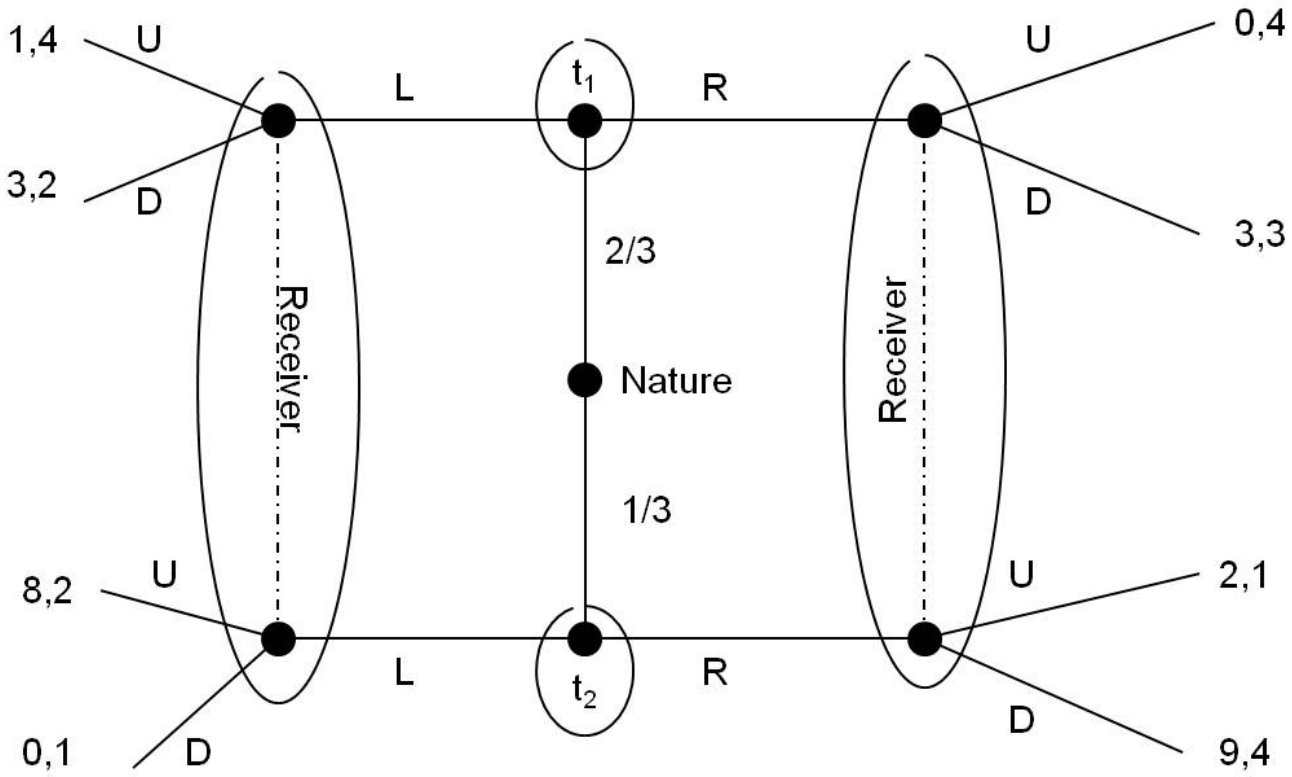
Problem Set 5 Answers

ECON 6206, Game Theory and Experiments

March 25, 2013

Directions: Answer each question completely. If you cannot determine the answer, explaining how you would arrive at the answer might earn you some points.

1. Consider the following game. Note that the probability of being a sender type t_1 is $\frac{2}{3}$ and the probability of being a sender type t_2 is $\frac{1}{3}$.



a Find all pure strategy separating perfect Bayesian equilibria. If there are none explain why not.

b Find all pure strategy pooling perfect Bayesian equilibria. If there are none explain why not.

2. A buyer and seller have valuations v_b and v_s. It is common knowledge that there are gains from trade (i.e. $v_b > v_s$), but the size of the gains is private information as follows: the seller's valuation is uniformly distributed on $[0, 1]$; the buyer's valuation $v_b = k * v_s$, where $k > 1$ is common knowledge; the seller knows v_s (and hence v_b) but the buyer does not know v_s or v_b. Suppose the buyer makes a single offer, p, which the seller either accepts or rejects. Based on Samuelson (1984).

a What is the perfect Bayesian equilibrium when $k < 2$?

b What is the perfect Bayesian equilibrium when $k > 2$?

3. There are two basic schools of thought on what exactly a college education is. The first school of thought is that students go to college to learn skills that will help them in their future career. The second school of thought is that a college degree is useful in the sense that it sends a signal to your prospective employers that you are able to be trained and willing to work hard enough for 4 (or 5 or 6 or 7 or perhaps only 3) years to get a degree, but that it does not really prepare you for your future career. Consider the following: There are two types of workers, high ability and low ability. The type of worker is randomly determined by chance with $\Pr(high)$ being the probability of a high type and $\Pr(low)$ being the probability of a low type, where $\Pr(low) = 1 - \Pr(high)$. The workers must make a decision regarding school. They can either go to college or not go to college. If a high ability worker attends college it costs him C^H. If a low ability worker attends college it costs him C^L. Assume that $C^L > C^H$ because the low ability worker will have to work harder in college than the high ability worker.

   There is an employer who must make a hiring decision. The employer is unable to observe type but is able to observe whether or not the worker has a college education. The employer must choose to either offer the worker the job or not offer the worker the job, and this must be made for both types of people – those who went to college and those who did not. If the employer offers a job, the wage is W regardless of worker type and whether or not they went to college. If the employer does not offer a job, then the worker receives his opportunity wage of M. If a job is offered the worker must decide whether or not to accept the job – if the job is accepted then the worker receives the wage of W (minus any costs incurred if he went to college) and if the job is not accepted the worker receives M (minus any costs incurred if he went to college). A high ability worker is worth R^H to the employer if he accepts the offer while a low ability worker is worth R^L to the employer, regardless of whether or not the worker went to college. Assume that $R^H > R^L$. If the worker does not accept the job then the employer receives 0.

   a Draw the extensive form of this game. Leave the payoffs as variables for now.

   b Propose a Perfect Bayesian Equilibrium that is a pooling equilibrium where all workers attend college. Verify that this is in fact a Perfect Bayesian Equilibrium by stating the relevant constraints.

   c Propose a Perfect Bayesian Equilibrium that is a separating equilibrium where only high ability workers attend college. Verify that this is in fact a Perfect Bayesian Equilibrium by stating the relevant constraints.

   d Suppose that $R^H = 20$, $R^L = 15$, $C^L = 12$, $C^H = 8$, $M = 12$, $W = 19$, and $\Pr(high) = 0.5$.

   - Will there be a pooling equilibrium or a separating equilibrium?
   - Which workers will attend college and which will not?