Problem Set 4

ECON 6206, Game Theory and Experiments

March 19, 2013

Directions: Answer each question completely. If you cannot determine the answer, explaining how you would arrive at the answer might earn you some points.

1. Consider the following game of incomplete information between Player 1 and Player 2. Player 1's type is known but Player 2 may be either an H type (with probability p) or an L type (with probability $1-p$). The payoffs to this simultaneous game of incomplete information are as follows:

		Player 2 (H type)				Player 2 (L type)	
		x	y			x	y
Player 1	a	$1,3$	$1,2$	Player 1	a	$3,2$	$1,3$
	b	$3,1$	$2,5$		b	$2,1$	$0,4$

a Suppose that $p = 0.75$. Find all pure strategy Bayes-Nash equilibria under this assumption.

b Find all pure strategy Bayes-Nash equilibria for each value of p (since p is a probability $p \in [0,1]$). **Hint 1**: There are no values of p such that there is more than one equilibrium for that value of p. **Hint 2**: It is best to find ranges of p for which a specific equilibrium exists. **Hint 3**: There is a range of p for which there are no pure strategy Bayes-Nash equilibria.

2. Consider two firms who compete by simultaneously choosing prices (a Bertrand game). If Firms 1 and 2 choose prices p_1 and p_2, respectively, the quantity that consumers demand from each firm is:

$$\begin{aligned} q_1(p_1, p_2) &= a - p_1 + bp_2 \\ q_2(p_1, p_2) &= a - p_2 + bp_1 \end{aligned}$$

Firm 2 has constant marginal cost c. With probability θ Firm 1 has constant marginal cost c_H while with probability $(1-\theta)$ Firm 1 has constant marginal cost c_L. Assume that there are no fixed costs. Prices must be nonnegative ($p_1 \geq 0$, $p_2 \geq 0$) and firms wish to maximize profit. **Important note**: Even though it is a Bertrand game, the demand functions are differentiable and continuous for both firms. Let $\theta = \frac{1}{2}$, $a = 108$, $b = \frac{1}{2}$, $c_H = 16$, $c_L = 8$, and $c = 12$.

a Find the best response functions for Firms 1 and 2.

b Find a pure strategy Bayes-Nash equilibrium to this game.

3. It is typical for the government to allocate construction contracts, such as repaving a highway, by holding an auction for the contract. The auction rules are as follows. Each bidder is to submit a sealed bid. The lowest bidder will win the contract, and the winning bidder will be paid an amount equal to the second lowest bid. Suppose that each bidder draws a cost, c_i, of completing the job from the uniform distribution over the interval $[\underline{c}, \overline{c}]$. The cost draws are made independently of each other. All bidders are aware of the common distribution of costs as well as the fact that cost draws are made independently of one another.

a Suppose that the bidders are risk-neutral. Find a Bayes-Nash equilibrium for this auction.

b Suppose that the bidders are risk-neutral. Suppose we changed the format so that the winning bidder, who is still the lowest bidder, now receives a payment equal to his bid if he wins (instead of a payment equal to the 2^{nd} lowest bid as before). Find a Bayes-Nash equilibrium for this new set of auction procedures.[1]

4. Recall that in the Battle of the Sexes game there are 2 players who are simultaneously deciding whether or not to attend a boxing match or an opera performance. However, now both players are uncertain about the value that the other player places on his or her favorite event. Thus, the normal form is:

		Player B	
		Opera	Boxing
Player A	Opera	$2+t_A,\ 1$	$0,\ 0$
	Boxing	$0,\ 0$	$1,\ 2+t_B$

where t_A is privately known by Player A and t_B is privately known by Player B. Both t_1 and t_2 are independent draws on a uniform distribution from $[0, x]$.

Let A be the critical value such that if $t_A \geq A$ then Player A plays Opera and if $t_A < A$ then Player A plays Boxing. Let B be the critical value such that if $t_B \geq B$ then Player B plays Boxing and if $t_B < B$ then Player B plays Opera. In this equilibrium, Player A plays Opera with probability $\frac{x-A}{x}$ and Player B plays Boxing with probability $\frac{x-B}{x}$.

a Find the critical values A and B. They will be a function of x (recall that x is the upper bound of the uniform distribution).

b Explain how the proposed equilibrium satisfies the criteria for a Bayes-Nash equilibrium.

[1] If you cannot find one for the distribution of values $[\underline{c}, \overline{c}]$ try the distribution $[0, 1]$.