

These notes essentially correspond to chapter 13 of the text.

The two market structures that we will study in this chapter, monopolistic competition and oligopoly, fall between the two extreme ends of perfect competition and monopoly. Firms operating in these market structures have some market power, but they do face competition from other firms in the industry.

1 Monopolistic Competition

Monopolistic competition captures elements of both monopoly and perfectly competitive markets. Three key assumptions for monopolistic competition markets are:

1. Large numbers of buyers and sellers
2. Differentiated products (or product heterogeneity) – there are many different fast-food establishments that have the hamburger as their primary product, but the hamburgers are all different (Wendy's, Burger King, McDonalds, etc.)
3. Barriers to entry are low

These assumptions are similar to those for a perfectly competitive firm, except that now firms are producing differentiated products. In perfectly competitive markets demand is perfectly elastic, while in monopolistic competition demand is highly elastic (note that some of my pictures do not really have "highly elastic" demand curves, but that is because it is easier to draw the pictures if the demand curves are slightly steeper). This difference is important, as the monopolistic competition will look like the monopoly model in the short run.

Sellers in these markets face competition from firms in existence and from firms that may want to enter the market. Burger King faces competition from McDonald's and Wendy's as well as any enterprising businesspeople who believe that they can make profits by opening a new hamburger establishment. Because of these facts competitive forces play a large part in monopolistic competition.

1.1 Price and output in the short run

Any firm in a monopolistic competition market will expand output until $MR = MC$. This result is standard across all market models.

In Figure 1, $MR = MC$ at the big blue dot and that quantity is given by Q^* . From here, we take the quantity that the firm wants to produce (Q^*) and find out the price that the firm will charge by going to the demand curve. The intersection of the Q^* and the demand curve is given by the small blue dot. The corresponding price is P^* . We have now found the profit-maximizing price and quantity for the firm. Because we have the ATC curve on this graph we can figure out the firm's profits. Remember that $\Pi = TR - TC$, and that $TR = P * Q$ while $TC = ATC * Q$, so all we need to do is find the ATC at the optimal quantity, which is given by ATC^* in the figure. Notice that this particular firm is earning a positive profit because $P^* > ATC^*$.

I have not put in a calculus example because in the short-run a firm in monopolistic competition faces the same problem as a monopoly.

1.2 Price and output in the long run

Because there are low barriers to entry in monopolistic competition, the price will be driven down in the long-run to where it equals a point on the ATC curve, so that a firm is earning zero-economic profit. Figure 2 shows the long-run equilibrium picture for monopolistic competition.

Comparing monopolistic competition to perfect competition and monopoly, we see that there are similarities with both markets. As with monopoly, there is deadweight loss in the market. However, unlike monopoly, these firms are earning zero-economic profit, which makes them similar to perfect competition though they are NOT producing a quantity such that $P = MC$ (they would need to be producing the quantity where MC intersects the demand curve), nor are they producing a quantity that minimizes ATC . Are these types of markets inefficient? They could be considered inefficient because there is deadweight loss

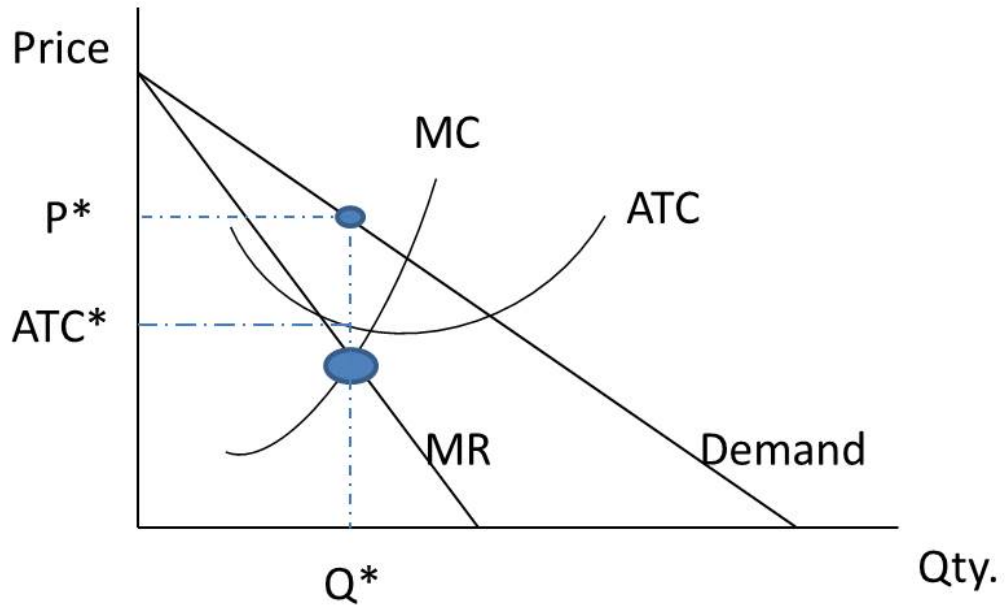


Figure 1: Short-run equilibrium for monopolistic competition.

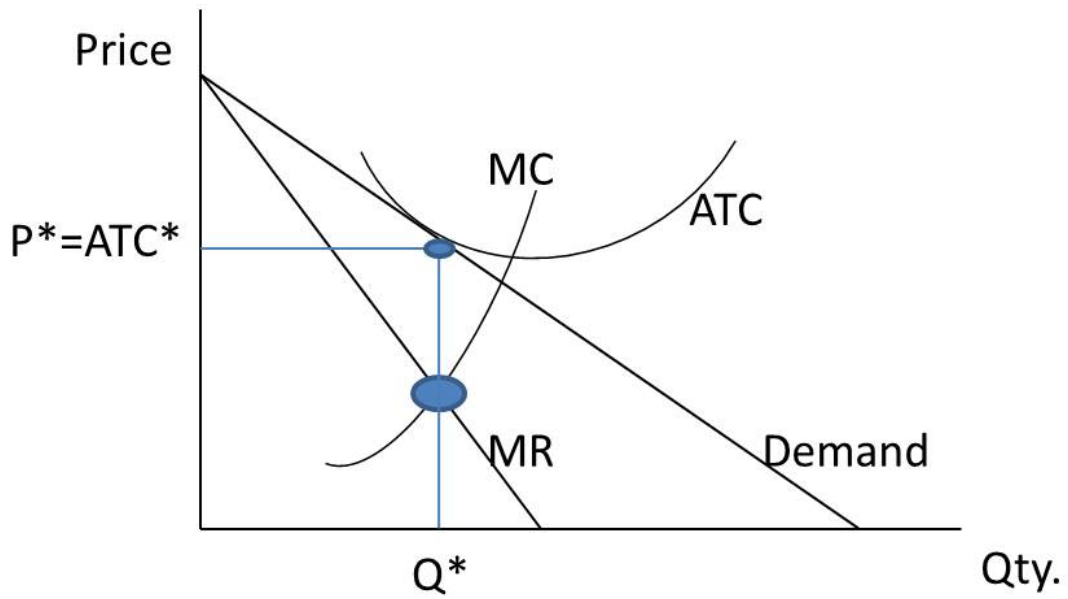


Figure 2: Long-run equilibrium for monopolistic competition

and because they are not minimizing ATC ; however, price is not the only variable to take into consideration. Consumers may value diversity and convenience of products as well as the price, and they may be willing to pay a little extra and sacrifice some efficiency for more variety.

Consider what has to hold in order for a firm in monopolistic competition to be in long-run equilibrium: the slope of the ATC has to be equal to the slope of the demand curve (as I mentioned earlier in class – tangency points). For a general inverse demand function, $P(Q) = a - bQ$, and a general cost function, $TC(Q) = x + yQ + zQ^2$ we would have:

$$\frac{\partial P}{\partial Q} = -b$$

and first we would need to find the ATC :

$$\begin{aligned} ATC &= \frac{x}{Q} + y + zQ \\ ATC &= xQ^{-1} + y + zQ \\ \frac{\partial ATC}{\partial Q} &= (-1)xQ^{-2} + z \end{aligned}$$

Now set:

$$\begin{aligned} \frac{\partial P}{\partial Q} &= \frac{\partial ATC}{\partial Q} \\ -b &= (-1)xQ^{-2} + z \\ b &= \frac{x}{Q^2} - z \\ (b+z)Q^2 &= x \\ Q &= \sqrt{\frac{x}{b+z}} \end{aligned}$$

Again, this result is specific to the structure of the problem. Using the demand function and total cost function from the book:

$$\begin{aligned} P(Q) &= 20,000 - 15.6Q \\ TC(Q) &= 400,000 + 4,640Q + 10Q^2 \end{aligned}$$

we would have $x = 400,000$, $b = 15.6$, and $z = 10$ which would lead to an optimal quantity of 125 (the same result is in the textbook, for the long-run high-price/low-output result).

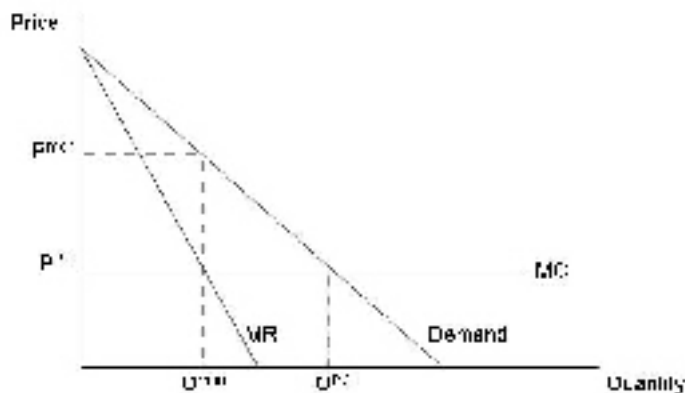
The high-price/low-output result that is discussed in the text assumes that there is a leftward shift (reduction) in demand for the firm's product due to new entrants (the positive economic profits attract new entrants, as they do in perfectly competitive markets). A different outcome would occur if the firm's demand curve became more elastic due to new entrants – in this case it might be possible to get to the actual perfectly competitive outcome where $P = MR = MC = ATC$.

2 Oligopoly

The key feature of the oligopoly (and to some extent, the monopolistically competitive market) market structure is that one firm's decision depends on the other firms' decisions. In other words, firm behavior is mutually interdependent. Note that in a monopoly there is no other firm on which behavior can depend, and in perfect competition no firm can affect the market price on its own, so firms do not have to worry about how much other firms produce as there will be no effect on the market price. Thus, while we had fairly robust results for the monopoly and the perfectly competitive markets, we will see that the results for the oligopoly market may vary greatly depending on the choice of strategic variable. Although there is a vast array of variables that firms may choose as their strategic variable (level of advertising, product quality, when to release a product, product type, etc.), the two standard choice variables are quantity and price. We will examine these two market games using a simultaneous game between 2 firms that produce identical products, face a linear inverse demand function, and have constant marginal costs.

We typically assume that oligopolies are small in number (while monopolistic competitors are larger) and that oligopolies are protected by some entry barrier (while free entry can occur under monopolistic competition). Products may either be identical or homogeneous in an oligopoly. OPEC is an oligopoly that produces oil (or petroleum if you want to be more precise), which is a fairly homogeneous product, while historically the big three auto manufacturers were an oligopoly that produce differentiated products (I say historically because they have less market power due to the recent influx of imports – they still produce differentiated products).¹

Before we begin the discussion it may be useful to consider the extremes of oligopoly behavior. At one extreme, the oligopolists could collude and act like a monopolist, choosing to produce a quantity that maximizes INDUSTRY profits. At the other extreme, the oligopolists could act like perfect competitors, driving price down to MC. The picture below shows the extreme forms of behavior.



The most likely outcome is that price and quantity will lie somewhere between the two extreme forms of behavior.

2.1 Quantity competition

Quantity games are also called Cournot games, after the author who is credited with first formalizing them in 1838. Cournot believed that firms competed by choosing quantities, with the inverse demand function determining the price in the market. Assume that there are 2 identical firms, Firm 1 and Firm 2, each of whom will simultaneously choose a quantity level (q_1 and q_2 respectively). The inverse demand function for this product is $P(Q) = a - bQ$, where Q is the total market quantity, which means $Q = q_1 + q_2$ for this example. Each firm's total cost is as follows: $TC_1 = c \cdot q_1$ and $TC_2 = c \cdot q_2$. Thus, each firm's marginal cost is: $MC_1 = MC_2 = c$. We will first show that the monopoly (or cartel) and perfectly competitive solutions are NOT equilibria in this market model, and then we will find the NE and compare it to the monopoly and perfectly competitive solutions.

2.1.1 Monopoly is NOT an equilibrium in quantity competition

Suppose that the two firms collude to form a cartel. The cartel's goal is to choose the quantity that will maximize industry profits. Each firm will produce $\frac{1}{2}$ of the monopoly quantity and receive the profits from producing that quantity. The monopolist will set $MR = MC$, where $MR = a - 2bQ$ and $MC = c$, so:

¹Note that technically the models we examine in this chapter are game theoretic models, though this text discusses these models before discussing game theory.

$$a - 2bQ = c$$

$$Q = \frac{a - c}{2b}$$

Thus, the total market quantity is $\frac{a-c}{2b}$, so each firm produces $\frac{a-c}{4b}$ (which is $\frac{1}{2} * \frac{a-c}{2b}$). Rather than work in the abstract, we can use some parameters to show that both firms would like to deviate from producing $\frac{a-c}{4b}$. Let $a = 120$, $b = 1$, and $c = 12$. There is nothing particular about these parameters, and these results hold for any parameter specification provided a , b , and c are all positive, and $a > c$. We need $a > c$ because otherwise the marginal cost will be above the highest point on the demand curve, which means a quantity of zero would be sold in the market since marginal cost would be greater than price for any units sold.

Using the parameters we find that: $Q = 54$ and $q_1 = q_2 = 27$. The price in the market is: $P(54) = 120 - (1) * 54 = 66$. The profit to each firm is: $\Pi_1 = \Pi_2 = 66 * 27 - 12 * 27 = 1458$.

Now, suppose that Firm 1 decides to cheat on the agreement and produces more than 27 units (so 28 units). If Firm 1 produces 28 units, then $Q = 55$ and $P(55) = 65$. Firm 1's profits are now: $\Pi_1 = 65 * 28 - 12 * 28 = 1484$, which is greater than the 1458 it was earning when it produced 27 units (to be complete, Firm 2's profits are: $\Pi_2 = 65 * 27 - 12 * 27 = 1431$). Because Firm 1 can earn a higher profit if it changes its strategy (chooses a quantity level greater than 27), the monopoly (or cartel) outcome is NOT an equilibrium.

2.1.2 Perfect competition is NOT an equilibrium in quantity competition

Suppose that firms act as perfect competitors. In this case, the firms will produce the total market quantity that corresponds to the point where MC crosses the demand curve. Because the two firms are identical, we will assume that each firm produces $\frac{1}{2}$ of this total market quantity. To find the total market quantity, set $MC = demand$ or $c = a - bQ$. Then $Q = \frac{a-c}{b}$, and $q_1 = q_2 = \frac{a-c}{2b}$. Using our parameters, we find that: $Q = 108$, and $q_1 = q_2 = 54$. Now, $P(108) = 120 - (1) * 108 = 12$. The profits to each firm are: $\Pi_1 = \Pi_2 = 12 * 54 - 12 * 54 = 0$. Notice that $P = MC$ and $\Pi_1 = \Pi_2 = 0$, both of which correspond to the theoretical predictions of a perfectly competitive market.

Now, suppose that Firm 1 decides to relax his stance on being competitive, and it produces 53 units rather than 54 units. If Firm 1 produces 53 units, then $Q = 107$ and $P(107) = 13$. Firm 1's profits are now: $\Pi_1 = 13 * 53 - 12 * 53 = 53$, which is greater than the 0 profit it was earning by acting competitively (to be complete, Firm 2's profits are: $\Pi_2 = 13 * 54 - 12 * 54 = 54$). Because Firm 1 can earn a higher profit if it changes its strategy (chooses a quantity level less than 54), the perfectly competitive outcome is NOT an equilibrium. The intuitive difference between this game and the perfectly competitive market is that each firm in this game has some impact on the price. If this were a true perfectly competitive market, then Firm 1 could NOT have caused the price to increase by reducing its quantity – however, in this game, Firm 1 can cause the price to increase by reducing its quantity.

2.1.3 The Cournot-Nash solution

We have seen that the 2 firms behaving like either extreme (cartel or perfect competition) is NOT an equilibrium. To solve for an equilibrium we will need to find each firm's reaction (or best-response) function. A best response function is a function that tells a firm the quantity level it should produce (or, more generally the strategy it should use) given the quantity level that the other firm produces. Thus, a firm's best response function will be a function of the other firm's quantity as well as the parameters of the problem.

Intuitively, we know that firms maximize their profit by setting $MR = MC$. Now, take Firm 1. We know that $MC = c$, so half of the equation is done for us. Finding MR is a little bit more difficult. We know that $P(Q) = a - bQ$, and that $Q = q_2 + q_1$, so $P(Q) = a - bq_2 - bq_1$. What we are trying to find is a function that tells us how much Firm 1 should produce for a GIVEN (or constant) level of q_2 . To find Firm 1's best response function simply take the partial derivative of Firm 1's profit function with respect to q_1 . A similar process can be used to find Firm 2's best response function. So:

$$\Pi_1 = (a - bq_1 - bq_2)q_1 - cq_1$$

Now, take the partial derivative of profit with respect to q_1 . We find:

$$\frac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c$$

Set this equal to zero to find the maximum (we know it's a maximum because the 2nd derivative is $(-2b)$, which is always negative for positive b). We get:

$$a - 2bq_1 - bq_2 - c = 0$$

Solving for q_1 :

$$q_1 = \frac{a - c - bq_2}{2b}$$

We can use a similar process to find that $q_2 = \frac{a - c - bq_1}{2b}$.

Before continuing on to find the actual quantity levels that each firm would produce I would like to point out one thing. Notice that if Firm 2 decides to produce $q_2 = 0$, then Firm 1's best response is to produce the entire monopoly quantity, which would be $q_1 = \frac{a - c}{2b}$. This result is consistent with the results that we have already seen.

As for finding the equilibrium quantities, we need to find a pair of quantities (one for each firm) such that neither firm will change its quantity choice. To find the equilibrium, we want to find the q_1 and q_2 that are best responses to one another. We can do this by substituting in the best response function for q_2 into the best response function for q_1 (essentially we have 2 equations and 2 unknowns, q_1 and q_2 , and we want to find the 2 unknowns). Substituting in we get:

$$q_1 = \frac{a - c - b\left(\frac{a - c - bq_1}{2b}\right)}{2b}$$

Simplifying:

$$2bq_1 = a - c - b\left(\frac{a - c - bq_1}{2b}\right)$$

Simplifying:

$$2bq_1 = a - c - \left(\frac{a - c - bq_1}{2}\right)$$

Simplifying:

$$4bq_1 = 2a - 2c - (a - c - bq_1)$$

Distributing the negative:

$$4bq_1 = 2a - 2c - a + c + bq_1$$

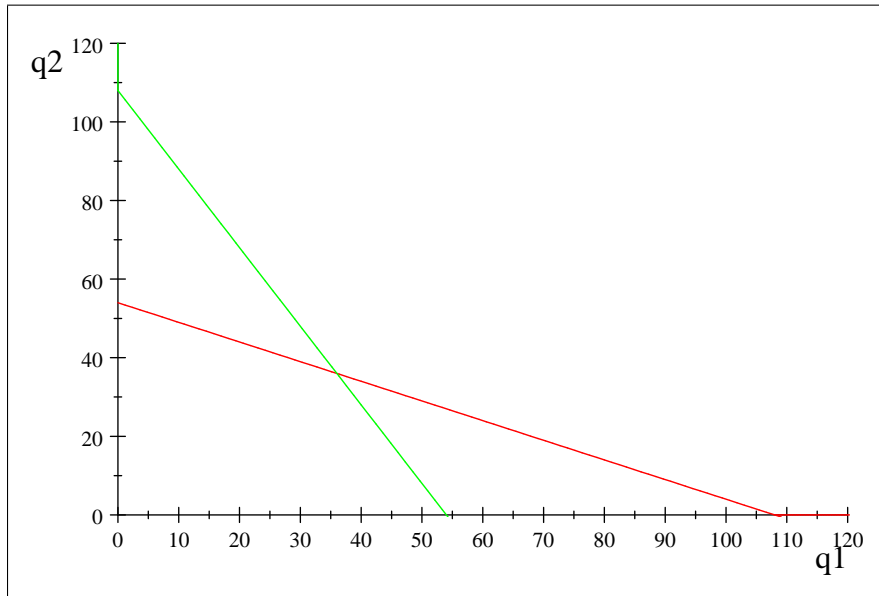
Solving for q_1 :

$$q_1 = \frac{a - c}{3b}$$

Thus, Firm 1 should produce $q_1 = \frac{a - c}{3b}$. We can solve for q_2 using a similar method to find that $q_2 = \frac{a - c}{3b}$. Thus, the equilibrium for this game is $q_1 = q_2 = \frac{a - c}{3b}$. Substituting in our numbers shows us that $q_1 = q_2 = \frac{120 - 12}{3} = 36$, so $Q = 72$ and $P(Q) = 120 - (1) * 72 = 48$. Thus, because both firms are identical and producing the same amount, $\Pi_1 = \Pi_2 = 48 * 36 - 12 * 36 = 1296$. If Firm 1 decides to deviate by producing a larger quantity (say 37), then $Q = 73$ and $P(73) = 47$. Firm 1's profits are:

$\Pi_1 = 47 * 37 - 12 * 37 = 1295$, which is less than the 1296 Firm 1 would earn if it produced 36 units. So producing a quantity greater than 36 is not more profitable than producing a quantity of 36.² Suppose Firm 1 decided to deviate by producing a lower quantity than 36 (say 35). Then $Q = 71$ and $P(71) = 49$. Firm 1's profits are: $\Pi_1 = 49 * 35 - 12 * 35 = 1295$, which is less than the 1296 Firm 1 would earn if it produced 36 units. So producing a quantity less than 36 is not more profitable than producing a quantity of 36. Thus, if Firm 2 produces 36 units then Firm 1's best response is to produce 36 units. If Firm 1 produces 36 units, then Firm 2's best response is to produce 36 units. Because each firm is using a strategy that is a best response to the other firm's strategy, we have an equilibrium.

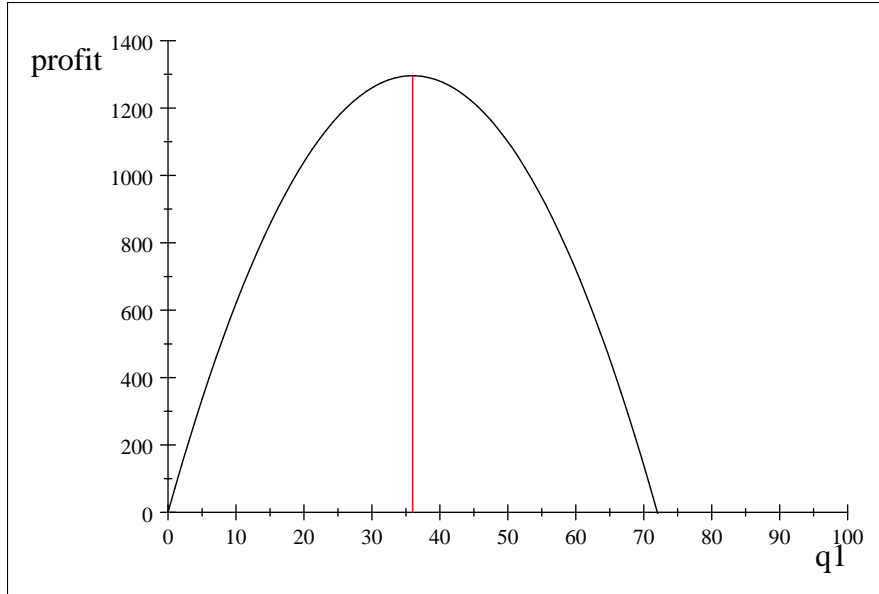
Graphical representations of the equilibrium Another way to find the equilibrium is to plot the best response functions. We can rewrite $q_1 = \frac{a-c-bq_2}{2b}$ and $q_2 = \frac{a-c-bq_1}{2b}$ as $q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2$ and $q_2 = \frac{a-c}{2b} - \frac{1}{2}q_1$. If we plot these on a graph we will get:



The red line (flatter line) is Firm 2's best response function and the green line (steeper line) is Firm 1's best response function. The point of intersection is the equilibrium point – it is where both firms are choosing their best responses to each other. Note that the lines intersect when $q_1 = 36$ and $q_2 = 36$.

Finally, we can look at Firm 1's profit when Firm 2 chooses 36 and Firm 2's profit when Firm 1 chooses 36.

²The intuition is that selling one more unit generates additional revenue of \$47 (since we sell one more unit), but the additional cost is the direct cost of selling one more unit (the \$12 *MC*) plus the decrease in revenue that occurs from selling the first 36 units at one dollar less than they were being sold before. Thus, the total additional cost is $12 + 36 = 48$, so the firm loses \$48 while only gaining \$47, which means it is less profitable to increase production.



The parabola is Firm 1's profit when $q_2 = 36$. The vertical red line corresponds to when $q_1 = 36$, which is the maximum of the profit function. Thus, when $q_2 = 36$, Firm 1 is maximizing its profit when $q_1 = 36$. Because the firms are identical, the same picture will result for Firm 2 (holding $q_1 = 36$).

2.1.4 Comparing the cartel, perfect competition, and Cournot outcomes

We began the discussion of oligopoly behavior by looking at the two extreme forms of behavior (cartel and perfect competition) and asserting that the real-world outcome was likely between those two. The table below compares the cartel, perfect competition, and Cournot outcomes using the parameters $a = 120$, $b = 1$, and $c = 12$.

	Q	q_1	q_2	$Price$	Π_1	Π_2
Cartel	54	27	27	66	1458	1458
Cournot	72	36	36	48	1296	1296
Perfect Competition	108	54	54	12	0	0

We can see that the price and quantity that result from Cournot competition falls between the extreme forms of behavior of the firms, which corresponds nicely to our assertion.

2.1.5 Cournot behavior and k firms

One other aspect of Cournot behavior that conforms with intuition is that as the number of firms increases the profit per firm decreases, and when there is an infinite number of firms profits become zero. Thus, if there is a very large number of firms then Cournot behavior approaches perfectly competitive behavior. We can show this by analyzing the profit a particular firm earns.

In the two-firm case the Cournot quantities are $\frac{a-c}{3b}$ for both firms, which leads to a total market quantity of $\frac{2a-2c}{3b}$. The price in the market is then:

$$P(Q) = a - b \left(\frac{2a - 2c}{3b} \right)$$

Simplifying this expression gives:

$$P(Q) = \frac{a + 2c}{3}$$

Firm profits are then:

$$\Pi_1 = \Pi_2 = \left(\frac{a + 2c}{3} \right) * \left(\frac{a - c}{3b} \right) - c * \left(\frac{a - c}{3b} \right)$$

Factoring out the $\left(\frac{a-c}{3b}\right)$ term gives:

$$\Pi_1 = \Pi_2 = \left(\left(\frac{a+2c}{3} \right) - c \right) * \left(\frac{a-c}{3b} \right)$$

Simplifying the first bracketed term, $\left(\left(\frac{a+2c}{3}\right) - c\right)$ gives:

$$\Pi_1 = \Pi_2 = \left(\frac{a-c}{3} \right) * \left(\frac{a-c}{3b} \right)$$

Or:

$$\Pi_1 = \Pi_2 = \frac{(a-c)^2}{9b}$$

Note that this is the profit for each firm in a duopoly. The general profit function for an oligopoly with k firms is:

$$\Pi_1 = \Pi_2 = \dots = \Pi_k = \frac{(a-c)^2}{(k+1)^2 b}$$

Notice that if we substitute in $k = 2$ we get the previous result, with $9b$ in the denominator. As k becomes very large, the profits to the firms fall, because we are dividing the same number, $(a-c)^2$ in this case, by an even larger number as k becomes bigger. Again, this result conforms with our previously held belief that if we have a large number of firms in the industry and the firms are in equilibrium then we should see zero economic profits.

2.2 Pricing games

About 50-60 years after Cournot, another economist (Bertrand) found fault with Cournot's work. Bertrand believed that firms competed by choosing prices, and then letting the market determine the quantity sold. Recall that if a monopolist wishes to maximize profit it can choose either price or quantity while allowing the market to determine the variable that the monopolist did not choose. The resulting price and quantity in the market is unaffected by the monopolist's decision of which variable to use as its strategic variable. We will see that this is not the case for a duopoly market.

The general structure of the game is as follows. There are identical 2 firms competing in the market – the firms produce identical products, have the same cost structure ($TC = c * q$ and $MC = c$), and face the same downward sloping inverse demand function, $P(Q) = a - bQ$. However, in this game it is more useful to structure the inverse demand function as an actual demand function (because the firms are choosing prices and allowing the market to determine the quantity sold), so we can rewrite the inverse demand function as a demand function, $Q(P) = \frac{a}{b} - \frac{1}{b}P$. Consumers have no brand or firm loyalty, and it is assumed that all consumers know the prices of both firms in the market. Consumers will purchase from the lowest priced producer according to the demand function. This last assumption means that each firm's quantity is determined by the table below (p_1 is Firm 1's price choice and p_2 is Firm 2's price choice):

	q_1	q_2
if $p_1 > p_2$	0	$\frac{a-p_2}{b}$
if $p_1 = p_2$	$\frac{1}{2} * \frac{a-p_1}{b}$	$\frac{1}{2} * \frac{a-p_2}{b}$
if $p_1 < p_2$	$\frac{a-p_1}{b}$	0

Thus, the firm with the lowest price will sell the entire market quantity at that price. If the firms have equal prices then they will each sell $\frac{1}{2}$ the total market quantity at that price. Now we will see what happens if the firms choose the monopoly, the Cournot, or the perfectly competitive price. These prices correspond to the ones derived in the section on the quantity games, using the parameter $a = 120$, $b = 1$, and $c = 12$.

2.2.1 Choosing the monopoly price

Suppose that the 2 firms both choose the monopoly price, which was \$66. Each then sells $\frac{1}{2}$ of the monopoly quantity, which means that $q_1 = q_2 = 27$. Firm profits are then $\Pi_1 = \Pi_2 = 1458$. Suppose that Firm 1 decides to cheat and chooses a lower price of \$65. Since $p_1 < p_2$, Firm 1 then produces $\frac{120-65}{1} = 55$ units. Firm 1's profits are: $\Pi_1 = 65 * 55 - 12 * 55 = 2915$, which is greater than 1458. So Firm 1 has the incentive to lower its price (as does Firm 2), which means choosing the monopoly price is NOT an equilibrium to Bertrand competition.

2.2.2 Choosing the Cournot price

Suppose that the 2 firms both choose the Cournot price, which was \$48. Each firm then sells $\frac{1}{2}$ of the total Cournot quantity, which means that $q_1 = q_2 = 36$. Firm profits are then $\Pi_1 = \Pi_2 = 1296$. Suppose that Firm 1 decides to cheat (just as a reminder, the firms are NOT jointly deciding to produce the Cournot quantity when playing the Cournot game – each is acting in its own self-interest) and chooses a lower price of \$47. Since $p_1 < p_2$, Firm 1 then produces $\frac{120-47}{1} = 73$ units. Firm 1's profits are now: $\Pi_1 = 47 * 73 - 12 * 73 = 2555$, which is greater than 1296. So Firm 1 has the incentive to lower its price (as does Firm 2), which means choosing the Cournot price is NOT an equilibrium to Bertrand competition.

2.2.3 Choosing the perfectly competitive price

Suppose that the 2 firms both choose the perfectly competitive price, which was \$12. Each firm then sells $\frac{1}{2}$ of the total perfect competition quantity, which means that $q_1 = q_2 = 54$. Firm profits are then $\Pi_1 = \Pi_2 = 0$. Suppose that Firm 1 wishes to change its strategy by lowering its price to \$11. It captures the entire market, and sells $\frac{120-11}{1} = 109$. Firm 1's profits are now: $\Pi_1 = 11 * 109 - 12 * 109 = (-109)$. Clearly, lowering the price makes Firm 1 worse off. If Firm 1 attempts to raise the price above \$12, then $p_2 < p_1$, and Firm 2 captures the entire market. This means that Firm 1's profit (if it raises the price to \$13) is still 0, so it did not make itself better off. Thus, the perfectly competitive outcome is the NE to this Bertrand game.

2.2.4 Comparing Cournot and Bertrand

Under Cournot competition each firm made a positive economic profit, and the perfectly competitive outcome is only achieved when the number of firms becomes large. Under Bertrand competition the perfectly competitive outcome is achieved with only two firms. Thus, we tend to assume that the Cournot outcome is more applicable in the framework we have been discussing – however, this Bertrand setting assumes identical products – if we were to assume differentiated products, or that firm's had capacity constraints, then we would reach an outcome that is more similar to the Cournot outcome.

3 Equilibrium in a sequential setting

We had been studying equilibrium in settings where firms make decisions at the same time. Now, we want to extend the analysis to include sequential settings, where one firm moves first, the second firm observes this decision, and then the second firm makes its decision. To analyze sequential settings, we will work from the end of the process (so we will start with the second mover), determine what the second mover would do for any choice that the first mover could make, and then incorporate that information into the first mover's decision.

Suppose that there are two firms (Firm A and Firm B) engaged in competition. Firm A will choose its quantity level first, and then Firm B will choose its quantity level after observing Firm A's choice. To keep this example simple, assume that the firms' quantity choices are restricted to be either 48 units or 64 units. If both firms choose to produce 64 units, then both firms will receive a payoff of \$4.1. If both firms choose to produce 48 units, then both firms will receive a payoff of \$4.6. If one firm chooses to produce 48 units and the other chooses to produce 64 units, the firm that produces 48 units receives a payoff of \$3.8 while

the firm that produces 64 units receives a payoff of \$5.1. This setting is sequential because Firm A chooses first and Firm B observes Firm A's decision.³

As I mentioned, to find an equilibrium in sequential settings we start from the end and work our way back towards the beginning. This is called backward induction. To find the equilibrium, we first determine what Firm B would do given a quantity choice by Firm A. In this example, Firm B would choose $Q_B = 64$ if Firm A chose $Q_A = 48$ because $\$5.1 > \4.6 . Also, Firm B would choose $Q_B = 64$ if Firm A chose $Q_A = 64$ because $\$4.1 > \3.8 . Thus, Firm B's strategy is: {Choose $Q_B = 64$ if Firm A chooses $Q_A = 48$; choose $Q_B = 64$ if Firm A chooses $Q_A = 64$ }. We now know what Firm B will do for any given choice by Firm A.

Firm A, knowing that Firm B will choose $Q_B = 64$ regardless of its quantity choice, can now disregard any payoffs that result when $Q_B = 48$. The reason that Firm A can disregard these payoffs is that it knows that it will never see the payoffs associated with Firm B choosing $Q_B = 48$ because Firm B will never choose $Q_B = 48$.

Firm A has one decision to make, produce a quantity of 48 or a quantity of 64. If it produces a quantity of 48, Firm B will produce 64, and Firm A will receive a payoff of \$3.8. If it produces a quantity of 64, Firm B will produce 64, and Firm A will receive a payoff of \$4.1. Because $\$4.1 > \3.8 , Firm A will choose $Q_A = 64$. Thus, the complete equilibrium in this market is:

Firm A: Choose $Q_A = 64$

Firm B: Choose $Q_B = 64$ if Firm A chooses $Q_A = 48$; choose $Q_B = 64$ if Firm A chooses $Q_A = 64$

Now, when the game is played only one payoff is received. To find this payoff just follow the path outlined by the equilibrium strategies. Firm A chooses $Q_A = 64$, and if Firm A chooses $Q_A = 64$ then Firm B chooses $Q_B = 64$, which leads to a payoff of \$4.1 for Firm A and \$4.1 for Firm B. Notice that we didn't use the fact that Firm B chooses $Q_B = 64$ if Firm A chooses $Q_A = 48$ because Firm A did not choose $Q_A = 48$. We still need to include that piece as part of our equilibrium strategy even though we don't use it when we find the path that the game actually follows.

4 Sequential Bertrand competition

Recall that in Bertrand competition the competing firms choose the price that they want to sell at in the market. The firm with the lowest price sells the quantity that corresponds to the entire market quantity at that price, while the firm with the higher price sells nothing. If the two firms choose the same price, then each firm sells $\frac{1}{2}$ the market quantity at that price. Assume that the firms are identical, and that each firm has constant MC equal to c . To make this sequential Bertrand competition, assume that Firm A chooses its price first, and then Firm B observes Firm A's choice and sets its own price.

Again, to find the solution of this game use backward induction. We want to find out what Firm B would do in response to any price choice that Firm A could make. Suppose that Firm A sets a really high price, above the MC of c . Firm B's best response would be to charge a slightly lower price and capture the entire market. Suppose that Firm A sets a really low price, less than the MC of c . Firm B's best response in this case is NOT to undercut Firm A. If it undercuts Firm A then it captures the entire market, but it captures the entire market at a price below cost which means it is making a loss, which it could avoid by not producing at all, which means that if Firm A chooses a price less than c that Firm B should choose a price greater than Firm A. We can assume that if Firm A chooses a price less than c that Firm B will choose to set its price equal to c to ensure that it does not make any losses. Suppose that Firm A chooses a price equal to the MC of c . If Firm B chooses a price below c then it captures the entire market, but at a price less than cost, which means that it is making a loss. Clearly, Firm B could do better if it decided to stay out of the market. If Firm B charges a price above c then it will not earn any profits as it allows Firm A to capture the entire market. If Firm B charges a price exactly equal to c , then it will still earn zero economic profit but at least it will then produce half of the market quantity. Formalizing this thought process into a strategy we can write down:

³In the real-world Firm A may actually choose a quantity before Firm B, but if Firm B gains no additional information from Firm A's decision (such as a change in the market price), then the setting is essentially one where Firm A and Firm B choose simultaneously.

$$P_B = \begin{cases} P_A - \varepsilon & \text{if } P_A > c \\ c & \text{if } P_A = c \\ c & \text{if } P_A < c \end{cases}$$

The term ε means the smallest possible amount by which Firm B can undercut Firm A's price (perhaps a penny). Firm A now knows that Firm B will use this strategy.⁴ Firm A then has to decide what it will do. If it prices below MC it will capture the entire market but will make a loss. If it prices above MC then Firm B will undercut its price and Firm A will sell nothing. If Firm A chooses to price at MC then it splits the market quantity with Firm B. Thus, Firm A chooses to set $P_A = c$, which means that Firm B will set $P_B = c$, which means that in sequential Bertrand competition the result is the same as in the simultaneous Bertrand game.⁵

5 Sequential quantity competition

Sequential quantity competition is called a Stackelberg game, after its "creator". In this setting one firm chooses its quantity first and then the other firm observes this quantity decision and chooses its quantity. We will assume the linear inverse demand function, $P(Q) = a - bQ$, where $Q = q_A + q_B$ and where firms costs are such that $TC = c * q_A$ and $MC = c$.

Again, begin with finding Firm B's strategy. When we worked the simultaneous Cournot game we found the best response functions for each firm. Firm B's best response function, for a given choice of q_A , was:

$$q_B = \frac{a - c - bq_A}{2b}$$

Because this problem has the same basic structure, Firm B's best response function is the same as it was in the Cournot game. Thus, for any choice of q_A we know the exact quantity amount that Firm B would choose. There is one slight caveat to this. If Firm A were to choose an amount of $q_A \geq \frac{a-c}{b}$, then Firm B would choose to produce 0. The reason why is that if Firm A chooses $q_A = \frac{a-c}{b}$, then it is choosing to produce the competitive quantity, where the price in the market equals marginal cost. If Firm A for some reason decides to produce a quantity $q_A > \frac{a-c}{b}$, then Firm A is producing a quantity such that the price in the market is LESS than MC . In this case, Firm B would opt out of the market and produce 0, as producing 0 ensures Firm B of receiving 0 profits, while producing any positive quantity will only force the price lower and ensure that Firm B earns a loss. To summarize, Firm B's strategy is:

$$q_B = \begin{cases} \frac{a-c-bq_A}{2b} & \text{if } 0 \leq q_A \leq \frac{a-c}{b} \\ 0 & \text{if } q_A > \frac{a-c}{b} \end{cases}$$

Firm A then takes Firm B's strategy as given. Firm A is like any other profit maximizing firm, and will set $MR = MC$. To do this we simply substitute Firm B's best response function in to Firm A's profit

⁴It's not that Firm B tells Firm A the strategy it will use, it's that Firm A knows the game that will be played and can also see what Firm B's best responses will be given Firm A's choice of price. Also, Firm B's strategy could have one more tier to it. If Firm A chose any price above the monopoly price, Firm B's best response would be to choose the monopoly price, not to undercut Firm A by a tiny amount. Then, for any price between the monopoly price and MC , Firm B's best strategy would be to undercut Firm A by the smallest possible amount. This, however, does not effect the result of the game.

⁵Technically, if the price space is discrete then there is an equilibrium where both firms choose a price at the lowest possible increment above MC . If $c = 12$, and firms must price in increments of pennies, then the equilibrium result is that both firms charge \$12.01 and make very, very small economic profits. This is true of the simultaneous game as well.

function and maximize profit.⁶ So:

$$\begin{aligned}\Pi_A &= (a - bq_A - bq_B)q_A - cq_A \\ \Pi_A &= \left(a - b \left(\frac{a - c - bq_A}{2b} \right) - bq_A \right) q_A - cq_A \\ \Pi_A &= \left(a - \frac{a}{2} + \frac{c}{2} + \frac{bq_A}{2} - bq_A \right) q_A - cq_A \\ \Pi_A &= \left(\frac{a}{2} + \frac{c}{2} - \frac{bq_A}{2} \right) q_A - cq_A\end{aligned}$$

Now differentiate Π_A with respect to q_A :

$$\begin{aligned}\frac{\partial \Pi_A}{\partial q_A} &= \frac{a}{2} + \frac{c}{2} - bq_A - c \\ \frac{a}{2} + \frac{c}{2} - bq_A - c &= 0 \\ a + c - 2bq_A - 2c &= 0 \\ a - c &= 2bq_A \\ \frac{a - c}{2b} &= q_A\end{aligned}$$

Thus, the first mover produces the monopoly quantity of $\frac{a-c}{2b}$. So the equilibrium to the Stackelberg game is:

$$q_A = \frac{a - c}{2b}$$

$$q_B = \begin{cases} \frac{a-c-bq_A}{2b} & \text{if } 0 \leq q_A \leq \frac{a-c}{b} \\ 0 & \text{if } q_A > \frac{a-c}{b} \end{cases}$$

We can find the payoffs to the firms of using these strategies by plugging $q_A = \frac{a-c}{2b}$ into Firm B's best response function to determine how much Firm B will produce.

$$q_B = \frac{a - c - b \left(\frac{a-c}{2b} \right)}{2b}$$

Or:

$$q_B = \frac{a - c - \left(\frac{a-c}{2} \right)}{2b}$$

Or:

$$q_B = \frac{a - c - \frac{a}{2} + \frac{c}{2}}{2b}$$

Or:

$$q_B = \frac{\frac{a}{2} - \frac{c}{2}}{2b}$$

Or:

$$q_B = \frac{a - c}{4b}$$

Thus, if Firm A produces $q_A = \frac{a-c}{2b}$, Firm B will produce $q_B = \frac{a-c}{4b}$. Note that this is NOT the equilibrium strategy for Firm B, just what the result is of Firm B using its equilibrium strategy. Total market quantity is then $q_A + q_B = \frac{a-c}{2b} + \frac{a-c}{4b} = \frac{3}{4} * \frac{a-c}{b}$, or $\frac{3}{4}$ of the perfectly competitive quantity.

⁶We know that the first mover will not choose to produce more than the competitive quantity because producing more than the competitive quantity results in $\Pi_A < 0$, and the firm can do better than this simply by producing 0, so we ignore the possibility that $q_A > \frac{a-c}{b}$.

5.1 Comparing the results

It is important to compare the results of the different market models. In the table below, I have used the values that we have been using in class, $a = 120$, $b = 1$, and $c = 12$ to compare the monopoly (or cartel), Cournot, Stackelberg, and perfectly competitive (or Bertrand, both simultaneous and sequential) outcomes. The column for CS stands for consumer surplus and the column for TS stands for total surplus, where total surplus is defined as the sum of the firm's profits and the consumer surplus.

	Q	q_A	q_B	$P(Q)$	Π_A	Π_B	CS	TS
Monopoly/Cartel	54	27	27	66	1458	1458	1458	4374
Cournot	72	36	36	48	1296	1296	2592	5184
Stackelberg	81	54	27	39	1458	729	3280.5	5467.5
Bertrand/Perfect competition	108	54	54	12	0	0	5832	5832

As should be clear from the total, consumers are made better off at the expense of the firms as we move down the table. It is interesting to note that the Cournot case, with two identical firms, is slightly less efficient than the Stackelberg case, with one large firm and one small firm (in terms of relative quantities produced). This raises the question of why antitrust policy may focus on the industry with one large firm and one small firm, rather than the one with two equal-sized firms.⁷ The reason has to do with the dynamic aspects of the markets.

⁷Also keep in mind that this is just one possible pair of demand curves and cost functions for these firms. Other settings may lead to slightly different orderings, but we will have monopoly as the "worst" outcome and perfect competition as the "best" outcome.