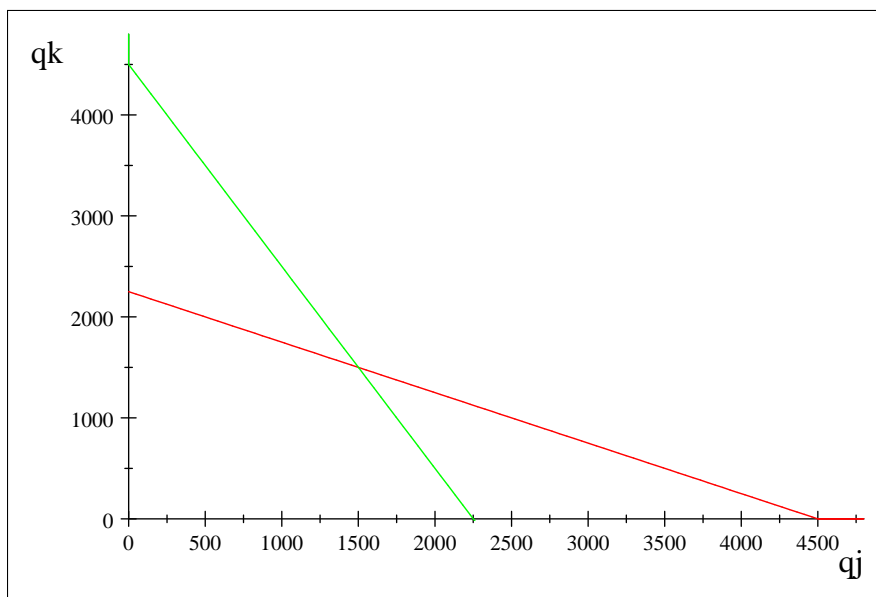
Best response functions for Cournot game.

The green line shows the best response function for firm j while the red line shows the best response function for firm k. The intersection point of the two best response functions is the NE for this game. To find this point, simply solve the system of equations for the two best response functions. We ignore the zero portion of $b_j(q_k) = Max\left[0, \frac{a - bq_k - c}{2b}\right]$ and $b_k(q_j) = Max\left[0, \frac{a - bq_j - c}{2b}\right]$ because neither firm will choose quantity greater than $\frac{a-c}{b}$ (note that choosing a quantity greater than this level is strictly dominated by choosing a quantity of 0 because if $q_i > \frac{a-c}{b}$ then profit for Firm i is negative).

$$q_j^* = \frac{a - bq_k^* - c}{2b}$$
$$q_k^* = \frac{a - bq_j^* - c}{2b}$$

Or:

$$2bq_j^* = a - b\left(\frac{a - bq_j^* - c}{2b}\right) - c$$
$$4bq_j^* = 2a - a + bq_j^* + c - 2c$$
$$3bq_j^* = a - c$$
$$q_j^* = \frac{a - c}{3b}$$

To find q_k^*:

$$q_k^* = \frac{a - b\left(\frac{a-c}{3b}\right) - c}{2b}$$
$$2bq_k^* = a - \left(\frac{a - c}{3}\right) - c$$
$$6bq_k^* = 3a - a + c - 3c$$
$$6bq_k^* = 2a - 2c$$
$$q_k^* = \frac{a - c}{3b}$$

Thus, the NE for this game is a pair of quantities $\left(q_j^*, q_k^*\right) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)$. Note that the total quantity in the market, Q, is equal to $\frac{2}{3}\frac{a-c}{b}$, which is greater than the monopoly quantity, $\frac{1}{2}\frac{a-c}{b}$, but less than the quantity from the purely competitive outcome, $\frac{a-c}{b}$. Thus, the firms in Cournot competition produce between the monopoly and the competitive level, which seems like a more intuitive result than the one we found in Bertrand competition.

The price in the market when this NE is played is $\frac{a+2c}{3}$. Profit to each firm is $\frac{(a-c)^2}{9b}$.

3.2 Sequential quantity choice

Now, consider the case of 2 firms that compete by choosing quantity levels, but one firm makes an observable quantity choice before the other. This is also a one-shot game and is due to von Stackelberg.[4] There are two symmetric firms with constant marginal cost of c. There is an inverse market demand function, $p(Q)$, where $Q = q_1 + q_2$. The function $p(\cdot)$ is differentiable, with $p'(q) < 0$ at all $q \geq 0$. We also have $p(0) > c$ (so that a market exists) and a unique output level $q^0 \in (0, \infty)$ such that $p\left(q^0\right) = c$. Let firm j be the first-mover and firm k be the second-mover. Again, the diagram of the extensive form game is similar to Figure 2 with q's replacing the p's.

Now, as for solving the game it is fairly similar. The first question to ask is what constitutes a strategy (in the Stackelberg game) for each player. The second-mover will have to specify the quantity he will produce given any quantity choice by the first-mover. Thus, the second-mover will need to have a best response function just like in the Cournot model. The first-mover will not need to have a best response function. The first-mover makes one decision – what quantity level do I choose? Thus, while the second-mover's strategy is a best response function, a first-mover's strategy is simply a quantity choice.

Solving the game we work backwards. Consider the second-mover's decision. The second-mover needs to specify a quantity choice for any decision made by the first-mover. This is the same problem as the Cournot problem – hold the first-mover's quantity choice constant and then maximize profit based on that quantity choice.

$$\max_{q_k \geq 0} \ p\left(\overline{q_j} + q_k\right) q_k - cq_k$$

This yields the first-order condition:

$$p'\left(\overline{q_j} + q_k\right) q_k + p\left(\overline{q_j} + q_k\right) \leq c, \text{ with equality if } q_k > 0.$$

[4]Now you get to read German.
von Stackelberg, H. (1934). Marktform und Gleichgewicht.

Note that this is the same first-order condition as we had in the Cournot problem. By a similar argument to the one made in the Cournot case we can assume that $q_k > 0$, so the first-order condition holds with equality. We can then specify a general best response function $b_k(\overline{q_j})$ which represents the quantity that firm k will produce given that firm j produces $\overline{q_j}$.

Now, firm j need only make a single quantity decision. Firm j will take into consideration firm k's best response function when making its decision, so that firm j's profit maximization problem is:

$$\max_{q_j \geq 0} \; p(q_j + b_k(q_j)) q_j - cq_j.$$

Thus, firm j now has the first-order condition:

$$p'(q_j + b_k(q_j)) b_k'(q_j) q_j + p(q_j + b_k(q_j)) \leq c, \text{ with equality if } q_j > 0.$$

Note that this first-order condition is different than the one in the Cournot model because firm j is now explicitly incorporating firm k's best response function into its profit function.

3.2.1 Linear inverse demand

Now, suppose that $p(Q) = a - bQ$, where $Q = q_j + q_k$. Firms still have constant marginal cost of c, with $a > c \geq 0$ and $b > 0$. Firm k's best response function in the Stackelberg game is identical to its best response function in the Cournot game, so:

$$b_k(\overline{q_j}) = Max\left[0, \frac{a - b\overline{q_j} - c}{2b}\right].$$

Again, recall that Firm j will then explicitly incorporate this best response function into its maximization problem. We focus on the part of the best response function where $\frac{a - b\overline{q_j} - c}{2b} > 0$. The reason for this is that firm k will only choose from the 0 portion of its best response function if firm j chooses $q_j > \frac{a-c}{b}$. Firm j will not choose $q_j > \frac{a-c}{b}$ because this will lead to a negative profit. Firm j's maximization problem is then:

$$\max_{q_j \geq 0} \; \left(a - b\left(q_j + \frac{a - bq_j - c}{2b}\right)\right) q_j - cq_j.$$

This yields the first-order condition:

$$a - 2bq_j - \frac{a}{2} + bq_j + \frac{c}{2} - c \leq 0, \text{ with equality if } q_j > 0.$$

We can easily check if $q_j > 0$ by imposing equality and determining whether or not profit is greater than or equal to zero. If profit is greater than or equal to zero when $q_j > 0$, then this is at least as good for the firm as when $q_j = 0$. Solving the first order condition gives:

$$\frac{1}{2}\frac{a - c}{b} = q_j.$$

Note that this is the monopoly quantity when the inverse demand function is linear. So an SPNE to this game is:

$$\left(q_j^*, b_k^*(\overline{q_j})\right) = \left(\frac{1}{2}\frac{a - c}{b}, Max\left[0, \frac{a - b\overline{q_j} - c}{2b}\right]\right).$$

The outcome from this SPNE is that firm j produces $q_j = \frac{1}{2}\frac{a-c}{b}$ and firm k produces $q_k = \frac{1}{4}\frac{a-c}{b}$, with $p(Q) = \frac{a+3c}{4}$ and $\pi_j = \frac{a+3c}{4}\frac{a-c}{2b} - c\frac{a-c}{2b} = \frac{(a-c)^2}{8b}$ and $\pi_k = \frac{a+3c}{4}\frac{a-c}{4b} - c\frac{a-c}{4b} = \frac{(a-c)^2}{16b}$. Note that firm j makes twice as much profit as firm k since firm j produces twice as much as firm k. In comparison with the Cournot outcome, note that market quantity in the Cournot model is $\frac{2}{3}\frac{a-c}{b}$ while market quantity in the Stackelberg model is $\frac{3}{4}\frac{a-c}{b}$. Thus, consumers are better off in the Stackelberg model than in the Cournot model because prices are lower ($\frac{a+3c}{4}$ versus $\frac{a+2c}{3}$).