Problems on regression and production/cost (chapters 5, 7, 8)

1. Getaway Tours, Inc., has estimated the following multiplicative demand function for packaged holiday tours in the East Lansing, Michigan, market using quarterly data covering the past four years (16 observations):

$$\begin{aligned} Q_y &= 10P_y^{-1.10}P_x^{0.5}A_y^{3.8}A_x^{2.5}I^{1.85} \\ R^2 &= 80\%, \, SEE = 20 \end{aligned}$$

Here, Q_y is the quantity of tours sold, P_y is average tour price, P_x is average price for some other good, A_y is tour advertising, A_x is advertising of some other good, and I is per capita disposable income. The standard errors of the exponents in the preceding multiplicative demand function are:

$$b_{P_y} = 0.04, \, b_{P_x} = 0.35, \, b_{A_y} = 0.5, \, b_{A_x} = 0.9, \, b_I = 0.45$$

General comment:

In this multiplicative setup, the exponents are the estimated coefficients and they represent elasticities.

a Is tour demand elastic with respect to price?

Answer:

From the exponent on P_y we see that demand looks elastic because $|-1.10| > 1$. The question we now face is if it is significantly greater than 1. First, this test should be a one-tailed test because we want to know if price elasticity is significant and greater than 1, not just different than one. Second, the null hypothesis does not involve zero now but one – our t-statistic is:

$$\frac{1.1 - 1}{0.04} = 2.5$$

Keep in mind there are only 10 degrees of freedom, and also that for a one-tailed test we cut the significance level in the table in half. Looking at the t-table for 10 degrees of freedom we see that that value for the 10% level is 1.812 and the value for the 5% level is 2.228. Because 2.5>2.228, this means that our estimate is significant at the 2.5% level (it is a one-tailed test so we cut the significance level in half).

b Are tours a normal good?

Answer:

A normal good requires $\varepsilon_I > 0$. Our estimate for ε_I is 1.85, and its standard error is 0.45. All we want to know now is if $\varepsilon_I > 0$, so again this is a one-tailed test, though our null hypothesis now involves zero rather than one (so this differs from part a). Our test statistic is:

$$\frac{1.85 - 0}{0.45} = 4.11$$

For the t-table with 10 degrees of freedom, the critical value for the 2% level is 2.764 and for the 1% level is 3.169. Because $4.5 > 3.169$, we can conclude that our estimated coefficient is significant at the 0.5% level (again, we cut the 1% in half because it is a one-tailed test).

c Is X a complement good or substitute good?

Answer:

The estimated coefficient is positive, so if the goods are going to be related they would likely be substitute goods. Again, this is a one-tailed test because we want to check if the coefficient is positive, not if it is significantly different than zero. Setting up our t-statistic we have:

$$\frac{0.5 - 0}{0.35} = 1.43$$

Now, looking at the t-table for 10 degrees of freedom, we see that the critical value for the 20% level is 1.372. Normally we would not test at the 20% level, but again, this is a one-tailed test, so we cut the significance value in half and conclude that the estimated coefficient is significant at the 10% level. Compare this result with the result in part e, which suggests that the estimated coefficient is NOT significantly different than zero when using a two-tailed test.

d Given your answer to part C, can you explain why the demand effects of Ay and Ax are both positive?

Answer:

What this result tells us is that even though X is a substitute good, advertising for X actually increases quantity demanded of tours. It is possible that a competitor's advertising increases awareness about your product – for instance, advertising for Disney World may increase sales at non-Disney related theme parks simply because people see the ads and think about taking a vacation to a theme park.

e Test for the statistical significance of the individual coefficient estimates. Use the null hypothesis that each coefficient estimate equals zero.

Answer:

Keep in mind that there are only 10 degrees of freedom in this problem, so the general rules of 1.96 and 2.57 do not apply.

	Coeff.	Std. error	t-stat	1% crit. val.	5% crit. val.	10% crit. val.	Significance
P_y	-1.10	0.04	2.75	3.169	2.228	1.812	5%
P_x	0.5	0.35	1.43	3.169	2.228	1.812	–
A_y	3.8	0.5	7.60	3.169	2.228	1.812	1%
A_x	2.5	0.9	2.78	3.169	2.228	1.812	5%
I	1.85	0.45	4.11	3.169	2.228	1.812	1%

While the critical values are all the same, I have put them in each row for easy comparison. Also, I am conducting two-tailed tests.

f Conduct an F-test to determine if all the independent variables, other than the constant, are jointly significant.

Answer:

Recall that:

$$\begin{aligned} F &= \left(\frac{n-k}{k-1}\right)\left(\frac{R^2}{1-R^2}\right) \\ F &= \left(\frac{16-6}{6-1}\right)\left(\frac{.8}{.2}\right) \\ F &= \frac{10}{5} * 4 \\ F &= 8 \end{aligned}$$

Now, this statistic is distributed $F_{k-1,n-k}$ or $F_{5,10}$ for this problem. The critical value for $F_{5,10}$ is 5.64. Because $8 > 5.64$, we can conclude that at least one $\beta \neq 0$.

2. Colorful Tile, Inc., is a rapidly growing chain of ceramic tile outlets that caters to the do-it-yourself home remodeling market. In 2007, 33 stores were operated in small to medium-size metropolitan markets. The equation for sales was estimated to be the following:

$$\begin{aligned} Q &= 4 - 5P + 2A + 0.2I + 0.25HF \\ R^2 &= 93\% \end{aligned}$$

Here, Q is tile sales (in thousands of cases), P is tile price (per case), A is advertising expenditures (in thousands of dollars), I is disposable income per household (in thousands of dollars), and HF is household formation (in hundreds). The table provides the coefficient estimate and standard error for each variable.

Variable	Coefficient estimate	Std. error
Intercept	4	3
P	-5	1.8
A	2	0.7
I	0.2	0.1
HF	0.25	0.1

a Fully evaluate and interpret these empirical results on an overall basis using R^2, $\overline{R}^2$, and F-statistic.

Answer:

What R^2 tells us is that 93% of the total variation in quantity is explained by this model. 93% is a very large portion of the variation.

To calculate $\overline{R}^2$ we need to know $n = 33$ (there are 33 stores) and $k = 5$ (because there are 5 independent variables). Now:

$$\begin{aligned}
\overline{R}^2 &= R^2 - \left(\frac{k-1}{n-k}\right)(1-R^2) \\
\overline{R}^2 &= 0.93 - \frac{4}{28} * (0.07) \\
\overline{R}^2 &= 0.93 - 0.01 \\
\overline{R}^2 &= 0.92
\end{aligned}$$

What $\overline{R}^2$ tells us is that 92% of the variation is explained by this model, once the sample size and number of estimated coefficients are controlled for. Because 0.92 is very close to 0.93, it is unlikely we are adding any insignificant independent variables to this model.

The F-statistic for this regression is given by:

$$\begin{aligned}
F &= \left(\frac{n-k}{k-1}\right)\left(\frac{R^2}{1-R^2}\right) \\
F &= \frac{28}{4}\left(\frac{0.93}{0.07}\right) \\
F &= 93
\end{aligned}$$

This value needs to be compared to the value from the F-table with 4 and 28 degrees of freedom. At the 1% level (or 99% confidence level), this critical value is 4.07. Because $93 > 4.07$ this means that we can reject the null hypothesis that all of our non-intercept βs are equal to zero.

b Is quantity demanded sensitive to "own" price?

Answer:

Looking at the t-statistic for own price, and using a null hypothesis that $\beta_p = 0$, we have: $\left|\frac{-5}{1.8}\right| = 2.78$. Because this value is greater than 2.57, quantity demanded is sensitive to own-price at the 1% level (99% confidence level).

c Austin, Texas, was a typical market covered by this analysis. During 2007 in the Austin market, price was \$5, advertising was \$30,000, income was an average \$55,000 per household, and the number of household formations was 4,000. Calculate and interpret the relevant advertising point elasticity.

Answer:

Recall that elasticity is the coefficient estimate for advertising multiplied by the ratio $\frac{A}{Q}$. We would need to find Q first. Using the model and the information in part c,

$$\begin{aligned} Q &= 4 - 5*5 + 2*30 + 0.2*55 + 0.25*40 \\ Q &= 4 - 25 + 60 + 11 + 10 \\ Q &= 60 \end{aligned}$$

Note that I have adjusted some of the numbers based upon the information about the units of measurement at the start of the problem. Also, because Q is measured in 1000s, the result is $Q = 60,000$.

The advertising elasticity is:

$$\begin{aligned} \varepsilon_A &= 2*\frac{30,000}{60,000} \\ \varepsilon_A &= 1 \end{aligned}$$

Thus, a 1% increase in advertising will lead to a 1% increase in quantity demanded.

d Assume that the preceding model and data are relevant for the coming period. Estimate the probability that the Austin store will make a profit during 2008 if total costs are projected to be $300,000.

Answer:

We did not really cover this question in class, but think about what it is asking. We know that the best prediction of sales is 60,000 and that price is equal to $5, so the best projection of total revenue is $300,000. Because costs are also projected to be $300,000 the projected total cost and total revenue are equal; this result means there is a 50% chance that the store will make a profit if costs are this high (because there is a 50% chance that TR is above $300,000, as well as a 50% chance that TR is below $300,000 – remember, these are estimates).

3. Determine whether the following production functions exhibit constant, increasing, or decreasing returns to scale.

General answer:

To check for constant, increasing, or decreasing returns to scale I simply multiply all inputs by 2 and check to see if quantity exactly doubled, more than doubled, or increased by less than double.

a $Q = 0.5X + 2Y + 40Z$

Answer:

$$\begin{aligned} Q &= 0.5X + 2Y + 40Z \\ Q_{new} &= 0.5*(2*X) + 2*(2*Y) + 40*(2*Z) \\ Q_{new} &= 2*(0.5X + 2Y + 40Z) \\ Q_{new} &= 2Q \end{aligned}$$

Output exactly doubled, so constant returns to scale.

b $Q = 3L + 10K + 500$

Answer:

$$
\begin{aligned}
Q &= 3L + 10K + 500 \\
Q_{new} &= 3*(2*L) + 10*(2*K) + 500 \\
Q_{new} &= 2*(3L + 10K + 250) \\
Q_{new} &< 2Q
\end{aligned}
$$

Note that this is LESS THAN double output – if it were double output there would be a $3L+10K+500$ in the bracket.

c $Q = 4A + 6B + 8AB$

Answer:

$$
\begin{aligned}
Q &= 4A + 6B + 8AB \\
Q_{new} &= 4*(2*A) + 6*(2*B) + 8*(2*A)*(2*B) \\
Q_{new} &= 2*(4A + 6B + 16AB) \\
Q_{new} &> 2Q
\end{aligned}
$$

Note that this is MORE THAN double output – the final term of the production function for Q_{new}, $16AB$, is greater than the final term in the original production function, $8AB$.

d $Q = 7L^2 + 5LK + 2K^2$

Answer:

$$
\begin{aligned}
Q &= 7L^2 + 5LK + 2K^2 \\
Q_{new} &= 7*(2*L)^2 + 5*(2*L)*(2*K) + 2*(2*K)^2 \\
Q_{new} &= 28L^2 + 20LK + 8K^2 \\
Q_{new} &= 4\left(7L^2 + 5LK + 2K^2\right) \\
Q_{new} &= 4Q > 2Q
\end{aligned}
$$

Be careful with this one – it looks like it is constant returns to scale but remember that we doubled inputs, but now output has increased 4 times, so we have increasing returns to scale.

e $Q = 10L^{0.5}K^{0.3}$

Answer:

This production function is a Cobb-Douglas production function. We can just add the exponents 0.5 and 0.3. Because they total $0.8 < 1$, this production function has decreasing returns to scale.

4. Optimal Input Level. Ticket Services, Inc., offers ticket promotion and handling services for concerts and sporting events. The Sherman Oaks, California, branch office makes heavy use of spot radio advertising on WHAM AM, with each 30-second ad costing \$100. During the past year, the following relation between advertising and ticket sales per event has been observed:

$$Sales\ (units) = 5,000 + 100A - 0.5A^2$$

Here, A represents a 30-second radio spot ad, and sales are measured in numbers of tickets.

Rachel Green, manager for the Sherman Oaks office, has been asked to recommend an appropriate level of advertising. In thinking about this problem, Green noted its resemblance to the optimal resource employment problem studied in a managerial economics course. The advertising/sales relation could be thought of as a production function, with advertising as an input and sales as the output. The problem is to determine the profit-maximizing level of employment for the input, advertising, in this "production" system. Green recognized that a measure of output value was needed to solve the problem. After reflection, Green determined that the value of output is $2 per ticket, the net marginal revenue earned by Ticket Services (price minus all marginal costs except advertising).

a Continuing with Green's production analogy, what is the marginal product of advertising?

Answer:

The marginal product of advertising is:

$$\frac{\partial Sales}{\partial A} = 100 - A$$

b What is the rule for determining the optimal amount of a resource to employ in a production system? Explain the logic underlying this rule.

Answer:

The rule is that the marginal benefit should equal the marginal cost. The marginal cost in this case is the price. The marginal benefit is how much production occurs times the value of that production (the product of these terms is known as "marginal revenue product"). We know

c Using the rule for optimal resource employment, determine the profit-maximizing number of ads.

Answer:

With the price equal to 100, and knowing that marginal product is $100 - A$, and that each ticket is worth $2 we have:

$$\begin{aligned} (100 - A)\,2 &= 100 \\ 200 - 2A &= 100 \\ 100 &= 2A \\ 50 &= A \end{aligned}$$

5. Consider the following Cobb-Douglas production function for bus service in a typical metropolitan area:

$$Q = b_0 L^{b_1} K^{b_2} F^{b_3}$$

Q = output in millions of passenger miles

L = labor input in worker hours

K = capital input in bus transit hours

F = fuel input in gallons

Each of the parameters of this model was estimated by regression analysis using monthly data over a recent three-year period. Results obtained were as follows:

Parameter	Estimate	Std. error
b_0	1.2	0.4
b_1	0.28	0.15
b_2	0.63	0.12
b_3	0.12	0.07

a Estimate the effect on output of a 4 percent decline in worker hours (holding K and F constant).

Answer:

The estimated coefficients for the Cobb-Douglas production function are the elasticities. Holding K and F constant, we know that the elasticity of L is 0.28. However, that is for a 1% change, and we want a 4% change (in particular, a 4% decrease). Multiply $0.28 * (-0.04) = -0.0112$ or -1.12%.

b Estimate the effect on output of a 3 percent reduction in fuel availability accompanied by a 4 percent decline in bus transit hours (holding L constant).

Answer:

Again, the estimated coefficients are elasticities. If fuel availability decreases 3%, then we will see $0.12*(-0.03) = -0.0036$ decline in miles. If bus transit hours declines by 4% we will see $0.63*(-0.04) = -0.0252$ decline in miles. Thus, the total decline would be $-0.0036 + -0.0252 = -0.0288$ or a -2.88% change in miles.

c Estimate the returns to scale for this production system.

Answer:

This production function is Cobb-Douglas, so we sum the estimated coefficients to find the returns to scale: $0.28 + 0.63 + 0.12 = 1.03$. Because their sum is greater than 1, this production function has increasing returns to scale.

6. The firm's production function is $q(K, L) = L^{\beta}K^{\alpha}$, so that the $MP_L = \beta L^{\beta-1}K^{\alpha}$ and the $MP_K = \alpha L^{\beta}K^{\alpha-1}$. Let $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. Let the slope of the isocost line be $-\frac{w}{r}$, and let $w = \$4$ and $r = \$27$.

a Find the marginal rate of technical substitution.

Answer:

The Marginal Rate of Technical Substitution can be found in one of two ways. The "easiest" way is that we know $MRTS = -\frac{MP_L}{MP_K}$. Since we have MP_L and MP_K,

$$MRTS = -\frac{MP_L}{MP_K} = -\frac{\beta L^{\beta-1}K^{\alpha}}{\alpha L^{\beta}K^{\alpha-1}}.$$

You could leave it like that, but it would be better (for later) to simplify it:

$$-\frac{\beta L^{\beta-1}K^{\alpha}}{\alpha L^{\beta}K^{\alpha-1}} = -\frac{\beta K}{\alpha L}.$$

Suppose that the firm wishes to produce 1080 units of the good.

b What is the lowest cost at which it can produce 1080 units?

Answer:

To answer this question you first need to find the cost-minimizing bundle for 1080 units of the good. This means that you will need to use 2 equations:

$$\begin{aligned} MRTS &= -\frac{w}{r} \\ q(K, L) &= L^{\beta}K^{\alpha} \end{aligned}$$

The first equation is the condition that needs to be met if the cost-minimizing bundle is an interior solution. The second equation is the production function that tells you the combinations of capital and labor that you can use to produce 1080 units. We know that $MRTS = \frac{K}{2L}$ from part 1 of this problem (this is where it is useful to have simplified the $MRTS$). Now we have:

$$\begin{aligned} \frac{K}{2L} &= \frac{w}{r} \\ q &= L^{\beta}K^{\alpha} \end{aligned}$$

The only 2 unknowns at this point are K and L, so just substitute in for all the other variables. Note that I have dropped the negative sign from both $\frac{-w}{r}$ and the $MRTS$ because both are negative numbers. Now, just solve for K using the first equation to get:

$$K = \frac{2Lw}{r}$$

and substitute this into the second equation to get:

$$\begin{aligned} q &= L^{\beta}\left(\frac{2Lw}{r}\right)^{\alpha} \\ q &= L^{\beta}\left(\frac{2w}{r}\right)^{\alpha} L^{\alpha} \\ q &= L^{\beta+\alpha}\left(\frac{2w}{r}\right)^{\alpha} \end{aligned}$$

Now we may as well go and substitute q, α, β, w, and r in to get:

$$\begin{aligned} 1080 &= L^{\frac{2}{3}+\frac{1}{3}}\left(\frac{2*4}{27}\right)^{2/3} \\ 1080 &= L\left(\frac{8}{27}\right)^{2/3} \end{aligned}$$

Now, $\left(\frac{8}{27}\right)^{2/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^2$. The cube root of 8 is 2 and the cube root of 27 is 3, so we have $\left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$. Thus we have:

$$\begin{aligned} 1080 &= L * \frac{4}{9} \\ \frac{9}{4} * 1080 &= L \\ 2430 &= L \end{aligned}$$

There are many, many places where we can substitute in our value of L to get K. I suggest using the easiest one – at some point in time we solved for K in terms of L. Thus we had:

$$K = \frac{2Lw}{r}$$

Now, substituting in everything we know gives:

$$\begin{aligned} K &= \frac{2*2430*4}{27} \\ K &= 720 \end{aligned}$$

Thus our cost minimizing bundle of producing 1080 units given our production function and input prices is 720 units of capital and 2430 units of labor (or $K = 720$, $L = 2430$). The question asks for the total cost at the cost-minimizing bundle which is given by:

$$\begin{aligned} TC &= w*L + r*K \\ TC &= 4*2430 + 27*720 \\ TC &= \$29,160 \end{aligned}$$

c What is the amount of capital used at the cost minimization bundle?

Answer:

This should be easy if you were able to do part b. It is simply the quantity of capital that you found in part 2. So $K = 720$.

d What is the amount of labor used at the cost minimization bundle?

Answer:

This is just the quantity of labor that you found in part b. So $L = 2430$.

Suppose that the price of capital increases, so that now $r = \$64$. The firm still wishes to produce 1080 units.

e Find the new cost minimizing bundle of inputs for this firm, and total cost, for this firm.

Answer:

This is basically the same problem as above, only now we have $r = \$64$. What you should realize is that by NOT substituting in all the numbers until very late in the solution this makes it very easy to go back in and change things. It was at this point in part 2:

$$q = L^{\beta+\alpha}\left(\frac{2w}{r}\right)^{\alpha}$$

that I began substituting in numbers. All the other steps up until this point (when the variables are left in) are the same. So now I just plug in the new numbers:

$$1080 = L^{2/3+1/3}\left(\frac{2*4}{64}\right)^{2/3}$$

and solve for L.

$$\begin{aligned}
1080 &= L*\left(\frac{8}{64}\right)^{2/3} \\
1080 &= L*\left(\frac{2}{4}\right)^{2} \\
1080 &= L*\left(\frac{1}{2}\right)^{2} \\
1080 &= L*\left(\frac{1}{4}\right) \\
4*1080 &= L \\
4320 &= L
\end{aligned}$$

Now plug $4320 = L$, as well as the other parameters, into:

$$\begin{aligned}
K &= \frac{2Lw}{r} \\
K &= \frac{2*4320*4}{64} \\
K &= 540
\end{aligned}$$

Thus the cost-minimizing bundle of inputs after the price of capital increased is $K = 540$ and $L = 4320$. The firm now has a total cost of:

$$\begin{aligned}
TC &= w*L + r*K \\
TC &= 4*4320 + 64*540 \\
TC &= \$51,840
\end{aligned}$$

7. Angelica Pickles is manager of a Quick Copy franchise in White Plains, New York. Pickles projects that by reducing copy charges from 5¢ to 4¢ each, Quick Copy's \$600-per-week profit contribution will increase by one-third.

a If average variable costs are 2¢ per copy, calculate Quick Copy's projected increase in weekly sales (volume).

Answer:

In order to do this we need to know the before and after volume of sales. We know that they are earning 3 cents on each sale before the change in price, and that they are earning \$600 in total, so they must be selling:

$$\begin{aligned} Q &= \frac{600}{0.03} \\ Q &= 20,000 \end{aligned}$$

If they reduce price to 4 cents, they are now earning 2 cents on each sale, but expect profits to be \$800 (because profit increases by one-third) so:

$$\begin{aligned} Q_{new} &= \frac{800}{0.02} \\ Q &= 40,000 \end{aligned}$$

Thus, sales will have increased by 20,000.

b What is Pickles' estimate of the *arc* price elasticity of demand for copies?

Answer:

For arc price elasticity we have (note that I have removed the "2" that is part of the formula because it is in both the denominator and the numerator and cancels, and I have also dropped the negative sign because it is price elasticity so it will be a negative number):

$$\begin{aligned} \varepsilon_P &= \frac{\%\Delta Q_D}{\%\Delta P} \\ \varepsilon_P &= \frac{\frac{Q_1 - Q_0}{Q_1 + Q_0}}{\frac{P_1 - P_0}{P_1 + P_0}} \\ \varepsilon_P &= \frac{Q_1 - Q_0}{P_1 - P_0} * \frac{P_1 + P_0}{Q_1 + Q_0} \\ \varepsilon_P &= \frac{40,000 - 20,000}{0.05 - 0.04} * \frac{0.05 + 0.04}{40,000 + 20,000} \\ \varepsilon_P &= \frac{20,000}{0.01} * \frac{0.09}{60,000} \\ \varepsilon_P &= \frac{1}{1} * \frac{9}{3} \\ \varepsilon_P &= 3 \end{aligned}$$

Thus demand for copies is elastic.

8. A bottling company uses two inputs to produce bottles of its best selling soft drink, Joltify: bottling machines, K, and workers, L. The isoquants are standard (meaning they are not special cases like perfect substitutes or perfect complements). A machine costs \$1,000 per day to run and workers earn \$200 per day. At current production, $MP_K = 200$ and $MP_L = 50$. Is this firm minimizing cost? Explain why or why not?

Answer:

Given that the isoquants are standard, we know the following result from cost minimization:

$$\frac{MP_K}{p_K} = \frac{MP_L}{p_L}$$

In our case:

$$\frac{MP_K}{p_K} = \frac{200}{1000} = \frac{1}{5}$$

and

$$\frac{MP_L}{p_L} = \frac{50}{200} = \frac{1}{4}$$

Because $\frac{1}{4} > \frac{1}{5}$, the firm should switch to more labor if it wishes to minimize cost.

9. Find average fixed cost (AFC), marginal cost (MC), average variable cost (AVC), and average total cost (ATC) for the following total cost functions:

a $TC = 10 + 10q$

Answer:

$$\begin{aligned} AFC &= \frac{10}{q} \\ MC &= 10 \\ AVC &= 10 \\ ATC &= \frac{10}{q} + 10 \end{aligned}$$

b $TC = 10 + q^2$

Answer:

$$\begin{aligned} AFC &= \frac{10}{q} \\ MC &= 2q \\ AVC &= q \\ ATC &= \frac{10}{q} + q \end{aligned}$$

c $TC = 10 + 10q - 4q^2 + q^3$

Answer:

$$\begin{aligned} AFC &= \frac{10}{q} \\ MC &= 10 - 8q + 3q^2 \\ AVC &= 10 - 4q + q^2 \\ ATC &= \frac{10}{q} + 10 - 4q + q^2 \end{aligned}$$

10. A U.S. manufacturer is considering producing abroad. Its production function is:

$$q = L^{0.7}K^{0.3}$$

In the U.S., the firm faces $r_{US} = \$3$ and $w_{US} = \$7$. At its international firm it will face a 30% higher cost of capital ($r_{int} = \$3.90$) but face a 30% lower cost of labor ($w_{int} = \$4.90$).

a How much labor and capital should the firm use if it wants to produce 100 units of the good in the U.S.?

Answer:

Again, to minimize cost simply set:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

First we need to find the marginal products:

$$\begin{aligned}
MP_L &= \frac{\partial q}{\partial L} \\
MP_L &= 0.7L^{-0.3}K^{0.3} \\
& \text{and} \\
MP_K &= \frac{\partial q}{\partial K} \\
MP_K &= 0.3L^{0.7}K^{-0.7}
\end{aligned}$$

Now I am going to rearrange our condition:

$$\begin{aligned}
\frac{MP_L}{MP_K} &= \frac{w}{r} \\
\frac{0.7L^{-0.3}K^{0.3}}{0.3L^{0.7}K^{-0.7}} &= \frac{7}{3} \\
\frac{K}{L} &= \frac{7 * 0.3}{3 * 0.7} \\
\frac{K}{L} &= 1 \\
K &= L
\end{aligned}$$

Now use the production function with $q = 100$ units to find:

$$\begin{aligned}
q &= L^{0.7}K^{0.3} \\
100 &= L^{0.7}L^{0.3} \\
100 &= L^{1} \\
100 &= L
\end{aligned}$$

So the firm should use 100 units of capital and labor given these prices.

b How much labor and capital should the firm use if it wants to produce 100 units of the good at its international location?

Answer:

It is the exact same process as part a except for when we get to inputting the prices so I will skip to:

$$\begin{aligned}
\frac{MP_L}{MP_K} &= \frac{w}{r} \\
\frac{0.7L^{-0.3}K^{0.3}}{0.3L^{0.7}K^{-0.7}} &= \frac{4.9}{3.9} \\
\frac{K}{L} &= \frac{4.9 * 0.3}{3.9 * 0.7} \\
\frac{K}{L} &= \frac{1.47}{2.73} \\
K &= \frac{1.47}{2.73}L
\end{aligned}$$

Not nearly as nice but now:

$$
\begin{aligned}
q &= L^{0.7}K^{0.3} \\
100 &= L^{0.7}\left(\frac{1.47}{2.73}L\right)^{0.3} \\
100 &= \left(\frac{1.47}{2.73}\right)^{0.3} L \\
\frac{100}{\left(\frac{1.47}{2.73}\right)^{0.3}} &= L \\
120.41 &\approx L
\end{aligned}
$$

Using the actual answer rather than the approximation, we find that:

$$
\begin{aligned}
K &= \frac{1.47}{2.73}L \\
K &= \frac{1.47}{2.73} * \frac{100}{\left(\frac{1.47}{2.73}\right)^{0.3}} \\
K &= \left(\frac{1.47}{2.73}\right)^{0.7} * 100 \\
K &\approx 64.83
\end{aligned}
$$

c What would the cost of production at the international location be if the firm had to use its optimal inputs for its U.S. plant?

Answer:

If the firm had to use its optimal U.S. inputs its cost would be:

$$
\begin{aligned}
TC &= 100 * 3.9 + 100 * 4.9 \\
TC &= 390 + 490 \\
TC &= 880
\end{aligned}
$$

For comparison, the cost (using $L = 121$ and $K = 65$ – it may be possible to get fractional units by only employing labor or capital part-time, but if the $L = 121$ and $K = 65$ bundle of inputs costs less than $K = 100$ and $L = 100$ then we know the optimal combination will cost less) at the international location is:

$$
\begin{aligned}
TC &= 65 * 3.9 + 121 * 4.9 \\
TC &= 253.50 + 592.90 \\
TC &= 846.40
\end{aligned}
$$