Test 1

MBAD6112M

Wednesday, October 29^{th}

For this exam answer all questions completely. Please provide ALL of your work – there are two reasons I ask this. One is so that I can see how you were attempting to solve a problem, and the other is so that I can attempt to identify any mistakes if you do not have the correct answer. I will award partial credit on problems, and if I can see your work then I will be able to determine where your mistake was and how close you came to answering the question correctly. For instance, if the correct answer to a question is "12" and you write down "21" it may be that you just switched the numbers, which is a very minor mistake and I may not even take off any points if I see your work; but if I do not see your work and I just see "21" instead of "12" then I would be forced to mark the answer wrong and deduct full points.

If you are having difficulty with the math, write down (in words) what you are attempting to do. Again, there is a big difference between a blank answer and an answer that does not have the correct numeric answer but does have the correct explanation of the process that would be used to get the numeric answer – the blank answer signals nothing while the description of how you would solve the problem at least signals that you know how to solve the problem but are just having difficulty with the math. Hopefully that will not be an issue, but there is an opportunity to earn partial credit if I know that you know the process to obtaining the answer.

1. Short answer explanation questions:

a Draw a picture of the following short-run cost curves: average fixed cost (AFC), average total cost (ATC), and marginal cost (MC) assuming they have their typical shapes. Explain why the typical shape of the **short-run** average total cost function is U-shaped.

Answer:

I have skipped drawing the picture. For low levels of quantity, high average fixed costs cause average total cost to be high, while for high levels of quantity high marginal costs cause average total cost to be high. The reason for the drop in ATC in the middle is because AFC is declining and MC is either decreasing or increasing but still below ATC.

b Explain why indifference curves cannot cross.

Answer:

Indifference curves cannot cross because that would mean that one bundle of goods was on two indifference curves. If that were true, then that one bundle of goods would be associated with two different levels of utility. Indifference curves that cross would violate transitivity.

c Is the law of diminishing marginal returns a short-run or long-run concept? Define diminishing marginal returns and give an example of why we have the law of diminishing marginal returns. This example can be a simple description or story.

Answer:

The law of diminishing marginal returns is a short-run concept; as we apply more and more variable factors to a fixed input output still increases, but at a decreasing rate. One example would be having a fixed piece of land and adding more farmers to the land; another would be having "too many cooks in the kitchen" because the space in the kitchen is the fixed input. We can see how eventually those farmers/cooks might not produce as much output as the previous farmer/cook because there is only so much available space with which to work.

d Why might we use $\bar{R}^2$ (also called adjusted R^2) instead of R^2 when evaluating how "good" our regression model is?

Answer:

We might want to use $\bar{R}^2$ instead of R^2 when evaluating how "good" our regression model is because R^2 will never decrease if we add more independent variables, which means we can maximize R^2 by adding all sorts of irrelevant variables. Because $\bar{R}^2$ penalizes the researcher for adding variables with little significance it forces the researcher to think about what variables are being included in the model.

e

2. Suppose that an individual has utility for goods A and B represented by the following utility function:

$$u(A,B) = \left(A^{1/2} + B^{1/2}\right)^2$$

Let w be the individual's wealth, and p_A and p_B be the prices for goods A and B, respectively. We know that w, p_A, and p_B are all greater than zero.

a Find the individual's marginal utility for good A.

Answer:

The marginal utility for good A is given by the partial derivative of the utility function with respect to A:

$$\begin{aligned}
\frac{\partial u(A,B)}{\partial A} &= 2\left(A^{1/2} + B^{1/2}\right)\frac{1}{2}A^{-1/2} \\
\frac{\partial u(A,B)}{\partial A} &= \frac{\left(A^{1/2} + B^{1/2}\right)}{A^{1/2}}
\end{aligned}$$

b Find the individual's marginal utility for good B.

Answer:

The marginal utility for good B is given by the partial derivative of the utility function with respect to B:

$$\begin{aligned}
\frac{\partial u(A,B)}{\partial B} &= 2\left(A^{1/2} + B^{1/2}\right)\frac{1}{2}B^{-1/2} \\
\frac{\partial u(A,B)}{\partial B} &= \frac{\left(A^{1/2} + B^{1/2}\right)}{B^{1/2}}
\end{aligned}$$

c Find the individual's demand functions for goods A and B. The demand functions will be functions of prices (p_A and p_B) and wealth (w). An alternative way to phrase this question is: find the consumer's optimal bundle for the utility maximization problem.

Answer:

I am going to use a shortcut, because at the optimal bundle:

$$\frac{MU_A}{p_A} = \frac{MU_B}{p_B}$$

Now:

$$\begin{aligned}
\frac{\frac{\left(A^{1/2}+B^{1/2}\right)}{A^{1/2}}}{p_A} &= \frac{\frac{\left(A^{1/2}+B^{1/2}\right)}{B^{1/2}}}{p_B} \\
\frac{\frac{\left(A^{1/2}+B^{1/2}\right)}{A^{1/2}}}{\frac{\left(A^{1/2}+B^{1/2}\right)}{B^{1/2}}} &= \frac{p_A}{p_B} \\
\frac{\left(A^{1/2}+B^{1/2}\right)}{A^{1/2}} * \frac{B^{1/2}}{\left(A^{1/2}+B^{1/2}\right)} &= \frac{p_A}{p_B} \\
\left(\frac{B}{A}\right)^{1/2} &= \frac{p_A}{p_B} \\
\frac{B}{A} &= \frac{p_A^2}{p_B^2} \\
B &= A\frac{p_A^2}{p_B^2}
\end{aligned}$$

Using the budget constraint and substituting in for B:

$$\begin{aligned}
w - p_A A - p_B B &= 0 \\
w - p_A A - p_B A\frac{p_A^2}{p_B^2} &= 0 \\
w p_B - p_A p_B A - p_A^2 A &= 0 \\
w p_B &= p_A p_B A + p_A^2 A \\
\frac{w p_B}{p_A p_B + p_A^2} &= A \\
\frac{w p_B}{p_A\left(p_B + p_A\right)} &= A
\end{aligned}$$

Finding B:

$$\begin{aligned}
B &= A\frac{p_A^2}{p_B^2} \\
B &= \frac{w p_B}{p_A\left(p_B + p_A\right)}\frac{p_A^2}{p_B^2} \\
B &= \frac{w p_A}{p_B\left(p_B + p_A\right)}
\end{aligned}$$

So we have:

$$\begin{aligned}
A &= \frac{w p_B}{p_A\left(p_B + p_A\right)} \\
B &= \frac{w p_A}{p_B\left(p_B + p_A\right)}
\end{aligned}$$

3. Consider a market with the the following supply and demand conditions:

$$\begin{aligned}
Q_D &= 920 - 10P_{own} - 20P_{other} + Y \\
Q_S &= 20 + 5P_{own} - 10P_{resource}
\end{aligned}$$

where Q_D is quantity demanded, Q_S is quantity supplied, P_{own} is the own-price, P_{other} is the price of another good, Y is income, and $P_{resource}$ is the price of a resource good. Suppose the average price of $P_{other} = 5$, the average income is $Y = 1000$, and the average price of the resource is $P_{resource} = 15$.

a Find the equilibrium price and quantity in this market.

Answer:

First substitute in the know values for P_{other}, Y, and $P_{resource}$ to find:

$$\begin{aligned} Q_D &= 920 - 10P_{own} - 20 * 5 + 1000 \\ Q_D &= 1820 - 10P_{own} \end{aligned}$$

and

$$\begin{aligned} Q_S &= 20 + 5P_{own} - 10 * 15 \\ Q_S &= -130 + 5P_{own} \end{aligned}$$

Then set $Q_D = Q_S$:

$$\begin{aligned} Q_D &= Q_S \\ 1820 - 10P_{own} &= -130 + 5P_{own} \\ 1950 &= 15P_{own} \\ 130 &= P_{own} \end{aligned}$$

Now substitute P_{own} into the Q_D and Q_S functions:

$$\begin{aligned} Q_D &= 1820 - 10 * 130 \\ Q_D &= 1820 - 1300 \\ Q_D &= 520 \\ & \text{and} \\ Q_S &= -130 + 5 * 130 \\ Q_S &= -130 + 650 \\ Q_S &= 520 \end{aligned}$$

So the equilibrium price and quantity are $P_{own} = 110$ and $Q_D = Q_S = 520$.

b Is this good a normal or inferior good? Explain how you know.

Answer:

This good is a normal good because the coefficient on income is 1, which is positive.

c Is the "other" good (with price of P_{other}) a substitute or complement? Explain how you know.

Answer:

This good and the "other" good are complements because the coefficient on P_{other} is negative. When the price of the "other" good increases, the quantity demanded of this good decreases.

d Suppose that $P_{resource}$ increases to 30. Calculate the new equilibrium price and quantity. Is the change in equilibrium price and quantity expected given the change in the market (a resource price increased)? Explain – it may be helpful to draw a simple supply and demand graph.

Answer:

None of the factors that affect demand have changed, so our demand function remains the same:

$$Q_D = 1820 - 10P_{own}$$

However, $P_{resource}$ does enter into the supply function so we need to recalculate it:

$$\begin{aligned} Q_S &= 20 + 5P_{own} - 10P_{resource} \\ Q_S &= 20 + 5P_{own} - 10 * 30 \\ Q_S &= -280 + 5P_{own} \end{aligned}$$

Again, now just set $Q_D = Q_S$ and solve for P_{own}:

$$\begin{aligned} Q_D &= Q_S \\ 1820 - 10P_{own} &= -280 + 5P_{own} \\ 2100 &= 15P_{own} \\ 140 &= P_{own} \end{aligned}$$

Now substituting into the demand and supply functions:

$$\begin{aligned} Q_D &= 1820 - 10P_{own} \\ Q_D &= 1820 - 10 * 140 \\ Q_D &= 420 \\ & \quad \text{and} \\ Q_S &= -280 + 5 * 140 \\ Q_S &= 420 \end{aligned}$$

The new equilibrium price and quantity are $P_{own} = 140$ and $Q_D = Q_S = 420$.

We see a decrease in quantity and an increase in price. Because demand remained the same, the increase in resource price caused the supply to decrease, which results in a higher price and a lower quantity, so this result is consistent with our economic intuition.

e Calculate the own-price elasticity of demand using the point elasticity formula, ε_P, using the original equilibrium price and quantity you found in part **a** and the new equilibrium price and quantity you found in part **d**. Is demand for this good elastic or inelastic?

Answer:

The price elasticity formula is:

$$\begin{aligned} \varepsilon_P &= \frac{Q_{new} - Q_{old}}{P_{new} - P_{old}} * \frac{P_{old}}{Q_{old}} \\ \varepsilon_P &= \frac{420 - 520}{140 - 130} * \frac{130}{520} \\ \varepsilon_P &= \frac{-100}{10} * \frac{1}{4} \\ \varepsilon_P &= \frac{-10}{4} = -\frac{5}{2} = -2.5 \end{aligned}$$

Demand for this good is elastic because $|-2.5| > 1$.

4. Consider a firm with the estimated production function:

$$q = 36K^{1/2}L_{skill}^{1/2}L_{unskill}^{1/3}$$

where K is the amount of capital, L_{skill} is the amount of skilled labor, and $L_{unskill}$ is the amount of unskilled labor.

a Does this production function have increasing, decreasing, or constant returns to scale?

Answer:

This production function is of the Cobb-Douglas type, so all we need to do is add the exponents: $\frac{1}{2} + \frac{1}{2} + \frac{1}{3} = \frac{4}{3}$. Because $\frac{4}{3} > 1$, this production function has increasing returns to scale.

b Find the marginal product (MP) for:

i capital (K)

Answer:

The marginal product of capital is given by the partial derivative of q with respect to K:

$$\frac{\partial q}{\partial K} = \frac{18L_{skill}^{1/2}L_{unskill}^{1/3}}{K^{1/2}}$$

ii skilled labor (L_{skill})

Answer:

The marginal product of skilled labor is given by the partial derivative of q with respect to L_{skill}:

$$\frac{\partial q}{\partial L_{skill}} = \frac{18K^{1/2}L_{unskill}^{1/3}}{L_{skill}^{1/2}}$$

iii unskilled labor ($L_{unskill}$)

Answer:

The marginal product of unskilled labor is given by the partial derivative of q with respect to $L_{unskill}$:

$$\frac{\partial q}{\partial L_{skill}} = \frac{12K^{1/2}L_{skill}^{1/2}}{L_{unskill}^{2/3}}$$

c Set up, but do NOT solve, the Lagrangian function for the cost-minimization problem. I repeat, only set up the Lagrangian function, do NOT solve it (you will see why in part **d**).

Answer:

The Lagrangian function for the cost-minimization problem is:

$$\min \mathcal{L}\left(K, L_{skill}, L_{unskill}, \lambda\right) = rK + w_{skill}L_{skill} + w_{unskill}L_{unskill} + \lambda\left(q - 36K^{1/2}L_{skill}^{1/2}L_{unskill}^{1/3}\right)$$

d Let $r = 72$, $w_{skill} = 8$, and $w_{unskill} = 6$ be the prices for capital, skilled labor, and unskilled labor, respectively. Suppose the firm wants to produce $q = 55,296$. Your optimization specialist, Dmitry, tells you that at these prices the cost-minimizing level of capital, skilled labor, and unskilled labor are: $K = 64$, $L_{skill} = 576$, and $L_{unskill} = 512$. Verify that he is correct. (**Hint**: There are at least two methods to verify if he is correct. One would be to actually solve the problem. The other would be to think about the conditions or relationships that need to hold when costs are minimized and then check those conditions.)

Answer:

We could go through the entire optimization problem. Or, we could remember that at the optimal bundle of cost-minimizing inputs the following conditions are true:

$$\begin{aligned}
\frac{MP_K}{r} &= \frac{MP_{skill}}{w_{skill}} \\
\frac{MP_K}{r} &= \frac{MP_{unskill}}{w_{unskill}} \\
\frac{MP_{skill}}{w_{skill}} &= \frac{MP_{unskill}}{w_{unskill}}
\end{aligned}$$

Alternatively, rearranging we have:

$$\begin{aligned}
\frac{MP_K}{MP_{skill}} &= \frac{r}{w_{skill}} \\
\frac{MP_K}{MP_{unskill}} &= \frac{r}{w_{unskill}} \\
\frac{MP_{skill}}{MP_{unskill}} &= \frac{w_{skill}}{w_{unskill}}
\end{aligned}$$

I am going to work with the first two, and I will work with the general MP formulas from part **b** because it will make life easy.

$$\begin{aligned}
\frac{MP_K}{MP_{skill}} &= \frac{r}{w_{skill}} \\
\frac{\frac{18L_{skill}^{1/2}L_{unskill}^{1/3}}{K^{1/2}}}{\frac{18K^{1/2}L_{unskill}^{1/3}}{L_{skill}^{1/2}}} &= \frac{r}{w_{skill}} \\
\frac{18L_{skill}^{1/2}L_{unskill}^{1/3}}{K^{1/2}} * \frac{L_{skill}^{1/2}}{18K^{1/2}L_{unskill}^{1/3}} &= \frac{r}{w_{skill}} \\
\frac{L_{skill}}{K} &= \frac{r}{w_{skill}} \\
\frac{576}{64} &= \frac{72}{8} \\
9 &= 9
\end{aligned}$$

The first condition holds. Now using the second condition:

$$\begin{aligned}
\frac{MP_K}{MP_{unskill}} &= \frac{r}{w_{unskill}} \\
\frac{\frac{18L_{skill}^{1/2}L_{unskill}^{1/3}}{K^{1/2}}}{\frac{12K^{1/2}L_{skill}^{1/2}}{L_{unskill}^{2/3}}} &= \frac{r}{w_{unskill}} \\
\frac{18L_{skill}^{1/2}L_{unskill}^{1/3}}{K^{1/2}} * \frac{L_{unskill}^{2/3}}{12K^{1/2}L_{skill}^{1/2}} &= \frac{r}{w_{unskill}} \\
\frac{3L_{unskill}}{2K} &= \frac{r}{w_{unskill}} \\
\frac{3*512}{2*64} &= \frac{72}{6} \\
\frac{1536}{128} &= 12 \\
12 &= 12
\end{aligned}$$

So the second condition also holds. We do not need to check the third condition because if $A = B$, and $A = C$, then $B = C$, but I will provide the math anyways:

$$\begin{aligned}
\frac{MP_{skill}}{MP_{unskill}} &= \frac{w_{skill}}{w_{unskill}} \\
\frac{\frac{18K^{1/2}L_{unskill}^{1/3}}{L_{skill}^{1/2}}}{\frac{12K^{1/2}L_{skill}^{1/2}}{L_{unskill}^{2/3}}} &= \frac{w_{skill}}{w_{unskill}} \\
\frac{18K^{1/2}L_{unskill}^{1/3}}{L_{skill}^{1/2}} * \frac{L_{unskill}^{2/3}}{12K^{1/2}L_{skill}^{1/2}} &= \frac{w_{skill}}{w_{unskill}} \\
\frac{3L_{unskill}}{2L_{skill}} &= \frac{w_{skill}}{w_{unskill}} \\
\frac{3 * 512}{2 * 576} &= \frac{8}{6} \\
\frac{1536}{1152} &= \frac{8}{6} \\
9216 &= 9216
\end{aligned}$$

Finally, to make sure we are producing $q = 55,296$:

$$\begin{aligned}
q &= 36K^{1/2}L_{skill}^{1/2}L_{unskill}^{1/3} \\
q &= 36 * (64)^{1/2} * (576)^{1/2} * (512)^{1/3} \\
q &= 36 * 8 * 24 * 8 \\
q &= 55,296
\end{aligned}$$

5. Consider the following three regression models:

$$\begin{aligned}
\text{Model 1} &: \ wage = \beta_1 + \beta_2 tenure + \beta_3 age + \varepsilon \\
\text{Model 2} &: \ \ln(wage) = \beta_1 + \beta_2 \ln(tenure) + \beta_3 \ln(age) + \varepsilon \\
\text{Model 3} &: \ wage = \beta_1 + \beta_2 tenure + \beta_3 age + \beta_4 school + \beta_5 male + \beta_6 union + \varepsilon
\end{aligned}$$

The variables are:

Variable	Description
wage	an individual's hourly wage (in U.S. dollars)
tenure	how long (in years) an individual has been employed at current job
age	how old (in years) the individual is
ln(wage)	natural logarithm of wage
ln(tenure)	natural logarithm of tenure
ln(age)	natural logarithm of age
school	how many years of schooling the individual has
male	a dummy variable equal to 1 if the individual is male and 0 if female
union	a dummy variable equal to 1 if the individual is in a union and 0 if not in a union

The summary statistics for each of the variables and coefficient estimates for each model are on the following pages.

a Calculate the point elasticities for tenure and age in all three models, evaluating the elasticities at the means of the variables when necessary. How similar (or different) are the elasticities in each model?

Answer:

The elasticities for tenure and age in Model 1 and Model 3 are given by: $\beta_2 * \frac{tenure}{wage}$ and $\beta_3 * \frac{age}{wage}$. For Model 2, β_2 and β_3 are the elasticities for tenure and age. The table below has the calculate elasticities:

	tenure	age
Model 1	0.161	0.213
Model 2	0.148	0.244
Model 3	0.125	0.243

These elasticities are slightly different across models, but they do not vary much.

b For Model 3, determine if each of the individual regression coefficients is statistically different than zero. Tell me what your t-statistic is and the critical value you are comparing each of your computed t-statistics to is.

Answer:

For testing the hypothesis that an estimated coefficient is different than zero we need to calculate the ratio of the coefficient estimate to the standard error for each variable. Then we need to compare that ratio to the critical value for the t-distribution with 9148 degrees of freedom (or ∞). The simple guideline that if the absolute value of the t-statistic>2.57 then the estimate is significant at the 1% level (or 99% confidence level) will work because we have a large number of observations. The table below has the calculated t-statistics for each variable. Note that your answers may differ slightly depending upon how you rounded/truncated the coefficient estimates and standard errors.

Variable	Coeff. Est.	Std. Error	t-stat=$\frac{Coeff.\ Est.}{Std.\ Error}$
Constant	-10.45	0.429	-24.3
Tenure	0.203	0.010	20.3
Age	0.080	0.007	11.42
School	1.23	0.026	47.31
Male	3.22	0.132	24.39
Union	0.855	0.176	4.86

Note that all of these t-statistics are greater than 2.57 (at least in absolute value), so they are all significant at the 1% level. Technically, all of them are greater than 3.291, which is the critical value for the 0.1% level (or 99.9% confidence level), so they are all significant at that level as well.

c For Model 3, interpret the coefficient estimates for each independent variable. By "interpret" I mean tell me what the estimated value of each β means (example: what does $\beta_4 = 1.231744$ tell us about the impact of the variable school on the individual's hourly wage).

Answer:

The constant, β_1, tells us that an individual who is female, not a union member, 0 years old, with 0 tenure and 0 years of schooling would have an hourly wage of $-\$10.46$.

The coefficient on tenure, β_2, tells us that each additional year of tenure at the firm adds 20 cents to the hourly wage.

The coefficient on age, β_3, tells us that each additional year of living adds about 8 cents to the hourly wage.

The coefficient on school, β_4, tells us that each additional year of school adds $1.23 to the hourly wage.

The coefficient on male, β_5, tells us that men earn an additional $3.22 per hour when compared to women.

The coefficient on union, β_6, tells us that union workers earn 85 cents more per hour when compared to non-union workers.

Keep in mind that there are MANY variables that are uncontrolled for in this model. For instance, we have no information (in the model) on the types of jobs that these individuals hold or what companies they work for.

d For this question, use Model 3 as the unrestricted model and Model 1 as the restricted model. Test for the joint significance of the variables school, male, and union. The null hypothesis for this statistical test is that $\beta_4 = \beta_5 = \beta_6 = 0$.

Answer:

To conduct this test we need to use the F-test. The formula for the F-test is:

$$\frac{\frac{R^2_{UR}-R^2_R}{k}}{\frac{1-R^2_{UR}}{n-k}} \tilde{} F_{k,n-k}$$

We have all of these pieces so:

$$\frac{\frac{0.3249-0.1116}{3}}{\frac{1-0.3249}{9151}} = 963.76 \tilde{} F_{3,9151}$$

Looking at the F-table, the critical value for $F_{3,9151}$ is 3.78. Because 963.76>3.78, we reject the null hypothesis.

e An employee enters your office and demands a raise, stating that because he is not a union member that it is his right to bargain on his own behalf. He believes he is underpaid at \$35 per hour given he is 50 years old, has been with the firm for 12 years, and has 17 years of schooling. Could you use regression analysis to aid you in determining if his request is reasonable?

Answer:

Using Model 3, we can see that employees with his characteristics would, on average, be earning:

$$\begin{aligned}
wage &= -10.45 + 0.20 * 12 + 0.08 * 50 + 1.23 * 17 + 3.22 * 1 + 0.85 * 0 \\
wage &= -10.45 + 2.4 + 4 + 20.91 + 3.22 + 0 \\
wage &= \$20.08
\end{aligned}$$

While every individual is different, the results from the regression analysis suggest that he is not underpaid given his characteristics.