Problems on market structure (chapters 10, 11, 12, and 13)

1. Bada Bing, Ltd., supplies standard 256 MB RAM chips to the U.S. computer and electronics industry. Like the output of its competitors, Bada Bing's chips must meet strict size, shape, and speed specifications. As a result, the chip-supply industry can be regarded as perfectly competitive. The total cost and marginal cost functions for Bada Bing are:

$$TC = \$1,000,000 + \$20Q + \$0.0001Q^2$$

where Q is the number of chips produced.

a Calculate Bada Bing's optimal output and profits if chip prices are stable at \$60 each.

Answer:
This industry is perfectly competitive so $P = MR = \$60$. The firm's marginal cost is:

$$\frac{\partial TC}{\partial Q} = MC = 20 + 0.0002Q$$

Setting $MR = MC$ we have:

$$\begin{aligned} MR &= MC \\ 60 &= 20 + 0.0002Q \\ 40 &= 0.0002Q \\ 200,000 &= Q \end{aligned}$$

Profit is $TR - TC$:

$$\begin{aligned} \Pi &= TR - TC \\ \Pi &= (200,000) * 60 - \left(1,000,000 + 20 * 200,000 + 0.0001 * 200,000^2\right) \\ \Pi &= \$3,000,000 \end{aligned}$$

b Calculate Bada Bing's optimal output and profits if chip prices fall to \$30 each.

Answer:
Again, because the industry is perfectly competitive we have $P = MR = \$30$, and we still have $MC = 20 + 0.0002Q$, so:

$$\begin{aligned} MR &= MC \\ 30 &= 20 + 0.0002Q \\ 10 &= 0.0002Q \\ 50,000 &= Q \end{aligned}$$

And profit is now:

$$\begin{aligned} \Pi &= (50,000) * 30 - \left(1,000,000 + 20 * 50,000 + 0.0001 * 50,000^2\right) \\ \Pi &= -\$750,000 \end{aligned}$$

c If Bada Bing is typical of firms in the industry, calculate the firm's long-run equilibrium output, price, and economic profit levels.

Answer:

Recall that in long-run equilibrium the firm will be producing a quantity equal to the minimum of the firm's ATC curve because the firm will be making zero profit. We have:

$$\begin{aligned} TC &= \$1,000,000 + \$20Q + \$0.0001Q^2 \\ ATC &= \frac{1,000,000}{Q} + 20 + 0.0001Q \end{aligned}$$

We also know that marginal cost intersects ATC at its minimum, so we set $MC = ATC$:

$$\begin{aligned} 20 + 0.0002Q &= \frac{1,000,000}{Q} + 20 + 0.0001Q \\ 0.0001Q &= \frac{1,000,000}{Q} \\ 0.0001Q^2 &= 1,000,000 \\ Q^2 &= \frac{1,000,000}{0.0001} \\ Q &= \sqrt{\frac{1,000,000}{0.0001}} \\ Q &= 100,000 \end{aligned}$$

When $Q = 100,000$, then we have:

$$\begin{aligned} P &= MC \\ P &= 20 + 0.0002Q \\ P &= 20 + 0.0002 * 100,000 \\ P &= 20 + 20 \\ P &= 40 \end{aligned}$$

At a price of \$40, the firm should be making zero economic profit:

$$\begin{aligned} \Pi &= (100,000) * 40 - \left(1,000,000 + 20 * 100,000 + 0.0001 * 100,000^2\right) \\ \Pi &= 4,000,000 - 1,000,000 - 2,000,000 - 1,000,000 \\ \Pi &= 0 \end{aligned}$$

2. In class we discussed various reasons why a particular store may choose to remain open 24 hours. Consider a rural Wal-Mart and an urban Wal-Mart.

Rural Wal-Mart

The rural Wal-Mart earns net revenues (net revenues here are total revenues minus product costs, so the revenue earned on all product sales minus their cost) of \$2,000 per day between the hours of 6am and 11pm. It can also earn \$100 in net revenues if it is open from 11pm to 6am. Electricity costs \$500 per day between 6am and 11pm if the store is open, and \$100 per day if the store is closed during those hours. Electricity costs \$200 per day between the hours of 11pm and 6am if the store is open, and \$40 per day if the store is closed during those hours. If the store is open from 6am-11pm then it must pay \$1190 to cashiers and if the store is open from 11pm-6am it must pay another \$84 to cashiers. The table below gives the net revenues for the hours of operation as well as the electricity and cashiers cost for these hours.

	Open 6am-11pm	Closed 6am-11pm	Open 11pm-6am	Closed 11pm-6am
Net Revenues	\$2000	\$0	\$100	\$0
Electricity	\$500	\$100	\$200	\$40
Cashiers	\$1190	\$0	\$84	\$0

Urban Wal-Mart

The urban Wal-Mart earns net revenues (again, net revenues here are total revenues minus product costs, so the revenue earned on all product sales minus their cost) of \$5,000 per day between the hours of 6am and 11pm. It can also earn \$700 in net revenues if it is open from 11pm to 6am. Electricity costs \$1000 per day between 6am and 11pm if the store is open, and \$200 per day if the store is closed during those hours. Electricity costs \$400 per day between the hours of 11pm and 6am if the store is open, and \$80 per day if the store is closed during those hours. If the store is open from 6am-11pm then it must pay \$2380 to cashiers and if the store is open from 11pm-6am it must pay another \$168 to cashiers. The table below gives the net revenues for the hours of operation as well as the electricity and cashiers cost for these hours.

	Open 6am-11pm	Closed 6am-11pm	Open 11pm-6am	Closed 11pm-6am
Net Revenues	\$5000	\$0	\$700	\$0
Electricity	\$1000	\$200	\$400	\$80
Cashiers	\$2380	\$0	\$168	\$0

a What hours should the rural Wal-Mart be open? Explain.

Answer:

There are 4 cases that could happen:

a Open day, open night

b Open day, close night

c Close day, open night

d Close day, close night

The profit from (open day, open night) is: $2000 - 500 - 1190 + 100 - 200 - 84 = 126$

The profit from (open day, close night) is: $2000 - 500 - 1190 - 40 = 270$

The profit from (close day, open night) is: $-100 + 100 - 200 - 84 = -284$

The profit from (close day, close night) is: $-100 - 40 = -140$

Since the profit from opening during the day and closing at night is the greatest, the rural Wal-Mart should be open from 6am–11pm. Note that the rural Wal-Mart cannot cover its TVC at night, so it makes more money by shutting down.

b What hours should the urban Wal-Mart be open? Explain.

Answer:

There are 4 cases that could happen:

a Open day, open night

b Open day, close night

c Close day, open night

d Close day, close night

The profit from (open day, open night) is: $5000 - 1000 - 2380 + 700 - 400 - 168 = 1752$

The profit from (open day, close night) is: $5000 - 1000 - 2380 - 80 = 1540$

The profit from (close day, open night) is: $-200 + 700 - 400 - 168 = -68$

The profit from (close day, close night) is: $-200 - 80 = -280$

The urban Wal-Mart maximizes its profit by remaining open all 24 hours. Note that it can cover its time period specific TVC with its time period specific revenue.

c The _urban_ Wal-Mart has to pay people (stockers) to stock the shelves. These stockers can either work from 6am-11pm (the day) or from 11pm-6am (the night). If they work during the day then Wal-Mart must pay them \$300. However, the stockers also cause some congestion in the store during the day and the store loses \$500 in net revenues during the day. If they work at night Wal-Mart must pay them \$500, but no net revenues are lost at night. One other factor to consider is electricity. Whenever the stockers work Wal-Mart must pay full electricity regardless of whether or not the store is actually open (so it must pay \$400 in electricity if the stockers work at night even if the store is closed). Assuming this urban Wal-Mart must hire stockers, answer the following questions:

i Will the urban Wal-Mart be making a profit or loss during the night hours if it hires the stockers to work at night and it opens?

Answer:

The urban Wal-Mart will be making a loss if it hires the stockers to work at night and it opens. It will have to pay \$500 for the stockers, \$400 for the electricity, and \$168 for the cashiers. The store does have net revenues of \$700 though, so its loss is \$368.

ii If the urban Wal-Mart hires the stockers to work at night should they open? Explain.

Answer:

The urban Wal-Mart should open if it hires the stockers to work at night. We already know (from part a) that the store will lose \$368 if it hires the stockers to work at night and it opens. If it hires the stockers to work at night and it closes, it will lose \$900, \$500 from paying the stockers and \$400 from keeping the lights on.

iii If Wal-Mart is a business intent on maximizing its profits, during which hours should it hire the stockers, and what should the store's hours of business be? Explain, making sure to reference the profit level of your decision as well as the profit level of other decisions that could have been made.

Answer:

The urban store should open all 24 hours and it should hire its stockers to work at night. The store will make a profit of $5000 - 1000 - 2380 + 700 - 400 - 168 - 500 = 1252$ if it follows this business plan.

There are a number of alternative plans the store could use, but we know one thing: if the store hires the stockers to work nights, they will open. We know this because parts a and b of the question show us that opening when the stockers work at night loses less money than being closed. So, here are the 8 cases, with their profit levels:

1 Open day (stockers), open night: $5000 - 1000 - 2380 - 300 - 500 + 700 - 400 - 168 = 952$

2 Open day, open night (stockers): $5000 - 1000 - 2380 + 700 - 400 - 168 - 500 = 1252$

3 Open day (stockers), close night: $5000 - 1000 - 2380 - 300 - 500 - 80 = 740$

4 Open day, close night (stockers): $5000 - 1000 - 2380 - 400 - 500 = 720$

5 Close day (stockers), open night: $-1000 - 300 + 700 - 400 - 168 = -1168$

6 Close day, open night (stockers): $-200 + 700 - 400 - 168 - 500 = -568$

7 Close day (stockers), close night: $-1000 - 300 - 80 = -1380$

8 Close day, close night (stockers): $-200 - 400 - 500 = -1100$

Notice that the highest profit is 1252 when the store is open 24 hours and hires the stockers to work at night. Even though the store makes a loss by hiring the stockers to work at night and opening (this is the \$368 loss in part b), this loss is less than the loss that would occur if it closed, and it is less than the loss that would occur if the store hired the stockers to work during the day (they would save \$200 in stocker costs, but would lose another \$500 in revenues from congestion, which is why the profit from opening 24 hours and hiring the stockers during the day is only \$952). So Wal-Mart may actually be earning a loss on its night-time operation, but that may be the best time for the store to have its stockers work.

3. Bates Gill is the sole developer of underwater operating systems. His firm is protected by considerable barriers to entry. Gill faces the following inverse demand function for his underwater operating systems:

$$P(Q) = 4320 - 12Q$$

His total cost function is:

$$TC = 4Q^2 + 200,000$$

a Write down Gill's MR function solely as a function of quantity.

Answer:

We know that:

$$\begin{aligned} MR &= \frac{\partial TR}{\partial Q} \\ TR &= (4320 - 12Q)\,Q \\ MR &= 4320 - 24Q \end{aligned}$$

b Find Gill's profit-maximizing quantity.

Answer:

All we need to do is set $MR = MC$.

$$4320 - 24Q = 8Q$$

$$4320 = 32Q$$

$$135 = Q$$

So Gill's profit-maximizing quantity is 135.

c Find Gill's price at the profit-maximizing quantity.

Answer:

To find the price at the profit-maximizing quantity simply plug the profit-maximizing quantity into the inverse demand function:

$$P(135) = 4320 - 12 * 135 = 2700$$

So the price is \$2700.

d What are Gill's profits at the profit-maximizing price and quantity?

Answer:

To find the profit simply subtract total cost from total revenue.

$$\Pi = TR - TC$$

$$\Pi = 2700 * 135 - \left(4 * (135)^2 + 200000\right) = 91600$$

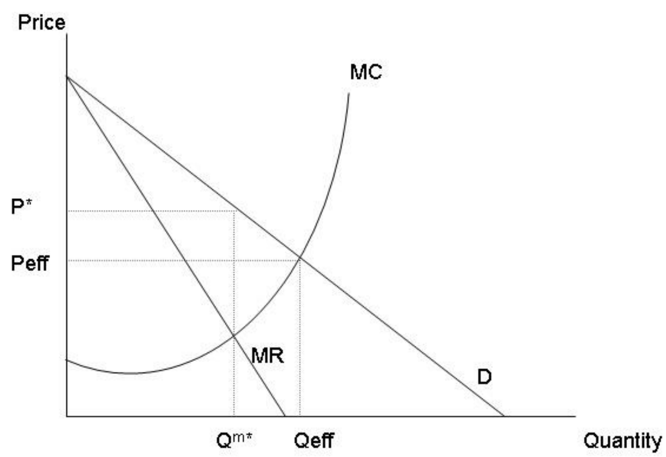
So Gill's profit at the profit-maximizing level is $\$91,600$.

Adding the government

The government realizes that Gill is a monopolist and that "considerable" deadweight loss is being created in the underwater operating systems market. Use the functions above to answer these questions.

e Draw a picture (just a rough sketch, not necessarily to scale) that illustrates Gill's profit-maximizing price and quantity as well as the quantity and the price that would be charged if the market were to operate efficiently (with no deadweight loss).

Answer:



f Find the price and quantity that correspond that will make this market completely efficient (no deadweight loss).

Answer:

Again, the reason I asked the question about the graph above was to aid you in answering this question. The socially efficient quantity occurs at the intersection of the demand curve and the MC. So, setting $P(Q) = MC$ we can find the socially efficient quantity.

$$4320 - 12Q = 8Q$$

$$4320 = 20Q$$

$$216 = Q$$

Now that we have the socially efficient quantity, we can find the socially efficient price by plugging that quantity into the inverse demand function:

$$P(216) = 4320 - 12 * 216 = 1728$$

So the socially efficient price in this market is $\$1,728$.

g Suppose that Gill had not yet entered into this market, but was merely planning to enter into the market. If the government were to tell Gill ahead of time that they would regulate his price at the efficient level, would Gill enter this market? Explain.

Answer:

No, Gill would not enter this market. There are a variety of methods to answer this question. One is to look at Gill's profit at the socially efficient price and quantity level:

$$\Pi = TR - TC$$

$$\Pi = 1728 * 216 - \left(4 * (216)^2 + 200000\right) = -13376$$

Since Gill's profit is less than 0, he would not enter this market.

Another method would be to compare the price and ATC at the socially efficient quantity. We know the price is 1728, and the ATC is given by:

$$ATC\,(Q) = 4Q + \frac{200,000}{Q}$$

$$ATC\,(216) = 4 * (216) + \frac{200,000}{216} = 1,789.93$$

Since $P < ATC$ at this quantity, Gill will not enter.

4. The market for a rare strain of apples is a perfectly competitive market, with 150 identical sellers. The market is a constant cost industry. Each seller has the following costs:

$$\begin{aligned} TC &= 4q^2 - 4q + 144 \\ ATC &= 4q - 4 + \frac{144}{q} \\ MC &= 8q - 4 \end{aligned}$$

The supply and demand conditions in the market are such that the market price is \$52 per apple (I told you they were rare).

a Find the profit-maximizing quantity for an individual firm in this market.

Answer:

The easiest way to find the profit-maximizing quantity was to find where $MR = MC$. Since this is a perfectly competitive market, $MR = P$, which means that $MR = 52$. Setting this equal to MC gives:

$$52 = 8q - 4$$

$$56 = 8q$$

$$7 = q$$

So each firm should produce 7 units in this market.

You could also have tried the table method that was used on the homework. It might not have been too bad for this question since the profit-maximizing quantity was 7 units, but if you tried this method on Bates Gill number 2 it probably did not work so well.

b What is the profit-level at the profit-maximizing quantity for an individual firm in this market.

Answer:

To find the profit-level for this firm, simply find the total revenue at the profit-maximizing quantity and the total cost at the profit-maximizing quantity, and subtract total cost from total revenue.

$$TR = P * Q = 52 * 7 = 364$$

$$TC = 4\,(7)^2 - 4\,(7) + 144 = 312$$

$$TR - TC = 364 - 312 = 52$$

So the profit at the profit-maximizing quantity is \$52.

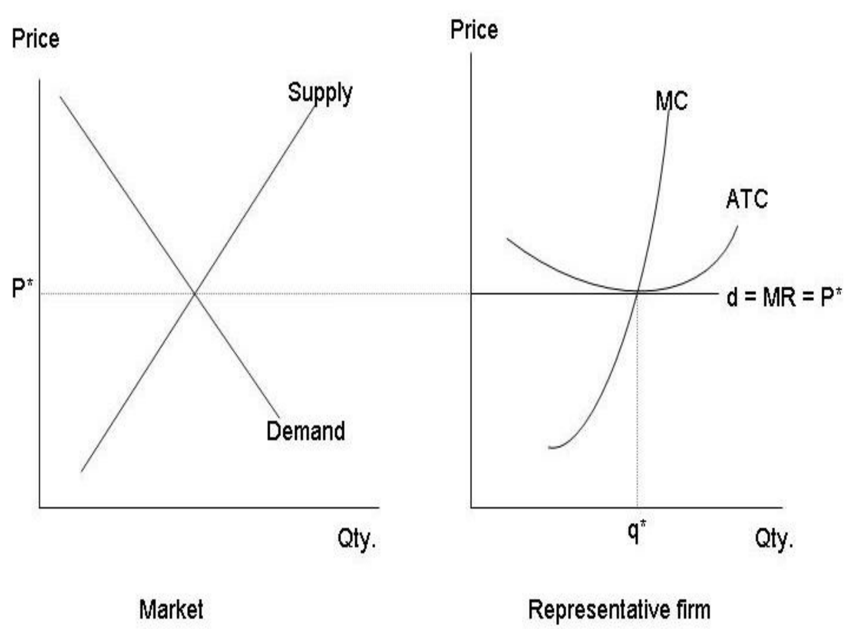
c What is the total MARKET quantity in this industry?

Answer:

To find the total market quantity, simply find the product of the quantity produced by each firm and the total number of firms. This method works because the firms are identical. So $150 * 7 = 1050$.

d Using the graphs below, draw a picture of the competitive market and firm in LR equilibrium. Be sure to label your graphs, and to include the firm's ATC, MC, and MR.

Answer:



e Find the price that would correspond to the apple market being in a LR equilibrium. Remember, the apple market is a constant cost industry.

Answer:

The reason that I asked the question about the graph (part **d**) was to provide you with an aid to answering this question. Looking at the graph in part **d**, notice that the LR equilibrium occurs at the quantity where $P = MC = ATC$. Because the price is the variable we wish to solve for we cannot use it to find the quantity. However, we can use the fact that the quantity the firm produces in LR equilibrium is the one that corresponds to the point where $MC = ATC$. Setting $MC = ATC$ gives:

$$8q - 4 = 4q - 4 + \frac{144}{q}$$

$$4q = \frac{144}{q}$$

$$4q^2 = 144$$

$$q^2 = 36$$

$$q = \pm 6$$

Now, we can't really have a quantity of (-6), so the answer has to be $q = 6$. Now we can find the price in the market by setting $P = MC$ (or $P = ATC$) when $q = 6$.

$$P = MC$$

$$P = 8(6) - 4 = 44$$

OR:

$$P = ATC$$

$$P = 4(6) - 4 + \frac{144}{6} = 44$$

Either way, the price in the market is \$44. If you really wanted to double check your work, you can find the profit-level at this price and quantity pair. $TR = P * Q = 44 * 6 = 264$. $TC = 4q^2 - 4q + 144 = 4(6)^2 - 4(6) + 144 = 264$. Because profit equals zero, this firm is in LR equilibrium.

f Explain why the fact that the apple market is a constant cost industry is a useful assumption in solving part **e**. If the apple market were a decreasing cost industry, how would this affect the resulting LR equilibrium price (would it be higher or lower than the price you found in part **e** and why).

Answer:

In the initial structure of the problem, with $P = \$52$, the firms in the market were making economic profits. This means that more firms will enter into the market, which will decrease the price and increase the total market quantity. If the constant cost industry assumption were not made, then the firms' cost curves would shift as the total market quantity increases (the cost curves would shift downward in a decreasing cost industry and upward in an increasing cost industry). And if the firms' cost curves are shifting, there is no possible way to answer number 5 because you don't know how much they would shift with the increase in total market quantity.

If this were a decreasing cost industry then the price would be lower than the one in question number 5, as the firms' cost curves would shift downward as market quantity increases.

5. To many upscale homeowners, no other flooring offers the warmth, beauty, and value of wood. New technology in stains and finishes call for regular cleaning that takes little more than sweeping and/or vacuuming, with occasional use of a professional wood floor cleaning product. Wood floors are also ecologically friendly because wood is both renewable and recyclable. Buyers looking for traditional oak, rustic pine, trendy mahogany, or bamboo can choose from a wide assortment.

   At the wholesale level, wood flooring is a commodity-like product sold with rigid product specifications. Price competition is ferocious among hundreds of domestic manufacturers and importers. Assume that market supply and demand conditions for mahogany wood flooring are:

$$\begin{aligned} Q_S &= -10 + 2P \\ Q_D &= 320 - 4P \end{aligned}$$

   where Q is output in square yards of floor covering (000), and P is the market price per square yard.

   a Calculate the equilibrium price/output solution before and after imposition of a $9 per unit tax on suppliers.

   Answer:

   Calculating the before-tax equilibrium price and quantity we simply need to set $Q_S = Q_D$:

$$\begin{aligned} -10 + 2P &= 320 - 4P \\ 6P &= 330 \\ P &= 55 \end{aligned}$$

   When $P = 55$, then $Q = 100$ (actually $100,000$ given the units of measurement).

   When the tax is imposed, we alter the supply function by subtracting the amount of the tax from the price:

$$\begin{aligned} Q_S &= -10 + 2(P - \tau) \\ Q_S &= -10 + 2(P - 9) \\ Q_S &= -28 + 2P \end{aligned}$$

   We subtract the amount of the tax because the price the suppliers receive is $P - \tau$. Now we set $Q_S = Q_D$ again:

$$\begin{aligned} -28 + 2P &= 320 - 4P \\ 6P &= 348 \\ P &= 58 \end{aligned}$$

   When $P = 58$, $Q = 88$ (actually $88,000$).

   b Calculate the deadweight loss to taxation caused by imposition of the $9 per unit tax. How much of this deadweight loss was suffered by consumers versus producers? Explain.

   Answer:

   There is a $9 tax and a loss of 12,000 units sold in the market due to the tax. Because the supply and demand functions are linear, the deadweight loss will be given by a triangle with a "base" equal to 9 and a "height" equal to 12,000. Using $\frac{1}{2} * base * height$ we have that the total deadweight loss is:

$$\begin{aligned} DWL &= \frac{1}{2} * 9 * 12,000 \\ DWL &= 54,000 \end{aligned}$$

   The easiest way to determine the burden on the consumers and the burden on the suppliers is to look at the after-tax price in relation to the before-tax price. The after-tax price is $3 higher than the before-tax price, so consumers are paying $\frac{1}{3}$ of the $9 tax. Thus the consumers suffer a DWL of 18,000, while the producers (who are bearing $6 of the $9 tax) are suffering a DWL of 36,000.

6. Each year, about 9 billion bushels of corn are harvested in the United States. The average market price of corn is a little over \$2 per bushel, but costs farmers about \$3 per bushel. Tax payers make up the difference. Under the 2002 \$190 billion, 10-year farm bill, American taxpayers will pay farmers \$4 billion a year to grow even more corn, despite the fact that every year the United States is faced with a corn surplus. Growing surplus corn also has unmeasured environmental costs. The production of corn requires more nitrogen fertilizer and pesticides than any other agricultural crop. Runoff from these chemicals seeps down into the groundwater supply, and into rivers and streams. Ag chemicals have been blamed for a 12,000-square-mile dead zone in the Gulf of Mexico. Overproduction of corn also increases U.S. reliance on foreign oil.

   To illustrate some of the cost in social welfare from agricultural price supports, assume the following market supply and demand conditions for corn:

$$\begin{aligned} Q_S &= -5,000 + 5,000P \\ Q_D &= 10,000 - 2,500P \end{aligned}$$

   where Q is output in bushels of corn (in millions), and P is the market price per bushel.

   a Calculate the equilibrium price/output solution.

   Setting $Q_S = Q_D$:

$$\begin{aligned} -5,000 + 5,000P &= 10,000 - 2,500P \\ 7,500P &= 15,000 \\ P &= 2 \end{aligned}$$

   When $P = 2$, then $Q = 5,000$ (in millions).

   b Calculate the amount of surplus production the government will be forced to buy if it imposes a price floor of \$2.50 per bushel.

   Answer:

   If $P = 2.50$, then we have:

$$\begin{aligned} Q_S &= -5,000 + 5,000 * 2.5 = 7,500 \text{ (in millions)} \\ Q_D &= 10,000 - 2,500 * 2.5 = 3,750 \text{ (in millions)} \end{aligned}$$

   So there will be a surplus of $3,750$ (in millions)

7. Calvin's Barber Shops, Inc. has a monopoly on barbershop services provided on the south side of Chicago because of restrictive licensing requirements, and not because of superior operating efficiency. As a monopoly, Calvin's provides all industry output. Assume $TC = 20Q$.

   Assume that:

$$P = \$80 - \$0.0008Q$$

   Where P is price per unit and Q is total firm output (haircuts).

   a Calculate the monopoly profit maximizing price/output combination, as well as profits for the monopolist.

   Answer:

   To find the profit-maximizing price and output combination, set $MR = MC$:

$$\begin{aligned} TR &= (\$80 - \$0.0008Q)\, Q \\ MR &= 80 - 0.0016Q \end{aligned}$$

Now:

$$\begin{aligned} MR &= MC \\ 80 - 0.0016Q &= 20 \\ 60 &= 0.0016Q \\ Q &= 37,500 \end{aligned}$$

When $Q = 37,500$, we have $P = 50$. Monopoly profits are given by $Q * (P - ATC) = 37,500 * (50 - 20) = 1,125,000$.

b What is the competitive market long-run equilibrium (price and quantity of haircuts)?

Answer:

For the perfectly competitive outcome, we need $P = MC = ATC$. Both MC and ATC equal 20, so:

$$\begin{aligned} \$80 - \$0.0008Q &= 20 \\ 60 &= 0.0008Q \\ Q &= 75,000 \end{aligned}$$

c Discuss the "monopoly problem" from a social perspective in this instance.

Answer:

The problem of monopoly is that prices are higher and output is lower than the competitive equilibrium. In this instance, output is half of what it would be in a competitive market, and prices are 2.5 times higher.

8. Consider a simultaneous quantity choice (Cournot) game between 2 firms. Each firm chooses a quantity, q_1 and q_2 respectively. The inverse market demand function is given by $P(Q) = 1434 - 2 * Q$, where $Q = q_1 + q_2$. Firm 1 has total cost function $TC(q_1) = 3 * (q_1)^2$ and Firm 2 has total cost function $TC(q_2) = 12 * (q_2)^2 - 12 * q_2$. Each firm wishes to maximize profit.

   a Set up the profit function for Firm 1 and Firm 2. Remember, this is a quantity choice game.

   Answer:

   Profit functions are simply total revenue for the firm minus total cost. For each firm their respective total revenue is given by the product of their quantity and the market price. For each firm their respective total cost is given by their total cost function. So we have:

$$\begin{aligned} \Pi_1 &= (1434 - 2q_1 - 2q_2)\, q_1 - 3q_1^2 \\ \Pi_2 &= (1434 - 2q_1 - 2q_2)\, q_2 - \left(12q_2^2 - 12q_2\right) \end{aligned}$$

   b Find the best response functions for Firms 1 and 2.

   Answer:

   Firm 1 maximizes:

$$\begin{aligned} \Pi_1 &= (1434 - 2q_1 - 2q_2)\, q_1 - 3q_1^2 \\ \frac{\partial \Pi_1}{\partial q_1} &= 1434 - 4q_1 - 2q_2 - 6q_1 \\ 0 &= 1434 - 10q_1 - 2q_2 \\ 10q_1 &= 1434 - 2q_2 \\ q_1 &= \frac{1434 - 2q_2}{10} \\ q_1 &= \frac{717 - q_2}{5} \end{aligned}$$

Technically, Firm 1's best response function is $q_1 = Max\left[0, \frac{717-q_2}{5}\right]$.
Firm 2 maximizes:

$$\begin{aligned}
\Pi_2 &= (1434 - 2q_1 - 2q_2)\, q_2 - \left(12q_2^2 - 12q_2\right) \\
\frac{\partial \Pi_2}{\partial q_2} &= 1434 - 2q_1 - 4q_2 - 24q_2 + 12 \\
0 &= 1446 - 2q_1 - 28q_2 \\
28q_2 &= 1446 - 2q_1 \\
q_2 &= \frac{1446 - 2q_1}{28} \\
q_2 &= \frac{723 - q_1}{14}
\end{aligned}$$

Technically, Firm 2's best response function is $q_2 = Max\left[0, \frac{723-q_1}{14}\right]$.

c Find the equilibrium to this game.

Answer:
To find the equilibrium to the game simply substitute one best response function into the other:

$$\begin{aligned}
q_1 &= \frac{717 - q_2}{5} \\
q_1 &= \frac{717 - \left(\frac{723-q_1}{14}\right)}{5} \\
5q_1 &= 717 - \left(\frac{723 - q_1}{14}\right) \\
70q_1 &= 10038 - 723 + q_1 \\
69q_1 &= 9315 \\
q_1 &= 135
\end{aligned}$$

Now to find Firm 2's quantity simply use $q_1 = 135$ in Firm 2's best response function:

$$\begin{aligned}
q_2 &= \frac{723 - q_1}{14} \\
q_2 &= \frac{723 - 135}{14} \\
q_2 &= 42
\end{aligned}$$

So the equilibrium to this game is $q_1 = 135$ and $q_2 = 42$.

d Find the (1) total market quantity, (2) price, and (3) profit for each firm.

Answer:
The market quantity is $Q = q_1 + q_2$, so $Q = 177$. The market price is $P(Q) = 1434 - 2Q$, so $P = 1080$. Each firm's profit is:

$$\begin{aligned}
\Pi_1 &= P * q_1 - TC(q_1) \\
\Pi_1 &= 1080 * 135 - 3 * 135^2 \\
\Pi_1 &= 91125 \\
&\text{and} \\
\Pi_2 &= P * q_1 - TC(q_2) \\
\Pi_2 &= 1080 * 42 - \left(12 * 42^2 - 12 * 42\right) \\
\Pi_2 &= 24696
\end{aligned}$$

e Assume Firm 1 is the only producer in the market now. Find Firm 1's monopoly (1) quantity, (2) price, and (3) profit.

Answer:

If Firm 1 is the only producer then Firm 1 is a monopolist so that $q_2 = 0$. There are many ways to go about this, the most direct being to set up Firm 1's profit function (assuming $q_2 = 0$) and solve for the monopoly quantity:

$$\begin{aligned}
\Pi_1 &= (1434 - 2q_1)\, q_1 - 3q_1^2 \\
\frac{d\Pi_1}{dq_1} &= 1434 - 4q_1 - 6q_1 \\
0 &= 1434 - 10q_1 \\
10q_1 &= 1434 \\
q_1 &= 143.4
\end{aligned}$$

We now know that Firm 1's monopoly quantity is 143.4 (we could also have used Firm 1's best response function from part **b** to figure this out), with price being equal to

$$\begin{aligned}
P(Q) &= 1434 - 2Q \\
P(143.4) &= 1434 - 2 * 143.4 \\
P &= 1147.2
\end{aligned}$$

Now knowing both P and q_1 we can find Firm 1's profit:

$$\begin{aligned}
\Pi_1 &= P * q_1 - TC(q_1) \\
\Pi_1 &= 1147.2 * 143.4 - 3 * (143.4)^2 \\
\Pi_1 &= 164508.48 - 61690.68 \\
\Pi_1 &= 102817.8
\end{aligned}$$

9. Consider two firms who compete by simultaneously choosing prices (a Bertrand game). If firms 1 and 2 choose prices p_1 and p_2, respectively, the quantity that consumers demand from firm i is

$$q_i(p_i, p_j) = a - p_i + bp_j, \text{ with } 0 < b < 2.$$

Assume that there are no fixed costs and that marginal cost is constant and equal to c, where $a > c > 0$. Prices must be nonnegative ($p_1 \geq 0$, $p_2 \geq 0$) and firms wish to maximize profit.

a Find the best response functions for firms 1 and 2.

Answer:

This problem is a straightforward maximization problem. Using the case-by-case analysis approach won't work here. Firm 1's maximization problem is:

$$\begin{aligned}
\max_{p_1 \geq 0} \Pi_1 &= q_1(p_1, p_2) * p_1 - c * q_1(p_1, p_2) \\
&\quad \text{or} \\
\max_{p_1 \geq 0} \Pi_1 &= (a - p_1 + bp_2) * p_1 - c * (a - p_1 + bp_2)
\end{aligned}$$

The first-order condition is:

$$\frac{\partial \Pi_1}{\partial p_1} = a - 2p_1 + bp_2 + c = 0$$

Firm 1's best response function is then:

$$p_1^* = \frac{a + bp_2 + c}{2}$$

Firm 2's problem is similar:

$$\begin{aligned}\max_{p_2 \geq 0} \Pi_2 &= q_2(p_1, p_2) * p_2 - c * q_2(p_1, p_2) \\ &\text{or} \\ \max_{p_2 \geq 0} \Pi_2 &= (a - p_2 + bp_1) * p_2 - c * (a - p_2 + bp_1)\end{aligned}$$

and yields a similar first-order condition:

$$\frac{\partial \Pi_2}{\partial p_2} = a - 2p_2 + bp_1 + c = 0$$

Firm 2's best response function is then:

$$p_2^* = \frac{a + bp_1 + c}{2}$$

b Find the equilibrium to this game.

Answer:

The equilibrium to this game is a price, p_1, for Firm 1 and a price, p_2, for Firm 2. There are 2 equations (the best response functions) and 2 unknowns (p_1 and p_2), so we need to solve for p_1 and p_2.

$$\begin{aligned}p_1 &= \frac{a + bp_2 + c}{2} \\ p_2 &= \frac{a + bp_1 + c}{2}\end{aligned}$$

Substituting leads to:

$$\begin{aligned}2p_1 &= a + b\left(\frac{a + bp_1 + c}{2}\right) + c \\ 4p_1 &= 2a + ab + b^2 p_1 + bc + 2c \\ 4p_1 - b^2 p_1 &= 2a + ab + bc + 2c \\ p_1 &= \frac{2a + ab + bc + 2c}{4 - b^2} \\ p_1 &= \frac{(2a + ab) + (bc + 2c)}{(2 - b)(2 + b)} \\ p_1 &= \frac{a * (2 + b) + c * (2 + b)}{(2 - b)(2 + b)} \\ p_1 &= \frac{a + c}{2 - b}\end{aligned}$$

Now substituting back into Firm 2's best response function, we get:

$$\begin{aligned}p_2 &= \frac{a + b * \left(\frac{a+c}{2-b}\right) + c}{2} \\ 2p_2 &= a + b\left(\frac{a - c}{2 - b}\right) + c \\ (2 - b) * 2p_2 &= (2 - b) * a + ba - bc + c * (2 - b) \\ (4 - 2b) * p_2 &= 2a - ba + ba - bc + 2c - bc \\ (4 - 2b) * p_2 &= 2a + 2c \\ p_2 &= \frac{2 * (a + c)}{2 * (2 - b)} \\ p_2 &= \frac{a + c}{2 - b}\end{aligned}$$

So the equilibrium is:

$$\begin{aligned} p_1^* &= \frac{a+c}{2-b} \\ p_2^* &= \frac{a+c}{2-b} \end{aligned}$$

c Explain why $b < 2$.

Answer:

Looking at the Nash equilibrium strategies, we see that if $b > 2$ then the best response by a firm would be a negative price, which is not allowable.

10. Consider a simultaneous quantity choice game between 2 firms. Each firm chooses a quantity, q_1 and q_2 respectively. The inverse market demand function is given by $P(Q) = 1980 - 6Q$, where $Q = q_1 + q_2$. Firm 1 has total cost function $TC(q_1) = 24q_1 + 624$ and Firm 2 has total cost function $TC(q_2) = 18(q_2)^2 + 12q_2 + 16$. The profit functions for each firm are:

$$\begin{aligned} \Pi_1 &= (1980 - 6q_1 - 6q_2)\, q_1 - (24q_1 + 624) \\ \Pi_2 &= (1980 - 6q_1 - 6q_2)\, q_2 - \left(18(q_2)^2 + 12q_2 + 16\right) \end{aligned}$$

a Find the best response functions for Firms 1 and 2.

Answer:

For Firm 1 we have:

$$\begin{aligned} \Pi_1 &= 1980q_1 - 6q_1^2 - 6q_1q_2 - 24q_1 - 624 \\ \Pi_1 &= 1956q_1 - 6q_1^2 - 6q_1q_2 - 624 \\ \frac{\partial \Pi_1}{\partial q_1} &= 1956 - 12q_1 - 6q_2 \\ 0 &= 1956 - 12q_1 - 6q_2 \\ 12q_1 &= 1956 - 6q_2 \\ q_1 &= \frac{1956 - 6q_2}{12} \end{aligned}$$

For Firm 2 we have:

$$\begin{aligned} \Pi_2 &= 1980q_2 - 6q_1q_2 - 6q_2^2 - 18q_2^2 - 12q_2 - 16 \\ \Pi_2 &= 1968q_2 - 24q_2^2 - 6q_1q_2 - 16 \\ \frac{\partial \Pi_2}{\partial q_2} &= 1968 - 48q_2 - 6q_1 \\ 0 &= 1968 - 48q_2 - 6q_1 \\ 48q_2 &= 1968 - 6q_1 \\ q_2 &= \frac{1968 - 6q_1}{48} \end{aligned}$$

So the best response functions for Firm 1 and 2 are, respectively: $q_1 = \frac{1956-6q_2}{12}$ and $q_2 = \frac{1968-6q_1}{48}$. Technically, the best responses are: $q_1 = Max\left[0, \frac{1956-6q_2}{12}\right]$ and $q_2 = Max\left[0, q_2 = \frac{1968-6q_1}{48}\right]$ because neither firm can produce a negative quantity.

b Find the Nash equilibrium to this game.

Answer:

Using the best response functions we have:

$$\begin{aligned}
q_1 &= \frac{1956-6q_2}{12} \\
q_1 &= \frac{1956-6\left(\frac{1968-6q_1}{48}\right)}{12} \\
12q_1 &= 1956-6\left(\frac{1968-6q_1}{48}\right) \\
12q_1 &= 1956-\left(\frac{1968-6q_1}{8}\right) \\
96q_1 &= 15648-1968+6q_1 \\
90q_1 &= 13680 \\
q_1 &= 152
\end{aligned}$$

Now using that $q_1 = 152$ we have:

$$\begin{aligned}
q_2 &= \frac{1968-6q_1}{48} \\
q_2 &= \frac{1968-6*152}{48} \\
q_2 &= 22
\end{aligned}$$

So the equilibrium to this game is: $q_1 = 152$ and $q_2 = 22$.

c Find (1) the total market quantity, (2) the market price, and (3) each firm's profit.

Answer:

The total market quantity is $Q = q_1 + q_2 = 174$.

The price is $P(Q) = 1980 - 6Q = 1980 - 6 * 174 = 936$.

Firm 1's profit is:

$$\begin{aligned}
\Pi_1 &= P * q_1 - (24q_1 + 624) \\
\Pi_1 &= 936 * 152 - (24 * 152 + 624) \\
\Pi_1 &= 142272 - 3648 - 624 \\
\Pi_1 &= 138000
\end{aligned}$$

Firm 2's profit is:

$$\begin{aligned}
\Pi_2 &= P * q_2 - \left(18q_2^2 + 12q_2 + 16\right) \\
\Pi_2 &= 936 * 22 - \left(18 * 22^2 + 12 * 22 + 16\right) \\
\Pi_2 &= 20592 - (8712 + 264 + 16) \\
\Pi_2 &= 20592 - 8992 \\
\Pi_2 &= 11600
\end{aligned}$$