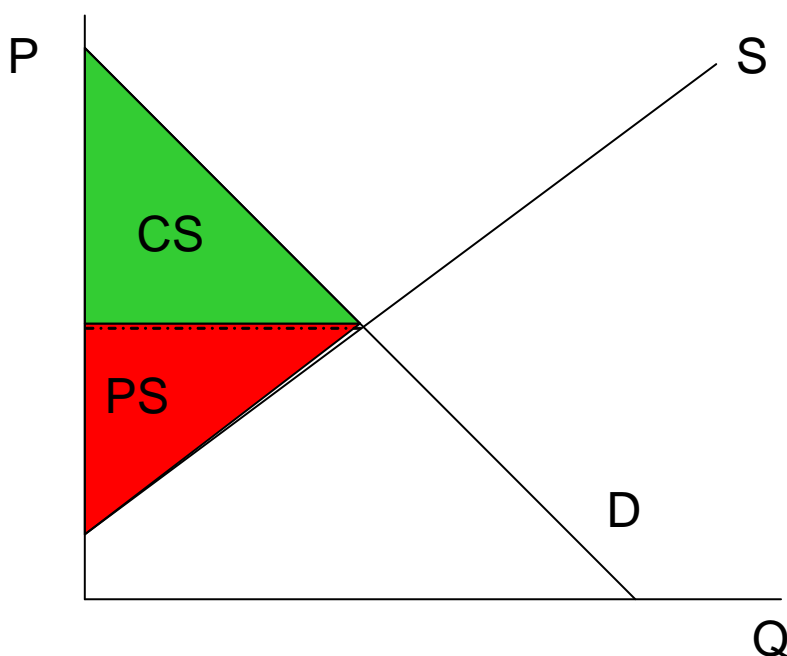


These notes essentially correspond to chapter 11 of the text.

1 Result of the price system

One result of the price system under perfectly competitive markets is that the gains from trade in a market are maximized. The gains from trade are the sum of consumer surplus and producer surplus. Consumer surplus (CS) is the net gain to consumers, which we can find as the area below the demand curve but above the equilibrium price. Consumer surplus defined in this manner is a somewhat abstract concept, but think about any time you have gone into a store and seen an item and said or thought “I can’t believe that only costs \$10; I’d gladly pay \$45 for it”. So the difference between someone’s willingness to pay and the price they actually pay is consumer surplus. Producer surplus (PS) is the net gain to producers, which we can find as the area above the supply curve but below the equilibrium price. It is similar to, but not the same as, profit, for reasons which we may discuss later in the course. The following picture illustrates consumer and producer surplus on the graph, with consumer surplus in green and producer surplus in red:



Mathematically, if we have a general inverse demand function $P_{own} = f(Q_D)$, then we have consumer surplus as:

$$CS = \int_0^{Q^*} f(Q_D) dQ - \int_0^{Q^*} P^* dQ$$

$$CS = \int_0^{Q^*} (f(Q_D) - P^*) dQ$$

where Q^* is the equilibrium quantity and P^* is the equilibrium price. For an inverse supply function, $P_{own} = f(Q_S)$, producer surplus can similarly be defined as:

$$PS = \int_0^{Q^*} P^* dQ - \int_0^{Q^*} f(Q_S) dQ$$

$$PS = \int_0^{Q^*} (P^* - f(Q_S)) dQ$$

2 Taxes

We will analyze the effects that a per-unit tax has on the equilibrium price and quantity in a market. To be specific, we will do a partial equilibrium analysis of the per-unit tax. A partial equilibrium analysis isolates a particular market (or sector of the economy) and considers any effects on other markets to be negligible. A general equilibrium analysis, which is a little more involved, would consider more markets.

Suppose that the government places a per-unit tax on a good. We know that the supply curve will decrease (shift to the left) and that the result will be a higher price and lower equilibrium quantity. What we want to do is see how the consumers, producers, and the government are affected.

2.1 Example

I will use the processed pork market, and we will suppose that the government wants to place a per-unit tax (use τ as the symbol for the tax) on the producers of processed pork. We know that the supply and demand conditions before the tax are:

$$\begin{aligned}Q_D &= 286 - 20P_{own} \\ Q_S &= 88 + 40P_{own}\end{aligned}$$

The initial equilibrium price and quantity in this market are \$3.30 and 220 units (see chapter 2 notes on how to get this result). When the government places this tax on the suppliers, nothing happens to the demand curve, so we still have $Q_D = 286 - 20P_{own}$. However, we know that for every unit sold the producer will have to give back τ to the government. The easiest way to incorporate τ into the supply function is to rewrite it as the inverse supply function:

$$P_{own} = -2.2 + .025Q_S$$

Now, all we need to do is subtract the tax, τ , from P_{own} because the producer will receive only $P_{own} - \tau$ for each unit. The after-tax inverse supply function is then:

$$P_{own} - \tau = -2.2 + .025Q_S$$

Rewriting this as the supply function, we get:

$$Q_S = 88 + 40(P_{own} - \tau)$$

Or,

$$Q_S = 88 + 40P_{own} - 40\tau$$

Now, if we know the amount of the tax, all we need to do is substitute in the amount of the tax and then solve for the equilibrium price and quantity as we did in chapter 2. Suppose the tax is \$1.05 per-unit sold. Then our supply function becomes: $Q_S = 88 + 40P_{own} - 42$. Subtracting 42 from 88 will give us supply and demand functions of:

$$\begin{aligned}Q_D &= 286 - 20P_{own} \\ Q_S &= 46 + 40P_{own}\end{aligned}$$

Now all we need to do is solve for the equilibrium price and quantity. Set $286 - 20P_{own} = 46 + 40P_{own}$ and solve for P_{own} . You should get $P_{own} = \$4$. Now substitute \$4 in for the price to find that $Q_S = Q_D = 206$. You should note that the buyers in this market pay \$4, but that the sellers only keep \$2.95 because they must give \$1.05 to the government.

2.2 Some additional calculations

Now that we have found the new equilibrium price and quantity, we can see how much revenue the government receives, how much society loses from the tax, and who actually bears the burden of the tax – the consumer or the producer.

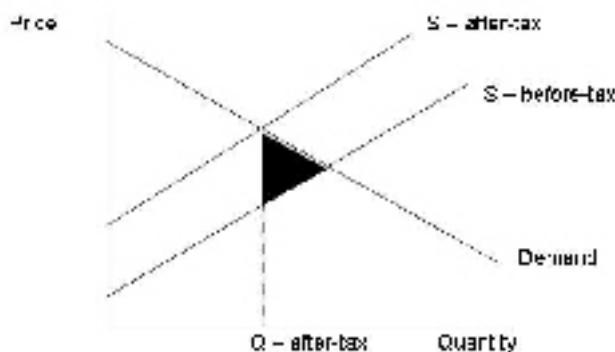
2.2.1 Government revenue

This is a fairly quick calculation to make with a per-unit tax. The government receives the amount of the tax, τ , for every unit sold in the after-tax market. So all we need is the new equilibrium quantity, which we found to be 206, and the tax rate, which was \$1.05 per-unit. Multiply the two of them together to find how much revenue the government generates. In this case, it is \$216.30.

2.2.2 Deadweight loss

If a market is allowed to operate unimpeded then all of the gains from trade should be captured by the buyers and sellers in the market. The gains from trade can be defined as the total amount of consumer and producer surplus that a market can generate. When a market has some restriction imposed on it, such as a price control or a tax, or if the market is not competitive, as in the case of a monopoly, the market is no longer unimpeded. This generates a deadweight loss to society. Deadweight loss is best defined as gains from trade that fail to be captured by the market due to some restriction in the market.

Look at the picture below, with a linear demand curve and a linear supply curve. The triangular area shaded in black as the deadweight loss due to the tax. When the market was untaxed, consumers and producers shared these gains. Now, however, since the tax is placed on the market, no one receives these potential gains from trade because no units beyond Q – after-tax are traded. Thus, this is society’s loss.



To calculate the deadweight loss given a linear demand curve and a linear supply curve, use the formula for a triangle, $\frac{1}{2} * base * height$. The best way to do this is to rotate the triangle 90 degrees. Then the base of the triangle will be the amount of the tax, τ , and the height of the triangle will be the change in quantity traded. Just subtract the new equilibrium quantity from the initial (pre-tax) equilibrium quantity. So we can rewrite the formula as $\frac{1}{2} * \tau * (Q^{before-tax} - Q^{after-tax})$. Plugging in the numbers from the example we get that the deadweight loss is $\frac{1}{2} * 1.05 * (220 - 206) = 7.35$.

2.2.3 Who bears the burden of the tax

When the government imposes a tax on the seller of a good there is a common misconception that the seller will pass the entire tax along to the consumer by raising the price of the product by the full amount of the tax. While the seller may do this, it will only be an equilibrium if demand is perfectly inelastic. Eventually market forces will bring the price back down, although not to its former level. The amount by which the price rises (and ultimately the percentage of the tax the consumer must actually pay) depends on the elasticity of the supply and demand curves. The simple method (if we already have the before-tax

and after-tax equilibrium prices) to calculating the percentage of the tax that the consumer must pay is to see how much the equilibrium price rises by and divide that by the amount of the tax. In our example, the price that the buyers paid rose from \$3.30 to \$4, so \$0.7. The amount of the tax was \$1.05. Divide 0.7 by 1.05 and we get that the consumers must bear $\frac{2}{3}$ of the burden of the tax.

An alternative method of finding the burden of the tax that the consumers must bear is to use the following formula:

$$\text{consumer burden} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_P}$$

You should note that ε_S is the price elasticity of supply at the before-tax equilibrium price, which was 0.6. Also, ε_P is the price elasticity of demand at the before-tax equilibrium price, which was -0.3 . Substituting these numbers in we get that the consumers must bear a burden of $\frac{.6}{.6 - (-.3)} = \frac{2}{3}$, which is exactly what we found up above. We can also use this formula to see what happens to the consumers burden as demand becomes more elastic, holding supply constant. Suppose the price elasticity of demand is now (-0.9) . The consumers burden is $\frac{.6}{.6 - (-.9)} = \frac{2}{5}$. So, as demand becomes more elastic the seller is not able to pass as much of the burden along to the consumer. Alternatively, if supply were to become more elastic, say to 0.9, then we would have the consumers burden as $\frac{.9}{.9 - (-.3)} = \frac{3}{4}$. Thus, if the supply is more elastic the producer can pass more of the burden on to the consumer.

2.2.4 Government revenue and elasticity

The government would prefer to tax goods with both inelastic demand and inelastic supply. The more inelastic the curves, the less the equilibrium quantity traded will fall after the tax is imposed. As an extreme example, suppose that either the supply curve or the demand curve is PERFECTLY inelastic at a quantity level of 220. Suppose the equilibrium price is \$4. If the government places a \$1 tax on the item, the quantity traded will not fall at all since one curve is perfectly inelastic. While all the burden will fall on whoever has the perfectly inelastic curve, the government will gain a great deal of revenue as the quantity will not fall.

To convince yourself of this fact, draw two markets on the same graph, one with a supply and demand curve that are very inelastic and another with a supply and demand curve that are very elastic. Make sure the initial equilibrium price and quantity in both markets is the same. Then place a tax of the same amount on the sellers in each market. You should see the quantity traded in the market with the elastic curves fall by a greater amount than the quantity traded in the market with the inelastic curves. Thus, the government will gain a greater amount of revenue in the market with the inelastic curves. Times when $Q_D \neq Q_S$

There are some cases when determining the equilibrium price and quantity cannot be done as described above. Typically, these cases involve some restriction imposed on either price or quantity in the market. We will work through an example of a price floor.

2.3 Price Floor example

Recall that a price control is a government mandated price. A price floor is a price set by the government which the market price cannot fall below. A price ceiling is a price set by the government which the market price cannot rise above.

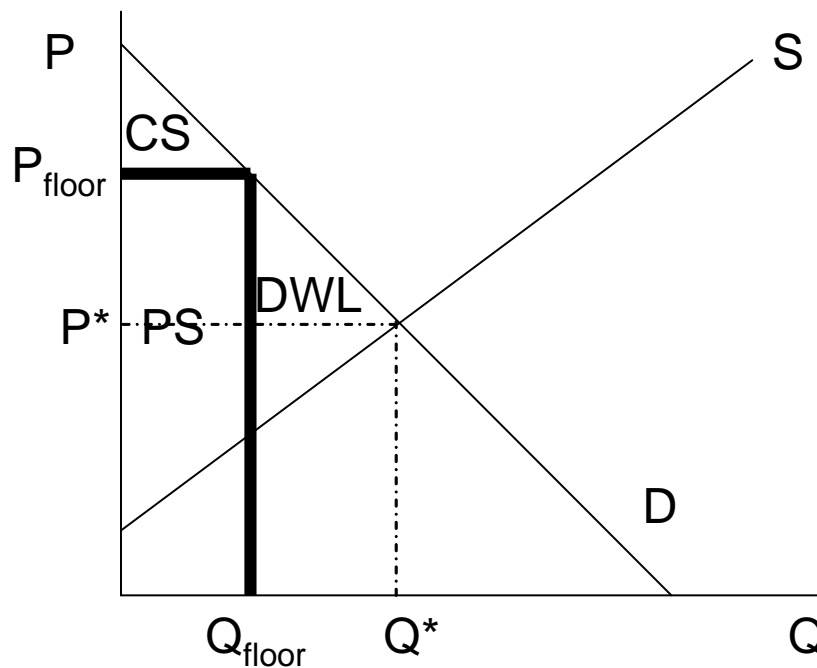
Suppose we have the following supply and demand functions:

$$\begin{aligned} Q_D &= 286 - 20P_{own} \\ Q_S &= 88 + 40P_{own} \end{aligned}$$

These are the same supply and demand functions from above, so the equilibrium price is \$3.30 and the equilibrium quantity is 220. Suppose the government imposes a price floor of \$4. We now know that the price cannot fall below \$4. How would we go about solving for the price and quantity traded (to be honest, it is really NOT an equilibrium quantity because $Q_D \neq Q_S$ which is why I call it the quantity traded) in the market? I propose the following steps:

1. First, calculate the equilibrium price and quantity without imposing the price floor. I say this because if the price floor is BELOW the equilibrium price, then the price floor does not bind because the market price is greater than the price floor. Thus, the equilibrium price and quantity would be the price and quantity traded in the market. So, if the government had decided to set a price floor of \$3 in this market, the outcome would just be the equilibrium price and quantity because $\$3 < \3.30 . (It is possible that a price floor set below the true equilibrium price may end up being a focal point in the market, but the “conventional wisdom” is that non-binding price controls have no effect on the market.)
2. If the price control does bind then you need to calculate Q_D and Q_S by substituting the value of the price control (\$4 in the example) into the demand and supply functions. We find that $Q_D = 206$ when $P_{own} = \$4$ and that $Q_S = 248$. Note that this is NOT an equilibrium solution because $Q_D \neq Q_S$.
3. Finally, choose the quantity level that is lower: in this case, $Q_D < Q_S$. The reason that we choose the lesser amount is that even if you have 248 units for sale, if people only want to buy 206 units at the price you are charging then only 206 units will be traded.

When a market cannot obtain equilibrium then the gains from trade are not maximized, which means there are gains from trade that are not realized because profitable transactions to both the buyer and seller do not occur. These “unrealized gains from trade” are known as deadweight loss. The following picture shows consumer surplus, producer surplus, and deadweight loss graphically.



Mathematically, deadweight loss is simply the area beneath the demand curve but above the supply curve for those units which are not traded, or:

$$DWL = \int_{Q^T}^{Q^*} (f(Q_D) - f(Q_S)) dQ.$$

Note that Q^T stands for the quantity traded in the market. This equation should make it clear that when the quantity traded in the market is equal to the equilibrium quantity that there is no deadweight loss.