

Problems on game theory and pricing practices (chapters 14 and 15)

- Two neighbors like to listen to their favorite recording artists, Black Eyed Peas and Linkin Park. There are only three possible volumes at which they can listen – low, medium, and high. Both neighbors prefer that the other neighbor listen to his music at a lower volume, although they prefer listening to their own music at the same volume as the neighbor than listening at a lower volume than the neighbor. The matrix for the game is as follows:

| | | | | |
|----------|--------|----------|--------|--------|
| | | Player 2 | | |
| | | High | Medium | Low |
| Player 1 | High | 2, 2 | 8, 1 | 15, -1 |
| | Medium | 1, 8 | 6, 6 | 12, 5 |
| | Low | -1, 15 | 5, 12 | 10, 10 |

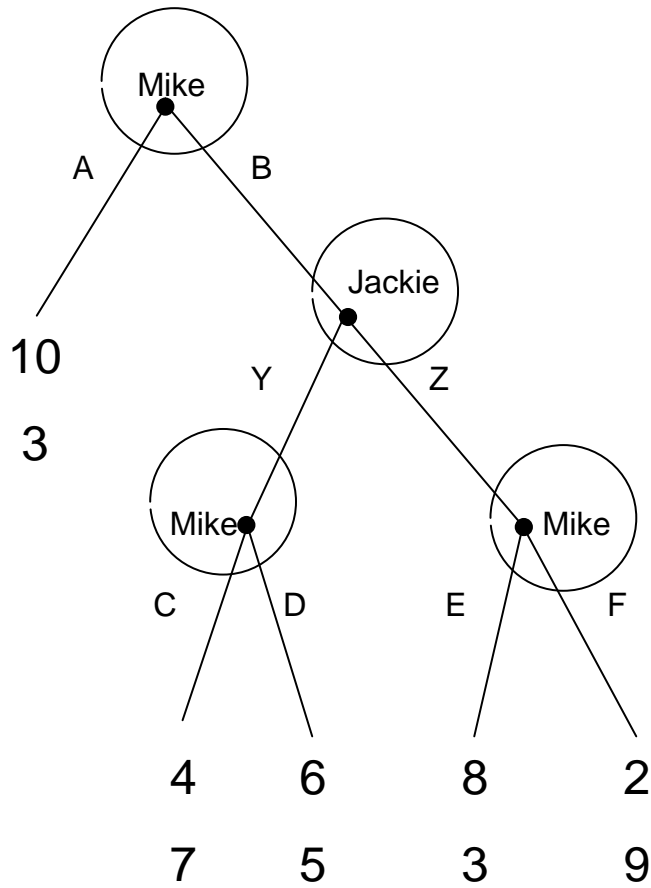
Find all Nash Equilibria of the game.

- There are two pigs in a pen, the Large Pig and the Little Pig. In order to receive food, one of the pigs must press a lever which is on one side of the pen. On the other side of the pen is a chute which deposits 6 pellets of food into the pen. Pressing the lever and then running across the pen to eat the food costs 0.5 pellets worth of energy. If both pigs go to press the lever then the Little Pig eats 1.5 pellets of food and the Large Pig eats 3.5 pellets of food (in total one pellet is lost in running across the pen). If the Little Pig presses the lever then the Large Pig is able to eat all 6 pellets of food prior to the Little Pig reaching the food, but it still costs the Little Pig 0.5 pellets worth of energy to run across the pen. If the Large Pig presses the lever then the Little Pig is able to eat 5 pellets of food before the Large Pig reaches the food, leaving the Large Pig with a net gain of 0.5 pellets of food. Finally, if neither pig presses the lever then no food is deposited into the pen and both pigs receive 0.

a Draw the matrix form version of this game.

b What is the Nash equilibrium of this game? Recall that Nash equilibria are strategies.

- Consider the following game tree:



Game 1

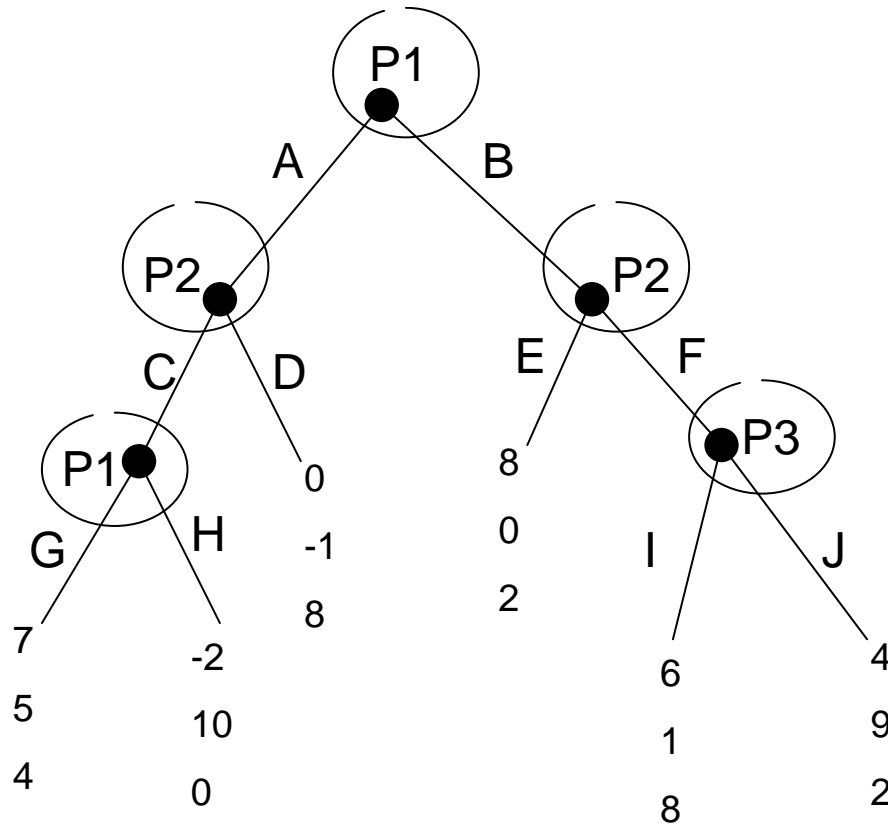
Find the subgame perfect Nash equilibrium to this game.

Answer:

If Mike gets to choose between C and D he will choose D because $6 > 4$. If Mike gets to choose between E and F he will choose E because $8 > 2$. If Jackie gets to choose between Y and Z she will receive 5 if she chooses Y (because Mike chooses D) and she will receive 3 if she chooses Z (because Mike chooses E), so she will choose Y because $5 > 3$. When Mike chooses between A and B he will get 10 if he chooses A and he will get 6 (because Jackie chooses Y and Mike then chooses D) if he chooses B, so he chooses A.

So the SPNE of this game is: Mike chooses strategy A, D, E, while Jackie chooses strategy Y.

4. Consider the following sequential game:



Find the subgame perfect Nash equilibrium to this game.

5. Consider two firms engaged in price (Bertrand) competition, such that the firm that charges the lowest price produces the entire market quantity. If the 2 firms charge the same price they split the market quantity evenly. This means that each firm has the following demand function:

| | q_1 | q_2 |
|----------------|-----------------------------------|-----------------------------------|
| if $p_1 > p_2$ | 0 | $\frac{500-p_2}{2}$ |
| if $p_1 = p_2$ | $\frac{1}{2} * \frac{500-p_1}{2}$ | $\frac{1}{2} * \frac{500-p_2}{2}$ |
| if $p_1 < p_2$ | $\frac{500-p_1}{2}$ | 0 |

Each firm has total cost equal to $TC(q_i) = 20 * q_i$, so that each firm has a constant marginal and average cost of production of 20.

- a Suppose that the pricing choices are made simultaneously. Find the pure strategy Nash equilibrium to this game.

Now suppose that Firm 1 announces the following policy: We are going to charge \$260 for our product. If any customer finds a lower price for this product than \$260 then tell us and we will not only match that price but offer a refund equal to the difference in the two prices. For instance, if another firm charges \$240, we will only charge \$220 (take $\$260 - \$240 = \$20$ and then deduct another \$20 so that the total amount deducted from our price of \$260 is \$40). This is known as a price-beating policy.

- b Find Firm 2's best response to Firm 1's policy announcement. *Reminder: This is only a one period game.*
- c Given the best response you found for Firm 2 in part b, is that best response and Firm 1's strategy (announced policy and price choice of \$260) a pure strategy Nash equilibrium to the game? Explain.

6. It is typical for the government to allocate construction contracts, such as repaving a highway, by holding an auction for the contract. The auction rules are as follows. Each bidder is to submit a sealed bid. The lowest bidder will win the contract, and the winning bidder will be paid an amount equal to the second lowest bid. Suppose that each bidder draws a cost, c_i , of completing the job from the uniform distribution over the interval $[0, 100]$. The cost draws are made independently of each other. All bidders are aware of the common distribution of costs as well as the fact that cost draws are made independently of one another.

a Suppose that the bidders are risk-neutral. Find a Nash equilibrium for this auction.

b Suppose that the bidders are risk-neutral. Suppose we changed the format so that the winning bidder, who is still the lowest bidder, now receives a payment equal to his bid if he wins (instead of a payment equal to the 2nd lowest bid as before). Would this bidder bid more than or less than the bidder's cost draw c_i ? Explain.¹

7. Coach Industries, Inc., is a leading manufacturer of recreational vehicle products. Its products include travel trailers, fifth-wheel trailers (towed behind pick-up trucks), and van campers, as well as parts and accessories. Coach offers its fifth-wheel trailers to both dealers (wholesale) and retail customers. Ernie Pantusso, Coach's controller, estimates that each fifth-wheel trailer costs the company \$10,000 in variable labor and material expenses. Demand relations for fifth-wheel trailers are

$$\begin{aligned} P_W &= \$15,000 - \$5Q_W \text{ (wholesale)} \\ P_R &= \$50,000 - \$20Q_R \text{ (retail)} \end{aligned}$$

a Assuming that the company can price discriminate between its two types of customers, calculate the profit-maximizing price, output, and profit contribution levels.

b Calculate point price elasticity for each customer type at the activity levels identified in part **a**. Are the differences in these elasticities consistent with your recommended price differences in part **a**? Why or why not?

8. The Heritage Club at Harbor Town offers elegant accommodations for discriminating vacationers on Hilton Head Island, South Carolina. Like many vacation resorts, Heritage Club has discovered the advantages of offering its services on an annual membership or "time-sharing" basis. To illustrate, assume that an individual vacationer's weekly demand curve can be written as:

$$P = \$6,500 - \$1,250Q$$

where P is the price of a single week of vacation time, and Q is the number of weeks of vacation time purchased during a given year. For simplicity, assume that the resort's marginal cost for a week of vacation time is \$1,500, and that fixed costs are zero. This gives the following total cost function:

$$TC = \$1,500Q$$

a Calculate the profit-maximizing price, output, profit, and consumer surplus assuming a uniform per unit price is charged each customer.

b Calculate the profit-maximizing price, output and profit assuming the optimal two-part pricing strategy is adopted for each customer.

9. Each ton of ore mined from the Baby Doe Mine in Leadville, Colorado, produces one ounce of silver and one pound of lead in a fixed 1:1 ratio. Marginal costs are \$10 per ton of ore mined. The demand curve for silver is:

$$P_S = \$11 - \$0.00003Q_S$$

and the demand curve for lead is:

$$P_L = \$0.4 - \$0.000005Q_L$$

where Q_S is ounces of silver and Q_L is pounds of lead.

¹Finding the actual equilibrium bid function for this type of auction involves solving a differential equation, which I am not going to make you all do.

- a** Calculate profit-maximizing sales quantities and prices for silver and lead.
- b** Now assume that wild speculation in the silver market has created a fivefold (or 500%) increase in silver demand. A 500% increase in silver demand means that our new demand curve is $P_S = 5(\$11 - \$0.00003Q_S)$. Calculate optimal sales quantities and prices for both silver and lead under these conditions.
10. The Bristol, Inc. is an elegant dining establishment that features French cuisine at dinner six nights per week, and brunch on weekends. In an effort to boost traffic from shoppers during the Christmas season, the Bristol offered Saturday customers \$4 off its \$16 regular price for brunch. The promotion proved successful, with brunch sales rising from 250 to 750 units per day.
- a** Calculate the arc price elasticity of demand for brunch at the Bristol.
- b** Assume that the arc price elasticity (from part A) is the best available estimate of the point price elasticity of demand. If marginal cost is \$8.56 per unit for labor and materials, calculate the Bristol's optimal markup on cost and its optimal price.
11. Brake-Checkup, Inc., offers automobile brake analysis and repair at a number of outlets in the Philadelphia area. The company recently initiated a policy of matching the lowest advertised competitor price. As a result, Brake-Checkup has been forced to reduce the average price for brake jobs by 3%, but it has enjoyed a 15% increase in customer traffic. Meanwhile, marginal costs have held steady at \$120 per brake job.
- a** Calculate the point price elasticity of demand for brake jobs.
- b** Calculate Brake-Checkup's optimal markup on cost as well as its optimal price.
12. Dr. John Dorian, chief of staff at the Northern Medical Center, has asked you to propose an appropriate markup pricing policy for various medical procedures performed in the hospital's emergency room. To help in this regard, you consult a trade industry publication that provides data about the price elasticity of demand for medical procedures. Unfortunately, the abrasive Dr. Dorian failed to mention whether he wanted you to calculate the optimal markup as a percentage of price or as a percentage of cost. To be safe, calculate the optimal markup on price and optimal markup on cost for each of the following procedures:

| Procedure | Price Elasticity | Optimal Markup on Cost | Optimal Markup on Price |
|-----------|------------------|------------------------|-------------------------|
| A | -1 | | |
| B | -2 | | |
| C | -3 | | |
| D | -4 | | |
| E | -5 | | |