

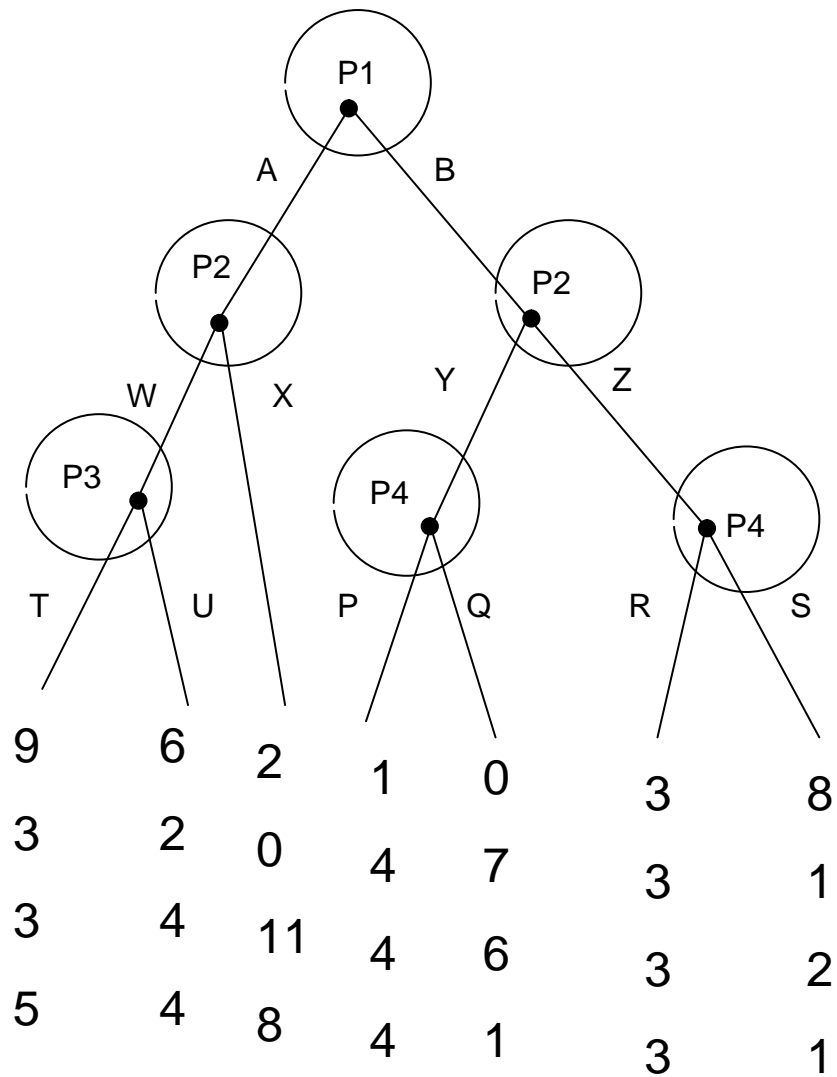
# Assignment 2

ECON6206, Game Theory and Experiments

January 29, 2013

**Directions:** Answer all questions as completely as possible. You may work in pairs on the assignment. If you do so, simply turn in one assignment per pair with both names on the assignment.

1. Consider the following game tree with 4 players:



Game 1

Note that the payoffs are listed in order, so the first payoff is for Player 1, the second for Player 2, etc.

- a How many subgames are in this extensive form game (not including the entire game as a subgame, even though it meets the properties of a subgame)?
- b How many strategies does each player have?
- c Find the subgame perfect Nash equilibrium to this extensive form game.

2. Consider the following game:

		Player 2		
		Low	Medium	High
Player 1	Low	7,5	2,7	1,1
	Medium	2,0	0,2	5,0
	High	8,1	3,4	6,2

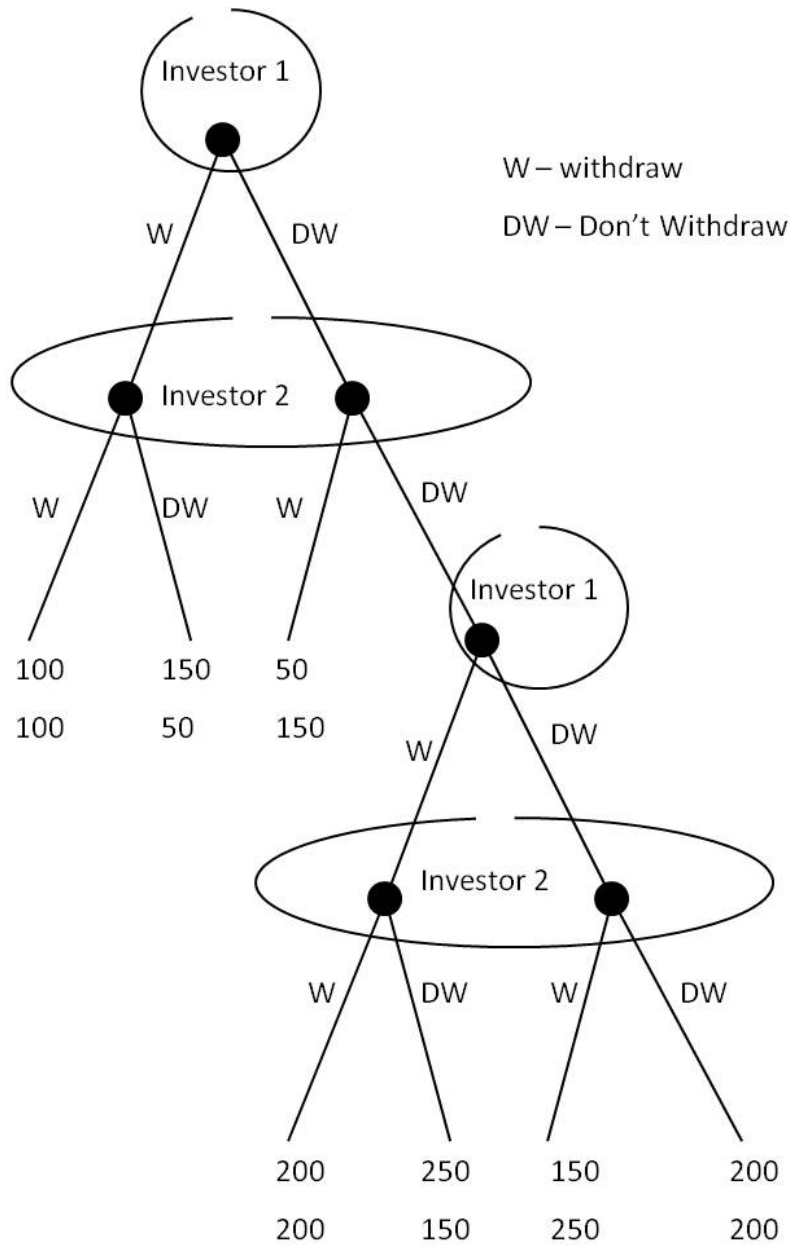
- a Suppose that this game is repeated 37 times. Find a pure strategy Nash equilibrium to this repeated game.
- b Is the Nash equilibrium you found for the finitely repeated game in part a unique? Explain why or why not.

Suppose now that the game is repeated *infinitely*.

- c Propose a set of strategies such that the outcome repeated in the stage game is the (7, 5) outcome when both players choose Low.
- d Determine the minimum discount rate needed by EACH player to ensure that the set of strategies you have suggested in part c is a subgame perfect Nash equilibrium to the game.

3. Consider the following game between two investors. Two investors have each deposited 150 with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of 200 can be recovered. If the bank allows the investment to reach maturity, however, the project will pay out a total of 400.

There are 2 dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1 then each receives 100 and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives 150, the other receives 50, and the game ends. Finally, if neither investor makes a withdrawal at date 1 then the project matures and both investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives 200 and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives 250, the other receives 150, and the game ends. If neither investor makes a withdrawal at date 2 then the bank returns 200 to each investor and the game ends. Note that neither player observes the withdrawal decision of the other player at either date (in other words, at date 1 the players simultaneously choose to withdraw or not, and the same at date 2 – obviously once date 2 is reached both players know what the other player chose at date 1). The figure below provides the game tree:



- a There are 2 subgames in this game, one of which is the entire game and the other of which is the game that begins at date 2. Write down the normal form (matrix) version of the subgame that begins at date 2.
- b Find the Nash equilibrium to the date 2 subgame in part a.
- c There are two subgame perfect Nash equilibria to this game. Find them. (Hint: You may want to use what you know from part b when solving for the SPNE).
- d While it is highly unlikely that I would ask you to do this on an exam due to time constraints, you can write down the entire game in normal form and find all PSNE.

4. Consider the following normal form game:

		Player 2			
		E	F	G	H
Player 1	A	22, 22	6, 4	2, 27	9, 2
	B	25, 10	15, 15	4, 3	2, 1
	C	17, 4	32, 6	8, 8	5, 3
	D	6, 1	9, 4	1, 2	1, 1

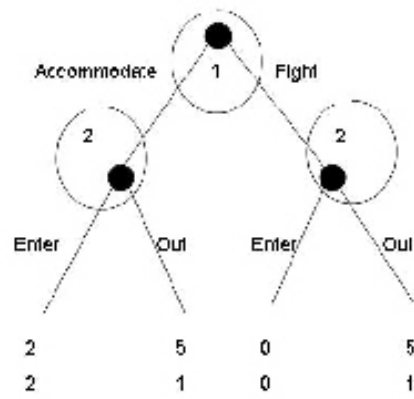
- a Find all pure and mixed strategy Nash equilibria to the one-shot version of this game. If there are no MSNE explain why there are none.
- b Given the Nash equilibria in part *a*, find a Pareto improving outcome (which means an outcome where both players are at least as well off as in the equilibrium, and at least one of the two is strictly better off). Suppose the game is repeated infinitely and that the players use a Nash reversion punishment strategy (meaning they punish the other player by playing the NE to the one-shot game forever). Find the minimum discount rates necessary to sustain an outcome path where your chosen Pareto improving outcome is the result of each stage game.
- c Note that Player 1's highest payoff is 32 and occurs when Player 1 plays C and Player 2 plays F. Again suppose the game is infinitely repeated. Explain how it is possible for the outcome path where this outcome (the C, F outcome) is repeated infinitely to be a SPNE. Be specific about Player 1's choice of punishment strategy.
5. Consider a developer who wishes to purchase  $k$  parcels of land. If the developer purchases all  $k$  parcels, the developer receives a payment of  $D$ . If the developer does not purchase all  $k$  parcels, the developer receives a payment of 0. The developer must purchase each parcel of land from the landowner who owns the land.

Consider  $k$  landowners who each own a parcel of land. That parcel has value of  $v_i$  to the landowner, where  $v_i \sim U[0, \frac{D}{k}]$ . The individual landowners know their own value for the land but the developer does not. Also, the landowners do NOT know the values of other landowners. Note that given these restrictions we have  $D \geq kv_i$  for any possible set of  $v_i$ .

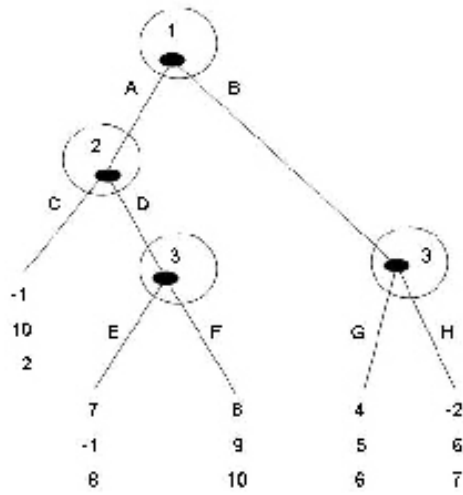
The game can be modeled as a sequential game. The developer makes an offer  $w_i$  to each landowner. Each landowner only observes his own  $w_i$  and must make a decision to accept or reject that  $w_i$ . If all  $k$  landowners accept their own offer  $w_i$ , then the landowners each receive  $w_i$  as a payment from the developer; the developer pays an amount  $\sum_{i=1}^k w_i$ , and the developer receives a payment of  $D$ . If ANY landowner chooses to reject  $w_i$ , then the developer makes no payment to any landowner and acquires no parcels of land – the developer receives 0 but pays 0. The landowners, who still own their land, receive  $v_i$ .

For simplicity, assume the seller sets  $w_i = w_j$  for all  $i, j$ . The developer maximizes expected utility, and receives  $D - kw$  if aggregation is successful (which occurs only if all  $k$  landowners accept the offer) and 0 if not. Note that  $\Pr(\tilde{v} > v)$  for the uniform distribution  $U[0, \frac{D}{k}]$  is  $\frac{\tilde{v}}{\frac{D}{k}}$ . Assume the developer is risk neutral. Find a subgame perfect Nash equilibrium to this game with  $k$  landowners. Be sure to set up the developer's expected utility function correctly.

6. Here are 2 extensive form games. Answer the questions below.



Game tree 1



Game tree 2

Find the SPNE for each game.