Problems on foundations of microeconomics (chapters 1-4)

1. Following are the supply and demand function for basketball shoes. Assume that the constant values for income and the price of basketballs are $Y = 2004$ and $P_{Basketballs} = 30$ respectively.

$$\begin{aligned} Q_D^{Shoes} &= 500 - 10P_{Shoes} + \frac{1}{2}Y - 20P_{Basketballs} \\ Q_S^{Shoes} &= 20 + 4P_{Shoes} \end{aligned}$$

a Find the equilibrium price and quantity for this market.

Answer:

Plugging in for Y and $P_{Basketballs}$ and setting $Q_D^{Shoes} = Q_S^{Shoes}$ we have:

$$\begin{aligned} 500 - 10P_{Shoes} + \frac{1}{2} * 1500 - 20 * 30 &= 20 + 4P_{Shoes} \\ 650 - 10P_{Shoes} &= 20 + 4P_{Shoes} \\ 630 &= 14P_{Shoes} \\ 45 &= P_{Shoes} \end{aligned}$$

Plugging back into the supply function we have:

$$\begin{aligned} Q_S^{Shoes} &= 20 + 4P_{Shoes} \\ Q_S^{Shoes} &= 20 + 4 * 45 \\ Q_S^{Shoes} &= 200 \end{aligned}$$

So the equilibrium price and quantity in this market is $Q^* = 200$, $P^* = 45$.

b Which measure of elasticity would you use to determine if basketballs and shoes are complements or substitutes based on this demand function? Calculate the elasticity you suggest at the equilibrium price and quantity. Are shoes and basketballs complements or substitutes?

Answer:

The elasticity measure that should be used is cross-price elasticity. Use the coefficient on $P_{Basketballs}$ and multiply that coefficient by the ratio of $\frac{P_{Basketballs}}{Q_{shoes}}$. You should find that the cross-price elasticity is $-20 * (\frac{30}{200}) = -3$. Since the cross-price elasticity of the goods is negative, these goods are complements.

c Which measure of elasticity would you use to determine if shoes are a normal good or an inferior good? Calculate the elasticity you suggest at the equilibrium price and quantity. Are Nike shoes a normal good or an inferior good?

Answer:

The elasticity measure that should be used is income elasticity. Use the coefficient on Y and multiply that coefficient by the ratio of $\frac{Y}{Q_{Shoes}}$. You should find that the income elasticity is $\frac{1}{2} * \frac{1500}{200} = \frac{15}{4} = 3.75$. Since the income elasticity is positive, shoes are a normal good.

d Suppose that a price floor is imposed at $P_{Shoes} = 75$. How will this alter the price and quantity of shoes in this market?

Answer:

If a price floor is imposed at $P_{Shoes} = 75$, then suppliers would wish to supply 320 units and consumers would wish to purchase -100 units. Thus, the market would cease to exist if this price floor were imposed because consumers are unwilling to purchase any shoes at that price.

2. How is the popular notion of business profit different from the economic profit concept? What role does the idea of normal profits play in this difference?

   Answer:

   The key distinction is that business or accounting profit provides a measure of the total return on capital investment, whereas economic profit refers to the return on capital in excess of that required (expected) by investors. Normal profit refers to the risk-adjusted rate of profit required by investors to attract and retain funds for capital investment.

3. Climate Control Devices, Inc., estimates that sales of defective thermostats cost the firm \$50 each for replacement or repair. Boone Carlyle, an independent engineering consultant, has recommended hiring quality control inspectors so that defective thermostats can be identified and corrected before shipping. The following schedule shows the expected relation between the number of quality control inspectors and the thermostat failure rate, defined in terms of the percentage of total shipments that prove to be defective.

Number of quality control inspectors	Thermostat failure rate (percent)	Number of failures	Marg. Failure Reduction	Marg. Value of Failure
0	5.0	12,500	–	–
1	4.0	10,000	2,500	\$125,000
2	3.2	8,000	2,000	\$100,00
3	2.6	6,500	1,500	\$75,000
4	2.2	5,500	1,000	\$50,000
5	2.0	5,000	500	\$25,000

   The firm expects to ship 250,000 thermostats during the coming year, and quality control inspectors each command a salary of \$60,000 per year.

   a Construct a table showing the marginal failure reduction (in units) and the dollar value of these reductions for each inspector hired.

   Answer:

   See above for answers. The third column is 250,000 times the second column. The fourth column is the difference in the number of failures in the third column when adding one more inspector. The fifth column is simply \$50 (the cost of replacement or repair of a defective thermostat) times the fourth column.

   b How many inspectors should the firm hire?

   Answer:

   Inspectors cost \$60,000, so the firm should hire 3 inspectors. The firm hires 3 inspectors because the marginal value of the 3^{rd} inspector (\$75,000) is greater than his cost (\$60,000) but does not hire a 4^{th} inspector because his marginal value is less than his cost ($\$50,000 < \$60,000$).

   c How many inspectors should be hired if additional indirect costs (lost customer goodwill and so on) were to average 30 percent of direct replacement or repair costs?

   Answer:

   A 30 percent increase in cost due to indirect costs would raise the marginal value of the fourth inspector to \$65,000. Thus, the firm should hire the fourth inspector as well.

4. Consider a coffee shop opening in the Charlotte area. On average, beverage customers spend \$4 on beverages with an 80% gross margin, and food customers spend \$5 on food with a 50% gross margin. Gross margin simply reflects price minus input cost and does not reflect variable labor and related expenses. The table below shows customer traffic throughout the day:

Time of Day	Beverage Customers	Food Customers	Profit (by hour)
06:00	150	50	605.00
07:00	250	100	1050.00
08:00	200	75	827.50
09:00	175	50	685.00
10:00	100	25	382.50
11:00	200	75	827.50
12:00	200	175	1077.50
13:00	125	150	775.00
14:00	75	75	427.50
15:00	50	50	285.00
16:00	100	25	382.50
17:00	75	50	365.00
18:00	50	75	347.50
19:00	50	25	222.50
20:00	25	25	142.50
21:00	25	10	105.00
22:00	25	10	105.00

a Assume labor, electricity, and other variable costs are \$175 per hour of operation. Which hours should the store remain open?

Answer:

I have added an additional column for "profit by hour." Each food customer contributes \$2.50 to profit, while each beverage customer contributes \$3.20. We can see that the marginal value (hourly profit) is greater than \$175 for all hours from 06:00–19:00. So the store should remain open from 06:00–20:00 (because the 19:00-20:00 hour is profitable, but the 20:00-21:00 hour is not).

b Assume the store is open 365 days per year and that the rental cost of the building is \$2 million per year. Should this site remain open?

Answer:

Assuming that the optimal hours are used, the firm would earn \$8,120 per day, minus its labor costs of \$2,450 per day, or \$5,810 per day. Multiplying \$5,810 by 365 we see that the yearly profit (before rental cost of the location) is \$2,120,650, and because this is larger than \$2 million the site should remain open.

c Suppose that instead of having 50 beverage customers and 50 food customers at 15:00 the store had 10 beverage customers and 20 food customers at that time. When analyzing profit for that hour, should the store remain open or close at 15:00 and reopen at 16:00? Now consider this decision from a more practical standpoint: What impact might closing down for an hour in the middle of the day have on sales throughout the day?

Answer:

From a strict profit analysis the store should close down between 15:00 and 16:00 because its hourly profit would not be as great as its variable costs. However, consider the impact of closing the store down for an hour and then reopening it. First, all workers would need to be sent home, and then someone would have to come in to fill a 4-hour shift (16:00-20:00 hours). Second, customers who are using the store would have to be asked to leave as you closed down for an hour. Third, closing and reopening the store takes some time, so you would likely be paying workers during the 15:00-16:00 hour anyways. These are just some potential problems with closing for an hour during the day – there's also the possibility of making customers who typically arrive at 14:55 angry if they arrive at 15:02 as they will not be able to get their food and beverages.

5. An insurance company has the following total cost function:

$$TC = \$41,000,000 + \$500Q + \$0.005Q^2$$

The annual premium of \$1,500 will remain stable for upcoming periods, so $MR = P = \$1,500$.

a Calculate the profit-maximizing quantity for this company.

Answer:
Marginal cost is simply:

$$\begin{aligned} MC &= \frac{\partial TC}{\partial Q} \\ MC &= 500 + 0.01Q \end{aligned}$$

Setting $MR = MC$:

$$\begin{aligned} MR &= MC \\ 1500 &= 500 + 0.01Q \\ 1000 &= 0.01Q \\ 100,000 &= Q \end{aligned}$$

b Calculate the company's optimal profit, and optimal profit as a percentage of sales revenue (profit margin).

Answer:
Sales revenue is simply $P * Q$:

$$\begin{aligned} TR &= P * Q \\ TR &= 1,500 * 100,000 \\ TR &= 150,000,000 \end{aligned}$$

Profit is $TR - TC$, and TC is:

$$\begin{aligned} TC &= \$41,000,000 + \$500Q + \$0.005Q^2 \\ TC &= \$141,000,000 \end{aligned}$$

So profit is \$9,000,000 and profit as a percentage of sales revenue is $\frac{9,000,000}{150,000,000} = \frac{9}{150} = \frac{3}{50} = 6\%$.

6. Consider a firm with the following total cost and total revenue functions:

$$\begin{aligned} TR &= \$1,800Q - \$0.006Q^2 \\ TC &= \$12,100,000 + \$800Q + \$0.004Q^2 \end{aligned}$$

a Calculate quantity, marginal cost, average cost, price and profit at the average cost minimizing quantity.

Answer:
Recall that MC intersects ATC at the minimum of ATC. So setting $MC = ATC$ we have:

$$\begin{aligned} MC &= ATC \\ 800 + 0.008Q &= \frac{12,100,000}{Q} + 800 + 0.004Q \\ 0.004Q &= \frac{12,100,000}{Q} \\ Q^2 &= \frac{12,100,000}{0.004} \\ Q &= 55,000 \end{aligned}$$

Because $Q = 55,000$, we have that:

$$
\begin{aligned}
MC &= 800 + 0.008 * 55000 \\
MC &= 1,240
\end{aligned}
$$

And:

$$
\begin{aligned}
ATC &= \frac{12,100,000}{55,000} + 800 + 0.004 * 55,000 \\
ATC &= \frac{12,100}{55} + 800 + 220 \\
ATC &= 1,240
\end{aligned}
$$

And then because $TR = 1800Q - 0.006Q^2$, we know that $P = 1800 - 0.006Q$, so:

$$
\begin{aligned}
P &= 1800 - 0.006 * 55,000 \\
P &= 1,470
\end{aligned}
$$

And then profit is simply $TR - TC$:

$$
\begin{aligned}
\Pi &= TR - TC \\
\Pi &= \left(1800Q - 0.006Q^2\right) - \left(\$12,100,000 + \$800Q + \$0.004Q^2\right) \\
\Pi &= 80,850,000 - 68,200,000 \\
\Pi &= 12,650,000
\end{aligned}
$$

b Calculate quantity, marginal cost, average cost, price and profit at the profit maximizing quantity.

Answer:
To find the profit maximizing quantity, set $MR = MC$:

$$
\begin{aligned}
1800 - 0.012Q &= 800 + 0.008Q \\
1000 &= 0.02Q \\
50,000 &= Q
\end{aligned}
$$

And:

$$
\begin{aligned}
MC &= 800 + 0.008 * 50000 \\
MC &= 1200
\end{aligned}
$$

And:

$$
\begin{aligned}
ATC &= \frac{12,100,000}{50,000} + 800 + 0.004 * 50,000 \\
ATC &= \frac{1210}{5} + 800 + 200 \\
ATC &= 242 + 1000 \\
ATC &= 1242
\end{aligned}
$$

And:

$$
\begin{aligned}
P &= 1800 - 0.006 * 50,000 \\
P &= 1500
\end{aligned}
$$

Our profit is then (using a shortcut, that $TR = P * Q$ and $TC = ATC * Q$; we have these numbers and know these relationships, so we may as well use them):

$$
\begin{aligned}
\Pi &= (1500 * 50,000) - (1242 * 50,000) \\
\Pi &= 258 * 50,000 \\
\Pi &= 12,900,000
\end{aligned}
$$

c Compare the results from parts **a** and **b**.

Answer:

Obviously profit will be at least as large under profit maximization (part b) than under cost minimization (part a) because otherwise profits would not be maximized at the profit maximizing quantity. While per unit costs are \$2 lower under cost minimization, price is \$30 higher under profit maximization, which more than offsets the increase in cost.

7. The owners of a van conversion company have fixed capital and labor expenses of \$1.2 million per year, and variable expenses that average \$2,000 per van conversion. The annual demand function for van conversions is given by:

$$Q = 1,000 - 0.1P$$

where Q is the quantity of van conversions and P is the price.

a Calculate the profit-maximizing quantity, price, and profit levels.

Answer:

We need to construct a TC function. We know that fixed costs are \$1.2 million, and that $AVC = \$2,000$. So $TVC = \$2,000Q$, which gives us:

$$TC = 1,200,000 + 2000Q$$

Our total revenue is simply $P * Q$, but we need P first:

$$\begin{aligned} Q &= 1,000 - 0.1P \\ 0.1P &= 1,000 - Q \\ P &= 10,000 - 10Q \end{aligned}$$

Now:

$$\begin{aligned} TR &= (10,000 - 10Q)\,Q \\ TR &= 10000Q - 10Q^2 \end{aligned}$$

We can now use $MR = MC$:

$$\begin{aligned} MR &= MC \\ 10,000 - 20Q &= 2000 \\ 8000 &= 20Q \\ 400 &= Q \end{aligned}$$

And:

$$\begin{aligned} P &= 10,000 - 10 * 400 \\ P &= 6,000 \end{aligned}$$

And:

$$\begin{aligned} \Pi &= 10000Q - 10Q^2 - (1,200,000 + 2000Q) \\ \Pi &= 8000Q - 10Q^2 - 1,200,000 \\ \Pi &= 3,200,000 - 1,600,000 - 1,200,000 \\ \Pi &= 400,000 \end{aligned}$$

b Using the Lagrangian method, calculate profit-maximizing quantity, price, and profit levels if quantity is limited to 300 van conversions (due to a parts shortage) in the upcoming year.

Answer:

Our objective is to maximize profit, which we know from above is:

$$\Pi = 8000Q - 10Q^2 - 1,200,000$$

Our constraint is $Q \leq 300$, which we would write as $0 \leq 300 - Q$. Note that we already know that this constraint will hold with equality – unconstrained they produce 400 van conversions per year. But the point is to use the Lagrangian method. Setting up the Lagrangian we have:

$$\mathcal{L}(Q, \lambda) = 8000Q - 10Q^2 - 1,200,000 + \lambda(300 - Q)$$

Taking partial derivatives we have:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q} &= 8000 - 20Q - \lambda \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 300 - Q
\end{aligned}$$

Setting these equal to zero we have:

$$\begin{aligned}
8000 - 20Q - \lambda &= 0 \\
300 - Q &= 0
\end{aligned}$$

Using the easy relationship first, we know that $Q = 300$. So price (using the price function in part a) is:

$$\begin{aligned}
P &= 10,000 - 10Q \\
P &= 10,000 - 10 * 300 \\
P &= 7,000
\end{aligned}$$

And profit is:

$$\begin{aligned}
\Pi &= 8000Q - 10Q^2 - 1,200,000 \\
\Pi &= 8000 * 300 - 10 * 300^2 - 1,200,000 \\
\Pi &= 2,400,000 - 2,100,000 \\
\Pi &= 300,000
\end{aligned}$$

c Calculate and interpret λ, the Lagrangian multiplier.

Answer:

We know:

$$\begin{aligned}
8000 - 20Q - \lambda &= 0 \\
8000 - 20Q &= \lambda \\
8000 - 20 * 300 &= \lambda \\
2000 &= \lambda
\end{aligned}$$

Looking at the second step, we know that $8000 - 20Q = \frac{\partial \Pi}{\partial Q}$ which is also equal to λ. Thus, λ represents the increase in marginal profits, which is \$2,000 in this case, so profit will increase by \$2,000 if output increased by one unit. Note that this is NOT true for every additional unit produced, just the 301^{st}. Our λ would change if the restriction limited them to 301 units of production.

d Calculate the value to the owners of having the parts shortage removed.

Answer:

They earned $400,000 when unrestricted and $300,000 when restricted, so they would gain $100,000 if the restriction was removed.

8. Demand and Supply Curves. The following relations describe monthly demand and supply relations for dry cleaning services in the metropolitan area:

$$\begin{aligned} Q_D &= 500,000 - 50,000P \text{ (Demand)} \\ Q_S &= -100,000 + 100,000P \text{ (Supply)} \end{aligned}$$

where Q is quantity measured by the number of items dry cleaned per month and P is average price in dollars.

a At what average price level would demand equal zero?

Answer:

Demand equals 0 when:

$$\begin{aligned} 0 &= 500,000 - 50,000P \\ 50,000P &= 500,000 \\ P &= 10 \end{aligned}$$

b At what average price level would supply equal zero?

Answer:

Supply equals 0 when:

$$\begin{aligned} 0 &= -100,000 + 100,000P \\ 100,000 &= 100,000P \\ 1 &= P \end{aligned}$$

c Calculate the equilibrium price/output combination.

Answer:

Set $Q_D = Q_S$ and solve for P:

$$\begin{aligned} 500,000 - 50,000P &= -100,000 + 100,000P \\ 600,000 &= 150,000P \\ 4 &= P \end{aligned}$$

Because $P = 4$, we have that $Q_D = Q_S = 300,000$.

9. Consider the following supply and demand functions for Ramen noodles. The variables are defined in the table below. Constant values are given for the last 2 variables.

Variable	**Meaning**	**Constant value**
Q_D	Quantity demanded of Ramen	
Q_S	Quantity supplied of Ramen	
P_{Ramen}	Price of Ramen	
P_{Kraft}	Price of Kraft Mac and Cheese	$0.99
Y	Consumer income	$11,500

$$Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + (100,000)\,P_{Kraft} - (20)\,Y$$
$$Q_S = -100,021 + (680,000)\,P_{Ramen}$$

a Write down the inverse demand function for Ramen noodles.

Answer:

To find the inverse demand function simply isolate P_{Ramen} on the left-hand side of the equation. The inverse demand function is:

$$P_{Ramen} = 0.425 - 0.000000373Q_D + 0.0373P_{Kraft} - 0.00000745Y$$

If you substituted in for P_{Kraft} and Y, which you did not have to do. I believe this altered the intercept to approximately 0.376, with the slope of the inverse demand function remaining the same.

b Find the equilibrium price and quantity in this market.

Answer:

Substitute in the constant values for income and the price of Kraft Macaroni and Cheese to find that:

$$Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + 99,000 - 230,000$$

We now have supply and demand functions:

$$Q_D = 1,010,000 - 2,683,700P_{Ramen}$$
$$Q_S = -100,021 + (680,000)\,P_{Ramen}$$

Now, set $Q_D = Q_S$ and then set the 2 equations equations equal to one another so that:

$$1,010,000 - 2,683,700P_{Ramen} = -100,021 + (680,000)\,P_{Ramen}$$

Now, solve for P_{Ramen}.

$$\frac{1,110,021}{3,363,700} = P_{Ramen}$$

We find that the price of Ramen noodles is \$0.33. To calculate the equilibrium quantity just plug 0.33 into either the supply or demand function. You should find that the equilibrium quantity is $124,379$.

c Suppose that P_{Kraft} increases to \$1.33. Recalculate the equilibrium price and quantity given this change.

Answer:

Because nothing has changed in the supply function from the original problem it is still the same:

$$Q_S = -100,021 + (680,000)\,P_{Ramen}$$

However, the change in the price of Kraft will cause a shift in the demand curve. To find the new demand function, plug in the new price of Kraft (as well as the old income value) to get:

$$Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + (100,000)(1.33) - (20)(11,500)$$

Simplifying,

$$Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + 133,000 - 230,000$$

$$Q_D = 1,044,000 - (2,683,700)\,P_{Ramen}$$

Now just set $Q_D = Q_S$ and solve for price.

$$-100,021 + (680,000)\,P_{Ramen} = 1,044,000 - (2,683,700)\,P_{Ramen}$$

$$3,363,700 P_{Ramen} = 1,144,021$$

We should find that $P_{Ramen} \approx 0.34$. Plugging this back into the demand function we find that $Q_D = 131,252.3835$ or a quantity of about 131,252.

NOTE: If you try to substitute 0.34 in as the equilibrium price you will get different numbers for Q_D and Q_S. You would have found $Q_D = 131,542$ and $Q_S = 131,179$.

d Calculate the own-price elasticity of demand. Use the equilibrium price and quantity as your initial price and quantity. Is demand elastic or inelastic at the equilibrium price and quantity?

Answer:

The own-price elasticity of demand is given by the following formula, where PED stands for price elasticity of demand:

$$PED = \frac{\Delta Q_D}{\Delta P_{own}} * \frac{P_{own}}{Q_D}$$

Recall that the first term, $\frac{\Delta Q_D}{\Delta P_{own}}$, is simply the coefficient on the own-price of the good in the demand function. So it is $-2,683,700$. We know that $P_{own} = 0.33$ and $Q_D = 124,379$. Plugging these numbers in gives us:

$$PED = -2,683,700 * \frac{0.33}{124,379} = -7.120342$$

So we find that the demand for Ramen noodles is elastic, since the PED is greater than 1 in absolute value.

e Calculate the cross-price elasticity for a 1% increase in the price of Kraft Macaroni and Cheese. Are Ramen noodles and Kraft Macaroni and Cheese substitutes or complements? Explain how you know whether they are substitutes or complements.

Answer:

The formula for the cross-price elasticity of demand is:

$$X - price = \frac{\Delta Q_D^A}{\Delta P_B} * \frac{P_B}{Q_D^A}$$

The term $\frac{\Delta Q_D^A}{\Delta P_B}$ is simply the coefficient on the price of good B (in our case, Kraft Macaroni and Cheese). We know that $P_B = 0.99$ and that $Q_D^A = 124,379$. Plugging in these values gives us:

$$X - price = 100000 * \frac{0.99}{124379} = 0.7959543$$

Ramen noodles and Kraft Macaroni and Cheese are substitutes. We can tell because the cross-price elasticity is positive between the quantity demanded of Ramen and the price of Kraft.

f Calculate the income elasticity for Ramen noodles. Use the equilibrium price and the constant value for income. Are Ramen noodles a normal good or an inferior good? How do you know? If it is a normal good, is it a necessity or a luxury?

Answer:

The formula for income elasticity is:

$$IE = \frac{\Delta Q_D}{\Delta Y} * \frac{Y}{Q_D}$$

The term $\frac{\Delta Q_D}{\Delta Y}$ is simply the coefficient on income in the demand function. We know that $Y = \$11,500$ and that $Q_D = 124,379$. Plugging in these values gives us:

$$IE = -20 * \frac{11500}{124379} = -1.849187$$

Ramen noodles are an inferior good because the income elasticity for Ramen noodles is negative.

10. Rob's utility function over goods a, b, and c is given by:

$$U(a, b, c) = 12a^2b^4\sqrt{c}$$

Rob has an income of $Y = 5200$ and the prices of goods a, b, and c are $p_a = 2$, $p_b = 8$, and $p_c = 4$ respectively. Find Rob's optimal bundle of goods a, b, and c.

Answer:

Note that we have an interior solution so that $a > 0$, $b > 0$, and $c > 0$ at the optimal bundle. To see this look at the utility function:

$$U(a, b, c) = 12a^2b^4\sqrt{c}$$

Note that if either $a = 0$, $b = 0$, or $c = 0$ then Rob's utility would equal zero, and he could easily increase his utility by consuming some small amount of the other one or two goods he is not consuming. Setting up the Lagrangian:

$$\max_{a,b,c} \mathcal{L}(a, b, c, \lambda) = U(a, b, c) + \lambda(Y - p_A a - p_B b - p_C c)$$

The resulting conditions are

$$\begin{aligned}
1 &: \frac{\partial \mathcal{L}}{\partial a} = 24ab^4c^{1/2} - \lambda p_A = 0 \\
2 &: \frac{\partial \mathcal{L}}{\partial b} = 48a^2b^3c^{1/2} - \lambda p_B = 0 \\
3 &: \frac{\partial \mathcal{L}}{\partial c} = 6a^2b^4c^{-1/2} - \lambda p_C = 0 \\
4 &: \frac{\partial \mathcal{L}}{\partial \lambda} = Y - p_A a - p_B b - p_C c = 0
\end{aligned}$$

I've set all 4 equal to zero because we know that $a > 0$, $b > 0$, $c > 0$, and $\lambda > 0$. I'm going to move the λp_X terms to the right hand side of the equations so we have:

$$\begin{aligned}
24ab^4c^{1/2} &= \lambda p_A \\
48a^2b^3c^{1/2} &= \lambda p_B \\
6a^2b^4c^{-1/2} &= \lambda p_C \\
Y - p_A a - p_B b - p_C c &= 0
\end{aligned}$$

If we take the ratio of the 2^{nd} equation to the first one we get:

$$\begin{aligned}
\frac{48a^2b^3c^{1/2}}{24ab^4c^{1/2}} &= \frac{\lambda p_B}{\lambda p_A} \\
\frac{2a}{b} &= \frac{p_B}{p_A}
\end{aligned}$$

Note that this is just $MRS_{BA} = MRT_{BA}$. Now take the ratio of the 3^{rd} equation to the 1^{st} one to get:

$$\begin{aligned}
\frac{6a^2b^4c^{-1/2}}{24ab^4c^{1/2}} &= \frac{\lambda p_C}{\lambda p_A} \\
\frac{a}{4c} &= \frac{p_C}{p_A}
\end{aligned}$$

This is just $MRS_{CA} = MRT_{CA}$. If we add in the budget constraint we have:

$$\begin{aligned}
\frac{2a}{b} &= \frac{p_B}{p_A} \\
\frac{a}{4c} &= \frac{p_C}{p_A} \\
Y &= p_A a + p_B b + p_C c
\end{aligned}$$

We can then find b and c in terms of a:

$$\begin{aligned}
b &= \frac{2ap_A}{p_B} \\
c &= \frac{ap_A}{4p_C}
\end{aligned}$$

Substituting into the budget constraint we have:

$$\begin{aligned}
Y &= p_A a + p_B\left(\frac{2ap_A}{p_B}\right) + p_C\left(\frac{ap_A}{4p_C}\right) \\
Y &= p_A a + 2ap_A + \frac{ap_A}{4} \\
4Y &= 4p_A a + 8p_A a + p_A a \\
4Y &= 13p_A a \\
\frac{4Y}{13p_A} &= a
\end{aligned}$$

With $Y = 5200$ and $p_a = 2$, $p_b = 8$, and $p_c = 4$ we get:

$$\begin{aligned}
\frac{4 * 5200}{13 * 2} &= a \\
800 &= a
\end{aligned}$$

Now, using our results for b and c in terms of a we have:

$$\begin{aligned}
b &= \frac{2ap_A}{p_B} \\
b &= \frac{2 * 800 * 2}{8} \\
b &= 400
\end{aligned}$$

and

$$\begin{aligned}
c &= \frac{ap_A}{4p_C} \\
c &= \frac{800 * 2}{4 * 4} \\
c &= 100
\end{aligned}$$

So $a = 800$, $b = 400$, and $c = 100$. To complete the problem I'll find λ, and it is:

$$\begin{aligned}
24ab^4c^{1/2} &= \lambda p_A \\
\frac{24ab^4c^{1/2}}{p_A} &= \lambda \\
\frac{24 * 800 * 400^4 * 100^{1/2}}{2} &= \lambda \\
12 * 400 * 400^3 * 200 * 5 &= \lambda
\end{aligned}$$

So $\lambda > 0$, but we already knew that from the equation $24ab^4c^{1/2} = \lambda p_A$ because all of the terms (a, b, c, p_A) are positive.

11. Holding all else equal, indicate how each of the following changes would affect a budget constraint that limits consumption of goods (Y) and services (X). Explain your answer.

a Deflation that uniformly drops the price of all goods and services.

Answer:

A parallel shift to the right (outward) because income has remained the same but all prices have dropped in the same proportion.

b Inflation that consistently increases the price of all goods and services.

Answer:

A parallel shift to the left (inward) because income has remained the same but all prices have increased in the same proportion.

c Technical change that reduces the price of goods, but leaves the price of services unchanged.

Answer:

If goods are on the Y axis, then the Y-intercept will increase because goods are less expensive but the X intercept will remain the same because the prices of services have not changed.

d Economic growth that boosts the level of disposable income.

Answer:

A parallel shift to the right (outward) because prices are now constant, but income itself has increased.

e Government-mandated health care coverage for workers that boosts the price of goods by 3% and increases the price of services by 5%.

Answer:

These changes will both have negative impacts on the amounts of goods and services the consumers can buy, but the impact will be worse for services because the increase is larger. Thus, both the x-intercept and y-intercept will decrease, but the x-intercept will decrease more.