These notes essentially correspond to chapter 15 of the text.

This chapter deals with various rules of thumb and pricing practices that can be used by firms who produce multiple products or operate in multiple markets.

1   Markup pricing

1.1   Markup on price

In chapter 12 we discussed markup on price formula:

$$Markup \ on \ Price = \frac{P - MC}{P}$$

We already discussed that the optimal markup on price is:

$$Markup \ on \ Price = \frac{-1}{\varepsilon_P}$$

If $\varepsilon_P = -1.5$, then the optimal markup on price is $\frac{2}{3}$. If we know the elasticity and the marginal cost we can back out the optimal price (rather than going through a profit-maximization problem). Assuming $MC = 25$:

$$\begin{aligned}
\frac{2}{3} &= \frac{P - 25}{P} \\
2P &= 3P - 75 \\
P &= 75
\end{aligned}$$

1.2   Markup on cost

Above we examined markup on price; an alternative is markup on cost:

$$Markup \ on \ cost = \frac{P - MC}{MC}$$

in which we are looking at how much the firm marks up its price in relation to cost. In chapter 12 we saw that:

$$\begin{aligned}
P\left(1 + \frac{1}{\varepsilon_P}\right) &= MC \\
&\text{so} \\
P &= \frac{MC}{\left(1 + \frac{1}{\varepsilon_P}\right)}
\end{aligned}$$

Substituting into the markup on cost equation we have:

$$\begin{aligned}
MC * (Markup \ on \ cost) &= P - MC \\
MC * (Markup \ on \ cost) &= \frac{MC}{\left(1 + \frac{1}{\varepsilon_P}\right)} - MC \\
Markup \ on \ cost &= \frac{1}{\left(1 + \frac{1}{\varepsilon_P}\right)} - 1 \\
Markup \ on \ cost &= \frac{\varepsilon_P}{\varepsilon_P + 1} - 1 \\
Markup \ on \ cost &= \frac{\varepsilon_P}{\varepsilon_P + 1} - \frac{\varepsilon_P + 1}{\varepsilon_P + 1} \\
Markup \ on \ cost &= \frac{-1}{\varepsilon_P + 1}
\end{aligned}$$

Thus, if we know the price elasticity of demand, we can determine the optimal markup on cost. If $\varepsilon_P = -1.5$, then the optimal markup on cost is 2 or 200%. So if the cost of our item was \$25, then we would need to find the price such that:

$$
\begin{aligned}
2 &= \frac{P - 25}{25} \\
75 &= P
\end{aligned}
$$

Note that both markup formulas are giving the same optimal price (assuming the same elasticity and marginal cost).

2 Price discrimination

When firms use price discrimination, they charge different prices to different groups of customers. We will discuss the three main categories of price discrimination. Keep in mind that while the word "discrimination" typically has a negative connotation (at least in the U.S.), price discrimination is not always bad and may sometimes allow a market to exist when it would not exist if only a single price were charged to all customers. All forms of price discrimination require that at least two groups (perhaps more) of customers are identifiable and have different elasticities for the good, and typically the firm would like to prevent resale from one party to another.

2.1 First degree price discrimination

In first degree price discrimination, it is as if the firm knows the maximum amount each individual will pay for each unit of the good and the firm charges each individual that maximum price (or just a little below the maximum price). Recall that in our discussion of monopoly, a monopolist's marginal revenue curve was beneath it's demand curve because the monopolist had to charge the same price to all customers, and that this difference between society's marginal benefit curve (the demand curve) and the monopolist's marginal benefit curve (the marginal revenue curve) led to the monopolist producing too low of a quantity. If the monopolist could first-degree price discriminate, then the demand curve would become the monopolist's marginal benefit curve and the monopolist would produce the efficient quantity (meaning there would be no deadweight loss in the market). In the process, the monopolist would also transfer all of the consumer surplus into producer surplus.

First degree price discrimination is more of a theoretical construct than a practical application, because the firm would be required to know each individual's demand curve to charge the optimal price. The most prevalent area for first degree price discrimination to occur is in negotiated sales, such as for a car, because each individual likely pays a different price than each other and than the sticker price for the car. It is also possible to use auctions in an attempt to first degree price discriminate, though there typically is some consumer surplus that will not be captured by the auctioneer.

2.2 Second degree price discrimination

In second degree price discrimination involves discounts for purchasers of large quantities. It is easy to see how groups of consumers are identified – the firm simply has to look at how much each consumer purchases and place them in the correct group. This practice can be beneficial to the firm because the volume of sales made to a large purchaser can offset the discount given to that purchaser. The firm may not be too concerned with whether or not the large quantity purchaser resells to other purchasers in this instance; in fact, it may just be part of the supply chain process (for example, Wal-Mart is able to buy in bulk from manufacturers who do not have any, or much, direct contact with final consumers, and Wal-Mart resells these products at a higher price).

2.3 Third degree price discrimination

Third degree price discrimination is probably the most common form of price discrimination. Discounts for senior citizens, lower prices for matinee (daytime) movie showings, discounts for early in the week (Monday-

Wednesday) dining out, and differences in airline flight tickets for business travelers and vacationers are just some examples of third degree price discrimination. In each instance, consider how the "groups" are identified: senior citizens are identified by age, people going to the movies during the day and those dining out early in the week are identified by when they are purchasing the good, and business travelers and vacationers are (usually) identified by the length of their trips. Resale in most of these instances is easily preventable because the good has to be consumed when purchased.

2.3.1 Price discrimination example

Suppose that college students are willing to pay $10 to see a movie, and that seniors are willing to pay $5 (we will assume that there are no seniors who are college students for this example). Also assume that the firm can sell one extra ticket to the movie at zero MC (meaning that the MC for the firm is a flat line at zero) and that the firm has no fixed costs, so that $TR = \Pi$. We will look at 3 policies: charging everyone $10, charging everyone $5, and charging college students $10 and seniors $5 (price-discrimination). The table below shows the profit from each group:

Pricing	10 college students	20 seniors	Total Π
All $5	$50	$100	$150
All $10	$100	$0	$100
$10 College, $5 Seniors	$100	$100	$200

The demand curve for movie tickets, based upon the patrons values, is also displayed below:
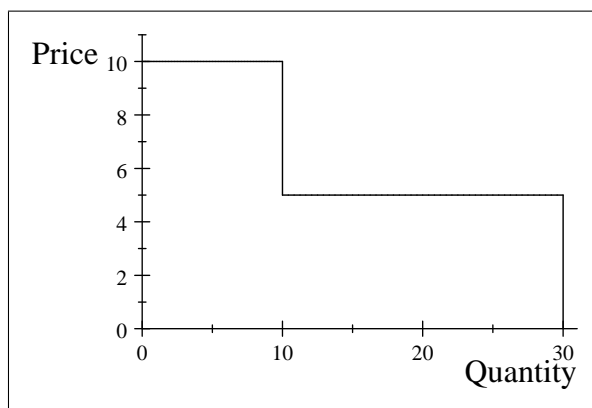
If the firm charges one price to all consumers, it will maximize profit by charging a price of $5. This will leave the college students with $50 of consumer surplus. However, the firm can earn more money by price discriminating and charging the college students $10 and the seniors $5. Thus, the firm takes the consumer surplus away from the college students, transferring the surplus to its own profit. Notice that this market was fully efficient even when a single price was charged, as the movie theater was selling a quantity of 30 tickets when it priced at $5. In this example, price discrimination allows for a transfer of consumer surplus to producer surplus (or profit because there are no costs in this example).

2.3.2 Another simple example

Keep the same parameters from above, only now there are only 5 seniors instead of 20. The table becomes:

Pricing	10 college students	5 seniors	Total Π
All $5	$50	$25	$75
All $10	$100	$0	$100
$10 College, $5 Seniors	$100	$25	$125

The demand curve for the tickets is now:
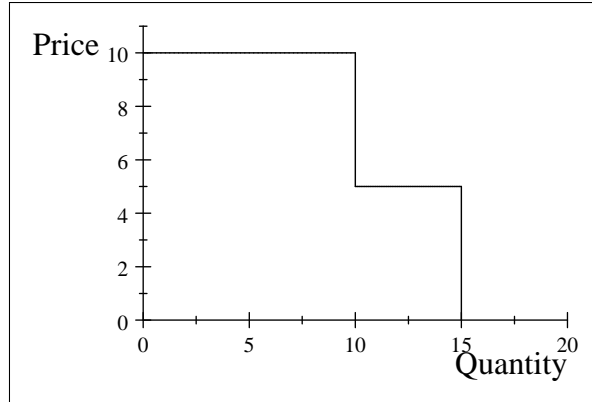
The profit-maximizing single price in this example is $10. Note that this excludes the seniors, which leads to a deadweight loss of $25. However, if the firm can price discriminate, then it will be able to sell the 5 additional tickets to the seniors, which will increase efficiency (while at the same time increasing the firm's own profits). So in this example the ability to price discriminate actually increases the efficiency in the market.

2.3.3   Multi-market price discrimination (mathematically)

The typical method the firm uses to price discriminate is similar to the method used above – break the consumers into 2 or more groups and then charge the consumer a price based upon the group into which he falls. Suppose that we have two groups, seniors and college students. College students have the inverse demand function: $P(Q) = 204 - 5Q$ and seniors have the inverse demand function $P(Q) = 152 - 2Q$. Suppose that the marginal cost per unit is constant at $4, or $MC = 4$ and that the $TC = 4Q$. If the monopolist can perfectly separate the two groups then it should act as a monopolist in both markets, and set the profit-maximizing single-price in EACH market. For the college students:

$$MC\,(college) = 4$$

$$204 - 10Q = 4$$

$$200 = 10Q$$

$$20 = Q$$

So the price for the college students is: $P(20) = 204 - 5(20) = 104$. The firm earns total revenue of $104 * 20 = 2080$ from the college students.

For the seniors:

$$MC\,(seniors) = 4$$

$$152 - 4Q = 4$$

$$148 = 4Q$$

$$37 = Q$$

The price for the seniors is: $P(37) = 152 - 2(37) = 78$. The firm earns total revenue of $37 * 78 = 2886$ from the seniors.

4

The firm's total cost is $TC = 4Q = 4*(37+20) = 228$. Its profit is then: $2886 + 2080 - 228 = 4738$. If the firm were to set one price in this market, it would set a price around \$85. It would sell about the same number of units, 57, but the firm's profit would only be 4631 as opposed to the 4738 above.

You should note that this type of analysis to find the profit-maximizing price *only* works when the marginal costs are constant. When the marginal costs are not constant, then one must sum the two demand curves and find the marginal revenue curve for this summed demand curve, and then find where $MR = MC$.

3 Two-part pricing[1]

With two-part pricing, a firm charges a lump sum fee to the consumer and then charges a per-use fee. Sam's Club is one example of two-part pricing: consumers pay an annual membership fee, and then they have to pay for the goods they purchase at the store. A gym membership could also be viewed as two-part pricing, with the monthly membership fee being the lump sum payment and then any additional services (personal training, coaching, etc.) being an additional payment beyond the membership fee. Again, the idea is that the firm will use this two-part pricing system to capture more of the consumer surplus (and possibly expand production). Similar to price discrimination, the firm must know consumer's demand and be able to prevent resale among consumers.

3.1 Identical consumers

Consider the case where all consumers have the same demand curve. Recall that a monopolist will not produce the efficient amount of output, where efficient is defined as the quantity where marginal cost intersects the demand curve. Consider a typical consumer who has the demand function:

$$P(Q) = 80 - Q$$

and a monopolist who has $TC = 10Q$. We will compare (1) the profit from charging the optimal single price, (2) the profit from charging the optimal two-part price, and (3) the profit from charging a suboptimal two-part price.

3.1.1 Optimal single price

The optimal single price is found by setting up the monopolist's profit function and solving for the optimal quantity:

$$\begin{aligned} \Pi &= (80 - Q)Q - 10Q \\ \frac{\partial \Pi}{\partial Q} &= 80 - 2Q - 10 \\ 0 &= 70 - 2Q \\ Q &= 35 \end{aligned}$$

When $Q = 35$, we have $P = 55$, and the monopolist's optimal profit is:

$$\begin{aligned} \Pi &= 55*35 - 10*35 \\ \Pi &= 45*35 \\ \Pi &= 1575 \end{aligned}$$

3.1.2 Optimal two-part price

The optimal two-part price, in the case of identical consumers, is to charge a fixed amount equal to the entire consumer surplus and then charge a per-unit price equal to marginal cost. In this problem, because of the constant marginal cost, the entire consumer surplus is the area of the right triangle given by $(80 - 10)$ (this is the y-intercept minus the marginal cost of 10) and the output when $P = MC$ (we know $MC = 10$,

[1]The example here is from Perloff, Jeffrey. (2004). *Microeconomics.* 3rd edition. Pearson Addison Wesley.

so when $P = 10$ we have $Q = 70$). The area of the triangle is $\frac{1}{2} * base * height = \frac{1}{2} * 70 * 70 = 2450$ in this instance. So the monopolist should charge the consumer a fixed fee of $2450 and then sell the consumer each unit of output at the level of marginal cost, so $P = MC = 10$. Note that the monopolist makes no profit on each individual unit sold, as all of the profit is from the fixed fee of $2450. Note that this profit is greater than the $1575 that the monopolist made from charging the optimal single per-unit price.

3.1.3 Suboptimal two-part price

Suppose that the monopolist now decides that it does not like making zero profit on each unit sold, and that it decides to charge a price above its marginal cost of 10. Suppose the monopolist decides to charge $20. Now, the consumer surplus in this market is still the area of a right triangle, but it is a right triangle with height 60 (the intercept of 80 minus the price of 20 being charged), and a quantity sold of 60 as well (because $P(Q) = 80 - Q$, so when $P = 20$, $Q = 60$). We still have $\frac{1}{2} * base * height = \frac{1}{2} * 60 * 60 = 1800$. So the optimal fixed fee, if the monopolist wishes to charge $P = \$20$, is $1800. Clearly this amount is less than the $2450 the monopolist earned in profit from the optimal fixed fee, but the monopolist also makes $10 per-unit on each unit sold when its price is $20. However, it only sells 60 units, so it makes $600 from the per-unit sales. When added to the $1800 fixed fee, the monopolist only makes $2400 from this consumer, which is less than the $2450 from the optimal two-part price.

3.2 Nonidentical consumers

Now consider a case with two representative consumers who have different demand curves. If the monopolist knows each individual's demand curve, and can prevent resale of the item, then the monopolist will maximize profit by charging each consumer type a different fixed fee. Let one consumer have the demand curve from above:

$$P(Q_1) = 80 - Q_1$$

while a second consumer has the following demand curve:

$$P(Q_2) = 100 - Q_2$$

We know that the optimal two-part price for the first consumer type is to charge $2450 as a fixed fee and then $10 per unit. For the second type the optimal fixed fee would be $\frac{1}{2} * 90 * 90 = 4050$ and the per-unit price would be $10. So the monopolist's profit if it knows each individual demand curve is $6500.

3.2.1 Single lump sum fee and optimal pricing

Suppose the monpolist knows the demand curves but must charge the same fee and per-unit price to each consumer (perhaps there are some legal restrictions). The monopolist will want to choose a lump sum fee equal to the consumer surplus of one of the two types. If it charges a lump sum fee equal to the potential consumer surplus of the second consumer type, then it will only sell to that type (the fee will be too expensive for the first consumer type because that type does not value the good that much) and so we know that the optimal fee would be $4050 and the optimal per-unit price would be $10.

Now consider a lump see fee being charged equal to the potential consumer surplus of the first type. We know that if only the first type is in the market the optimal lump sum fee is $2450 and the optimal per-unit price is $10. Because there are two types of consumers the consumer would receive $4900 in payments from the two types, which is better than the $4050 it would receive from charging a lump sum fee equal to the consumer surplus of the second type. The question is, can the firm do better by lowering its lump sum fee and charging a higher per-unit price?

Consider what the firm's profit is. The firm will receive the lump sum fee from each type of consumer, and then it will sell units to each type of consumer. The monopolist's profit function is then:

$$\Pi = 2 * CS_1 + (P * Q_1 - C * Q_1) + (P * Q_2 - C * Q_2)$$

where CS_1 is the potential consumer surplus to the first consumer type and C is the marginal cost. The potential consumer surplus to the first type is given by:

$$CS_1 = \frac{1}{2}(80 - P)Q_1$$

$$CS_1 = \frac{1}{2}(80 - P)(80 - P)$$

$$CS_1 = \frac{1}{2}(80 - P)^2$$

Recall that the potential consumer surplus is just the area of the triangle under the demand curve but above the price. Now we have:

$$\Pi = 2 * \frac{1}{2}(80 - P)^2 + (P * Q_1 - C * Q_1) + (P * Q_2 - C * Q_2)$$

$$\Pi = (80 - P)^2 + (P - C)(Q_1 + Q_2)$$

Substituting in for Q_1 and Q_2 we have:

$$\Pi = (80 - P)^2 + (P - C)(80 - P + 100 - P)$$

$$\Pi = (80 - P)^2 + (P - C)(180 - 2P)$$

Now we can just differentiate with respect to P, set the derivative equal to zero, and solve for the optimal P:

$$\frac{\partial \Pi}{\partial P} = 2(80 - P)(-1) + (180 - 2P) + (P - C)(-2)$$

$$0 = -160 + 2P + 180 - 2P - 2P + 2C$$

$$0 = 20 - 2P + 2C$$

$$P = 10 + C$$

Because $C = 10$, we know that $P = 20$. We also know that the lump sum fee is:

$$CS_1 = \frac{1}{2}(80 - P)^2$$

$$CS_1 = \frac{1}{2}60^2$$

$$CS_1 = 1800$$

What is the profit to the firm for setting the optimal lump sum fee and per-unit price? The firm receives $1800 from each consumer type so that is $3600 in lump sum fees. The firm is also pricing above marginal cost (which in this example is the same as average cost), so the firm will receive $10 in profit from each per-unit sale. At a price of $20, the firm sells 60 units to the type 1 consumer, and sells 80 units to the type 2 consumer. So there is an additional $1400 in profit from per-unit sales, meaning the firm has a total profit of $3600 + $1400 = $5000. Note that this profit is more than the profit the firm earns from setting a lump sum amount of $2450 and charging $10 per-unit (profit in that case was $4900).

4 Multiple product pricing

If a firm produces products that are not interrelated in the final market (the cross-price elasticity between the two goods is zero) and do not use the same production facilities, then the goods can be analyzed separately from the firm's point of view. The firm would choose the optimal production quantity based on the $MR = MC$ rule for each good and the firm's profit would be maximized.

However, if the firm produces products that are interrelated in either demand or production, then a complete analysis of each individual product market would have to incorporate the impact of each product in all markets. For instance, Kraft produces Kraft Macaroni and Cheese as well as Kool-Aid (among many

other items). If sales of Kraft Macaroni and Cheese impact sales of Kool-Aid, and vice versa, then a complete analysis of the Kraft Macaroni and Cheese market needs to incorporate its impact on the Kool-Aid market. Assuming that only these two goods are being produced, the marginal revenue for each product would be:

$$
\begin{aligned}
MR_{Mac} &= \frac{\partial TR}{\partial Q_{Mac}} = \frac{\partial TR_{Mac}}{\partial Q_{Mac}} + \frac{\partial TR_{K-A}}{\partial Q_{Mac}} \\
MR_{K-A} &= \frac{\partial TR}{\partial Q_{K-A}} = \frac{\partial TR_{K-A}}{\partial Q_{K-A}} + \frac{\partial TR_{Mac}}{\partial Q_{K-A}}
\end{aligned}
$$

4.1 Fixed proportions

In some instances production will occur in fixed proportions. This result could occur if the production of a certain amount of one good leads to a specific amount of "waste" that can be used to produce a specific amount of a second good. We will consider the case where all units of both products will be sold (there will be no excess waste).

Consider the total cost function:

$$
TC = 2,000,000 + 50Q + 0.01Q^2
$$

While we are considering markets for two goods, because these goods are produced in the same proportion we only need to consider the cost with producing Q units. The demand for goods A and B are:

$$
\begin{aligned}
P_A &= 400 - 0.01Q_A \\
P_B &= 350 - 0.015Q_B
\end{aligned}
$$

Total revenue for the firm is given by:

$$
\begin{aligned}
TR &= TR_A + TR_B \\
TR &= P_A Q_A + P_B Q_B
\end{aligned}
$$

We can now construct a profit function:

$$
\begin{aligned}
\Pi &= P_A Q_A + P_B Q_B - \left(2,000,000 + 50Q + 0.01Q^2\right) \\
\Pi &= \left(400 - 0.01Q_A\right)Q_A + \left(350 - 0.015Q_B\right)Q_B - \left(2,000,000 + 50Q + 0.01Q^2\right)
\end{aligned}
$$

Because the two goods are produced in the same proportion, we can substitute Q for Q_A and Q_B. This substitution leads to:

$$
\begin{aligned}
\Pi &= \left(400 - 0.01Q\right)Q + \left(350 - 0.015Q\right)Q - \left(2,000,000 + 50Q + 0.01Q^2\right) \\
\Pi &= 400Q - 0.01Q^2 + 350Q - 0.015Q^2 - 2,000,000 - 50Q - 0.01Q^2 \\
\Pi &= 700Q - 0.035Q^2 - 2,000,000
\end{aligned}
$$

Differentiating the profit function with respect to Q, setting the derivative equal to zero, and solving we have:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial Q} &= 700 - 0.07Q \\
0 &= 700 - 0.07Q \\
Q &= 10,000
\end{aligned}
$$

When $Q = 10,000$, we have $P_A = \$300$ and $P_B = \$200$, and the firm will earn $\$1,500,000$ in profit.

Before concluding that the firm has found the optimal quantity and prices we need to make sure that the marginal revenue for each product is greater than or equal to zero. If the marginal revenue were less

than zero, then we would be losing revenue with the last sale of whichever product had negative marginal revenue. In this example we have:

$$
\begin{aligned}
MR_A &= 400 - 0.02Q \\
MR_A &= 400 - 0.02 * 10,000 \\
MR_A &= 200 \\
&\text{and} \\
MR_B &= 350 - 0.03Q \\
MR_B &= 350 - 0.03 * 10,000 \\
MR_B &= 50
\end{aligned}
$$

4.1.1 When $MR < 0$ for one product

Now consider the case where $MR_B < 0$ when solving the firm's joint production profit maximization problem. To see this, consider that the demand for product B has fallen while cost and the demand for product A remains as above:

$$
\begin{aligned}
TC &= 2,000,000 + 50Q + 0.01Q^2 \\
P_A &= 400 - 0.01Q_A \\
P_B &= 290 - 0.02Q_B
\end{aligned}
$$

Setting up the new profit function we have:

$$
\begin{aligned}
\Pi &= P_A Q_A + P_B Q_B - \left(2,000,000 + 50Q + 0.01Q^2\right) \\
\Pi &= (400 - 0.01Q_A)\, Q_A + (290 - 0.02Q_B)\, Q_B - \left(2,000,000 + 50Q + 0.01Q^2\right)
\end{aligned}
$$

Again, substituting in Q for Q_A and Q_B we have:

$$
\begin{aligned}
\Pi &= (400 - 0.01Q)\, Q + (290 - 0.02Q)\, Q - \left(2,000,000 + 50Q + 0.01Q^2\right) \\
\Pi &= 400Q - 0.01Q^2 + 290Q - 0.02Q^2 - 2,000,000 - 50Q - 0.01Q^2 \\
\Pi &= 640Q - 0.04Q^2 - 2,000,000
\end{aligned}
$$

Differentiating with respect to Q, setting the result equal to zero and solving:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial Q} &= 640 - 0.08Q \\
0 &= 640 - 0.08Q \\
Q &= 8,000
\end{aligned}
$$

When $Q = 8,000$, we have $P_A = \$320$ and $P_B = \$130$. Profit for the firm will be $\$560,000$ if these quantities are sold at these prices.

Looking at MR_A and MR_B we have:

$$
\begin{aligned}
MR_A &= 400 - 0.02Q_A \\
MR_A &= 400 - 0.02 * 8,000 \\
MR_A &= \$240 \\
&\text{and} \\
MR_B &= 290 - 0.04Q_B \\
MR_B &= 290 - 0.04 * 8,000 \\
MR_B &= -\$30
\end{aligned}
$$

Thinking about what is happening, the firm's marginal cost at a production level of $8,000$ is $50 + 0.02 * 8000 = 210$. So when our firm adds the marginal revenue from the last unit of goods A and B they equal $\$210$,

but product B is not contributing a positive marginal revenue. This means that the firm could do better by looking only at product A as that is the only product making a positive contribution towards covering marginal cost.

When the firm focuses only on maximizing profit for good A we have:

$$
\begin{aligned}
\Pi_A &= (400 - 0.01Q_A)\,Q_A - (2,000,000 + 50Q_A + 0.01Q_A^2) \\
\Pi_A &= 400Q_A - 0.01Q_A^2 - 2,000,000 - 50Q_A - 0.01Q_A^2 \\
\Pi_A &= 350Q_A - 0.02Q_A^2 - 2,000,000
\end{aligned}
$$

Again, differentiating with respect to Q_A, setting the result equal to zero and solving we have:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial Q_A} &= 350 - 0.04Q_A \\
0 &= 350 - 0.04Q_A \\
Q_A &= 8,750
\end{aligned}
$$

Now, the firm will sell all of its Q_A at the price \$312.50. However, it also has $8,750$ units of good B. If the firm sells all of its units of good B then we have:

$$
\begin{aligned}
P_B &= 290 - 0.02Q_B \\
P_B &= 290 - 0.02 * 8750 \\
P_B &= 115
\end{aligned}
$$

So the firm's profit (from both products) would be:

$$
\begin{aligned}
\Pi &= 640Q - 0.04Q^2 - 2,000,000 \\
\Pi &= 537,500
\end{aligned}
$$

Note that this profit is lower than the profit we originally found when producing only $8,000$ units! However, the firm should NOT sell all of its units of product B. The marginal cost for product B is effectively zero because it is being produced while also producing product A. Setting marginal revenue of product B equal to its marginal cost we find that:

$$
\begin{aligned}
MR_B &= MC_B \\
290 - 0.04Q_B &= 0 \\
Q_B &= 7,250
\end{aligned}
$$

Thus, the firm should only sell $7,250$ of its $8,750$ units of good B. We will then have $P_B = 145$, and the firm's total profit will be given by:

$$
\begin{aligned}
\Pi &= (400 - 0.01Q_A)\,Q_A + (290 - 0.02Q_B)\,Q_B - (2,000,000 + 50Q_A + 0.01Q_A^2) \\
\Pi &= \$582,500
\end{aligned}
$$

We use Q_A to determine the total cost because the firm is producing $Q = Q_A = Q_B$ units (8,750) but is going to withhold some units of good B. By withholding these units (or destroying them), the firm is able to increase the price for firm B and actually increase its profit. While it seems like a lot of work to determine the exact quantities that should be sold, the firm is able to increase its profit by $22,500$.