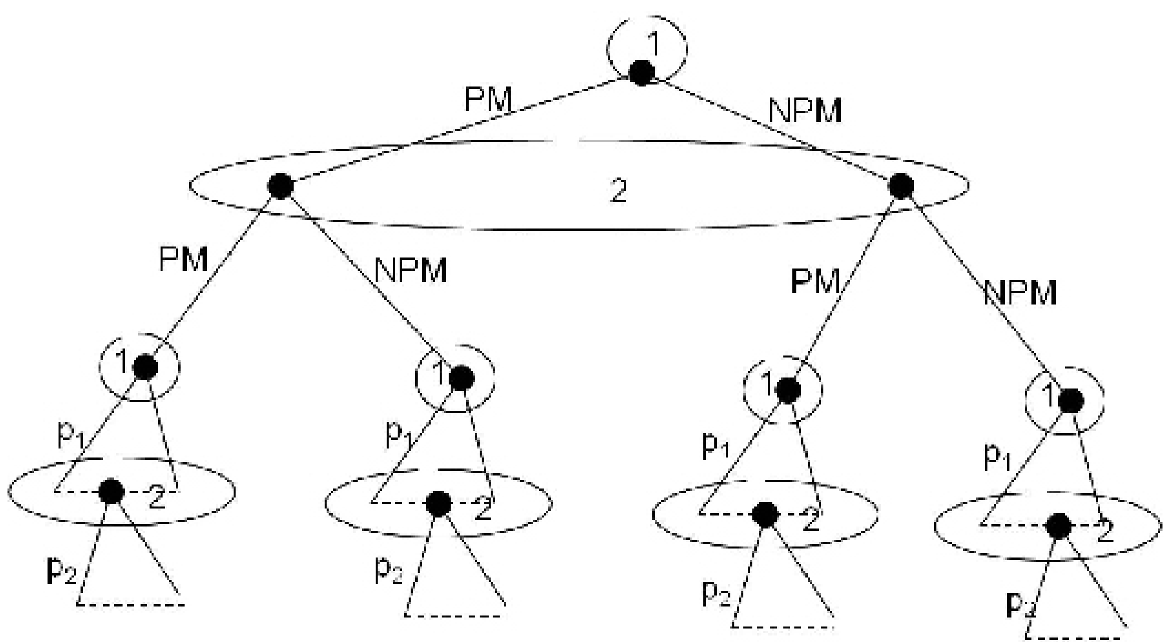
Problem Set 3

ECON 6206, Game Theory and Experiments

February 22, 2013

Directions: Answer each question completely. If you cannot determine the answer, explaining how you would arrive at the answer might earn you some points.

1. In the standard simultaneous Bertrand (pricing) game with two firms there is a unique pure strategy Nash equilibrium in which both firms choose a price equal to marginal cost c. Now suppose that before the firms simultaneously choose price in the standard Bertrand game they simultaneously choose whether or not to implement a price-matching policy. If a firm implements a price-matching policy this means that firm will match the lowest price in the market. Firms observe each other's price-matching decision and then simultaneously choose their respective prices (under the assumptions of the standard Bertrand pricing game), where a price can be any real number. A basic depiction of the game is below, where PM means "price-matching", NPM means "no price-matching", and p_1 and p_2 are the price choices of firms 1 and 2 respectively:



a Is there an equilibrium (or equilibria) to this game in which neither firm chooses to implement a price-matching policy in the first stage? If so find it (describe the set if there are multiple equilibria) and if not explain why not.

b Is there an equilibrium (or equilibria) to this game in which exactly one firm chooses to implement a price-matching policy in the first stage? If so find it (describe the set if there are multiple equilibria) and if not explain why not.

c Is there an equilibrium (or equilibria) to this game in which both firms choose to implement a price-matching policy in the first stage? If so find it (describe the set if there are multiple equilibria) and if not explain why not.

2. Consider a simultaneous quantity choice game between 2 firms. Each firm chooses a quantity, q_1 and q_2 respectively. The inverse market demand function is given by $P(Q) = 1434 - 2 * Q$, where $Q = q_1 + q_2$. Firm 1 has total cost function $TC(q_1) = 3 * (q_1)^2$ and Firm 2 has total cost function $TC(q_2) = 12 * (q_2)^2 - 12q_2$. Each firm wishes to maximize profit.

   a Find the best response functions for Firms 1 and 2.

   b Find the Nash equilibrium to this game.

   c Find (1) the total market quantity, (2) the market price, and (3) each firm's profit.

3. The citizens of Circleburg live in a city that is laid out in a perfect circle. The circumference of the circle is 12 miles. Residents live in houses which are distributed uniformly over the 12 miles. There are two competing gas stations, Chi Station and Epsilon Station. They are attempting to determine where to locate their respective stations. They know that residents of Circleburg will go to the gas station closest to their home. Assume that gas stations are concerned with maximizing the number of customers who visit their station. You may want to use a diagram to aid you when answering the questions. A pure strategy Nash Equilibrium (PSNE) for this game is a set of locations for the gas stations. Note that gas stations may locate at the same point on the circle. Also note that if two stations are equidistant to a customer then they each receive $\frac{1}{2}$ of a customer. **Note: Customers and gas stations can only locate on the perimeter of the circle. Assume that the interior of the circle is a huge chasm or a steep mountain. Someone always tries to locate stations/customers in the middle of the circle - do NOT do that.**

   a Assuming that residents of Circleburg are able to drive clockwise and counterclockwise around the circle, describe the set of PSNE.

   b The citizens of Circleburg have decided to outlaw backward thinking, which includes counterclockwise driving. The 2 gas stations are allowed to move their businesses from their previous locations. Now, residents of Circleburg will stop at the first gas station they see when they leave their home. If there is a PSNE to this game, find it. If there are multiple PSNE, describe the set of equilibria. If there are no PSNE, prove why there are none.

4. Consider a capacity-constrained duopoly pricing game. Firm j's capacity is q_j for $j = 1, 2$, and each firm has the same constant cost per unit of output of $c \geq 0$ up to this capacity limit. Assume that the market demand function $x(p)$ is continuous and sttrictly decreasing at all p such that $x(p) > 0$ and that there exists a price $\widetilde{p}$ such that $x(\widetilde{p}) = q_1 + q_2$. Suppose also that $x(p)$ is concave. Let $p(\cdot) = x^{-1}(\cdot)$ denote the inverse demand function.

   Given a pair of prices charged, sales are determined as follows: consumers try to buy at the low-priced firm first. If demand exceeds this firm's capacity, consumers are served in order of their valuations, starting with high-valuation consumers. If prices are the same, demand is split evenly unless one firm's demand exceeds its capacity, in which case the extra demand spills over to the other firm. Formally, the firms' sales are given by the functions $x_1(p_1, p_2)$ and $x_2(p_1, p_2)$ satisfying:

$$\text{If } p_j > p_i: \qquad \begin{cases} x_i(p_1, p_2) = Min\{q_i, x(p_i)\} \\ x_j(p_1, p_2) = Min\{q_j, Max\{x(p_j) - q_i, 0\}\} \end{cases}$$

$$If p_2 = p_1 = p: \qquad \left\{ x_i(p_1, p_2) = Min\left\{q_i, Max\left\{\frac{x(p)}{2}, x(p) - q_j\right\}\right\}\right\}$$

   a Suppose that $q_1 < b_c(q_2)$ and $q_2 < b_c(q_1)$, where $b_c(\cdot)$ is the best-response function for a firm with constant marginal costs of c. Show that $p_1^* = p_2^* = p(q_1 + q_2)$ is a Nash Equilibrium of this game.

   b Argue that if either $q_1 > b_c(q_2)$ or $q_2 > b_c(q_1)$, then no PSNE exists.