

Problems on regression and production/cost (chapters 5, 7, 8)

1. Getaway Tours, Inc., has estimated the following multiplicative demand function for packaged holiday tours in the East Lansing, Michigan, market using quarterly data covering the past four years (16 observations):

$$\begin{aligned} Q_y &= 10P_y^{-1.10}P_x^{0.5}A_y^{3.8}A_x^{2.5}I^{1.85} \\ R^2 &= 80\%, \text{ } SEE = 20 \end{aligned}$$

Here, Q_y is the quantity of tours sold, P_y is average tour price, P_x is average price for some other good, A_y is tour advertising, A_x is advertising of some other good, and I is per capita disposable income. The standard errors of the exponents in the preceding multiplicative demand function are:

$$b_{P_y} = 0.04, b_{P_x} = 0.35, b_{A_y} = 0.5, b_{A_x} = 0.9, b_I = 0.45$$

- Is tour demand elastic with respect to price?
 - Are tours a normal good?
 - Is X a complement good or substitute good?
 - Given your answer to part C, can you explain why the demand effects of A_y and A_x are both positive?
 - Test for the statistical significance of the individual coefficient estimates. Use the null hypothesis that each coefficient estimate equals zero.
 - Conduct an F-test to determine if all the independent variables, other than the constant, are jointly significant.
2. Colorful Tile, Inc., is a rapidly growing chain of ceramic tile outlets that caters to the do-it-yourself home remodeling market. In 2007, 33 stores were operated in small to medium-size metropolitan markets. The equation for sales was estimated to be the following:

$$\begin{aligned} Q &= 4 - 5P + 2A + 0.2I + 0.25HF \\ R^2 &= 93\% \end{aligned}$$

Here, Q is tile sales (in thousands of cases), P is tile price (per case), A is advertising expenditures (in thousands of dollars), I is disposable income per household (in thousands of dollars), and HF is household formation (in hundreds). The table provides the coefficient estimate and standard error for each variable.

Variable	Coefficient estimate	Std. error
Intercept	4	3
P	-5	1.8
A	2	0.7
I	0.2	0.1
HF	0.25	0.1

- Fully evaluate and interpret these empirical results on an overall basis using R^2 , \bar{R}^2 , and F-statistic.
- Is quantity demanded sensitive to “own” price?
- Austin, Texas, was a typical market covered by this analysis. During 2007 in the Austin market, price was \$5, advertising was \$30,000, income was an average \$55,000 per household, and the number of household formations was 4,000. Calculate and interpret the relevant advertising point elasticity.
- Assume that the preceding model and data are relevant for the coming period. Estimate the probability that the Austin store will make a profit during 2008 if total costs are projected to be \$300,000.

3. Determine whether the following production functions exhibit constant, increasing, or decreasing returns to scale.

a $Q = 0.5X + 2Y + 40Z$

b $Q = 3L + 10K + 500$

c $Q = 4A + 6B + 8AB$

d $Q = 7L^2 + 5LK + 2K^2$

e $Q = 10L^{0.5}K^{0.3}$

4. **Optimal Input Level.** Ticket Services, Inc., offers ticket promotion and handling services for concerts and sporting events. The Sherman Oaks, California, branch office makes heavy use of spot radio advertising on WHAM AM, with each 30-second ad costing \$100. During the past year, the following relation between advertising and ticket sales per event has been observed:

$$\text{Sales (units)} = 5,000 + 100A - 0.5A^2$$

Here, A represents a 30-second radio spot ad, and sales are measured in numbers of tickets.

Rachel Green, manager for the Sherman Oaks office, has been asked to recommend an appropriate level of advertising. In thinking about this problem, Green noted its resemblance to the optimal resource employment problem studied in a managerial economics course. The advertising/sales relation could be thought of as a production function, with advertising as an input and sales as the output. The problem is to determine the profit-maximizing level of employment for the input, advertising, in this “production” system. Green recognized that a measure of output value was needed to solve the problem. After reflection, Green determined that the value of output is \$2 per ticket, the net marginal revenue earned by Ticket Services (price minus all marginal costs except advertising).

- a** Continuing with Green’s production analogy, what is the marginal product of advertising?
b What is the rule for determining the optimal amount of a resource to employ in a production system? Explain the logic underlying this rule.
c Using the rule for optimal resource employment, determine the profit-maximizing number of ads.
5. Consider the following Cobb-Douglas production function for bus service in a typical metropolitan area:

$$Q = b_0L^{b_1}K^{b_2}F^{b_3}$$

Q = output in millions of passenger miles

L = labor input in worker hours

K = capital input in bus transit hours

F = fuel input in gallons

Each of the parameters of this model was estimated by regression analysis using monthly data over a recent three-year period. Results obtained were as follows:

Parameter	Estimate	Std. error
b_0	1.2	0.4
b_1	0.28	0.15
b_2	0.63	0.12
b_3	0.12	0.07

- a** Estimate the effect on output of a 4 percent decline in worker hours (holding K and F constant).
b Estimate the effect on output of a 3 percent reduction in fuel availability accompanied by a 4 percent decline in bus transit hours (holding L constant).
c Estimate the returns to scale for this production system.

6. The firm's production function is $q(K, L) = L^\beta K^\alpha$, so that the $MP_L = \beta L^{\beta-1} K^\alpha$ and the $MP_K = \alpha L^\beta K^{\alpha-1}$. Let $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. Let the slope of the isocost line be $-\frac{w}{r}$, and let $w = \$4$ and $r = \$27$.

a Find the marginal rate of technical substitution.

Suppose that the firm wishes to produce 1080 units of the good.

b What is the lowest cost at which it can produce 1080 units?

c What is the amount of capital used at the cost minimization bundle?

d What is the amount of labor used at the cost minimization bundle?

Suppose that the price of capital increases, so that now $r = \$64$. The firm still wishes to produce 1080 units.

e Find the new cost minimizing bundle of inputs for this firm, and total cost, for this firm.

7. Angelica Pickles is manager of a Quick Copy franchise in White Plains, New York. Pickles projects that by reducing copy charges from 5¢ to 4¢ each, Quick Copy's \$600-per-week profit contribution will increase by one-third.

a If average variable costs are 2¢ per copy, calculate Quick Copy's projected increase in weekly sales (volume).

b What is Pickles' estimate of the *arc* price elasticity of demand for copies?

8. A bottling company uses two inputs to produce bottles of its best selling soft drink, Joltify: bottling machines, K , and workers, L . The isoquants are standard (meaning they are not special cases like perfect substitutes or perfect complements). A machine costs \$1,000 per day to run and workers earn \$200 per day. At current production, $MP_K = 200$ and $MP_L = 50$. Is this firm minimizing cost? Explain why or why not?

9. Find average fixed cost (AFC), marginal cost (MC), average variable cost (AVC), and average total cost (ATC) for the following total cost functions:

a $TC = 10 + 10q$

b $TC = 10 + q^2$

c $TC = 10 + 10q - 4q^2 + q^3$

10. A U.S. manufacturer is considering producing abroad. Its production function is:

$$q = L^{0.7} K^{0.3}$$

In the U.S., the firm faces $r_{US} = \$3$ and $w_{US} = \$7$. At its international firm it will face a 30% higher cost of capital ($r_{int} = \$3.90$) but face a 30% lower cost of labor ($w_{int} = \$4.90$).

a How much labor and capital should the firm use if it wants to produce 100 units of the good in the U.S.?

b How much labor and capital should the firm use if it wants to produce 100 units of the good at its international location?

c What would the cost of production at the international location be if the firm had to use its optimal inputs for its U.S. plant?