PSNE and MSNE questions

Game Theory & Experiments, ECON 6206-001

January 15, 2013

Directions: Answer all questions as completely as possible. You may work in pairs on the assignment. If you do so, simply turn in one assignment per pair with both names on the assignment.

1. Consider the game represented by the following matrix:

		Player B		
		L	M	R
Player A	U	$2,1$	$1,1$	$0,0$
	C	$1,2$	$3,3$	$2,1$
	D	$2,-2$	$1,0$	$-1,-1$

   Find all pure and mixed strategy Nash equilibria for this game.

2. Suppose that there are 2 hunters, Fred and Barney. The hunters can go after big game or small game. If a hunter goes after small game then he catches small game for a payoff of 1. If he goes after big game and he hunts alone he fails to catch anything, for a payoff of 0. However, if a hunter hunts for big game and both hunters are hunting big game, then they have a hunting party and catch the big game for a payoff of 3 each.

   a Write down the normal form version of this 2-player game.

   b What is (are) the pure strategy Nash equilibria (PSNE) of this game?

   c Is there a mixed strategy Nash equilibrium to this game? If so find it.

   Now suppose that there are 3 hunters: Fred, Barney, and Bamm-Bamm. If each goes after small game then each gets a payoff of 1. If one hunter goes after big game and the other two go after small game, then the small game hunters receive a payoff of 1 and the big game hunter receives a payoff of 0. If 2 hunters hunt big game and 1 hunter hunts small game, the 2 hunters who hunted big game each receive a payoff of 3, while the hunter who hunted small game receives a payoff of 5 (there is some big game left over and the big game hunters share it with the small game hunter). If all 3 hunters hunt big game, then all 3 hunters receive a payoff of 3.

   d What is (are) the PSNE of this game?

   e Suppose that there are $N > 3$ hunters. If they all hunt small then they all receive a payoff of 1. If at least 2 of the hunters hunt big game, then those who hunt big game receive a payoff of 3 and the ones who hunt small game receive a payoff of 5. If only 1 hunter hunts big game then that hunter receives a payoff of 0 while the other hunters (who are all hunting small game) receive a payoff of 1. Qualitatively describe (how many hunt big game, how many hunt small game) the PSNE for the game with an unknown number of hunters, N.

3. Find all pure and mixed strategy Nash equilibria to the following game. If there are none of either type explain why there are none:

		Player 2				
		F	G	H	I	J
Player 1	A	$18,11$	$9,12$	$6,1$	$8,0$	$7,1$
	B	$6,6$	$7,8$	$5,7$	$9,11$	$4,5$
	C	$9,0$	$4,5$	$14,4$	$4,10$	$5,16$
	D	$3,4$	$3,6$	$2,3$	$6,7$	$1,9$
	E	$0,0$	$4,2$	$7,1$	$7,4$	$8,6$

4. Consider a three player game where the three players all simultaneously choose whether to use Beta or VHS. If all three choose Beta, then all three receive a payoff of 1. If all three choose VHS, then all three receive a payoff of 1. However, if there is any player who chooses differently than the other two then all three players receive 0. A normal form representation of the game is here:

		Player 2	
		Beta	VHS
Player 1	Beta	$1,1,1$	$0,0,0$
	VHS	$0,0,0$	$0,0,0$

↖ Beta

↖ Player 3

		Player 2	
		Beta	VHS
Player 1	Beta	$0,0,0$	$0,0,0$
	VHS	$0,0,0$	$2,2,2$

VHS ↗

↗

Note that Player 3 is essentially choosing which of the two matrices to play, the one on the left (if he chooses Beta) and the one on the right (if he chooses VHS). The payoffs are listed as: row player, column player, matrix choice player (or Player 1, Player 2, and Player 3 in this example).

a Find both pure strategy Nash equilibria in this game.

b Find the mixed strategy Nash equilibrium to this game.

c Suppose now there are four players, and the payoffs are such that if all 4 coordinate on Beta then they all receive 0, if all 4 coordinate on VHS then they all receive 2, and if there is any miscoordination then all players receive 0. What are the **pure strategy** Nash equilibria in the four player game? Do they differ from those in the three player game (and if they differ how do they differ)?

5. There are 5 players each with 5 tokens. The players will play a simultaneous game. Players can either keep the tokens for themselves or contribute tokens to the pot. Any tokens kept by the player pay the player who kept the tokens, and only that player, \$1 each. So if a player keeps 4 tokens he receives \$4, but the other players receive nothing from those 4 tokens. The tokens in the pot are counted and every player in the group receives \$0.5 for each token in the pot. So, if there are 6 tokens in the pot then each player receives a payout of \$3 from the pot, regardless of how many he or she contributed. Thus the player's total payoff is \$1 for every token kept plus \$0.5 times every token in the pot.

a Do the players have a strictly or weakly dominant strategy? Is so, what is the strictly or weakly dominant strategy? If not, explain why not.

b Find a pure strategy Nash Equilibrium to this game.

Now, consider the case of those 5 people, each with their 5 tokens, attempting to privately finance a library. The pot can now be thought of as the library fund. If the total contribution to the pot is greater than or equal to 10 tokens, then the library is funded and if there are less than 10 tokens then the library is not funded. If the library is funded then each player receives a benefit of \$15 from the library, regardless of how many tokens that player contributed. In addition to that \$15, any remaining tokens that they have also add \$1 to their benefit. So if the library is funded and a player kept 5 tokens, his payoff would be \$20. If the library is not funded then players only receive a benefit equal to the amount of tokens that they kept.

c Find 2 pure strategy Nash equilibria to this game – one where the library is funded and another where it is not.