Test 1 Answers, ECON 6090-92, Summer 2013

Directions: Answer all questions as completely as possible.If you cannot solve the problem, explaining how you would solve the problem may earn you some points. Point totals are in parentheses.

1. Short answer questions (20)

a (5) In the inequality constrained optimization process, there are "complementary slackness" conditions to the problem. For instance, with respect to the budget constraint, the complementary slackness condition is:

$$\lambda * [y - p_1 x_1 - p_2 x_2] = 0.$$

Why are these conditions imposed?

Answer:

The consumer's objective is to maximize the utility function, which is $U(x_1, x_2)$. When solving the problem we maximize the Lagrangian, which is $U(x_1, x_2) + \lambda [y - p_1 x_1 - p_2 x_2]$. By imposing $\lambda * [y - p_1 x_1 - p_2 x_2] = 0$ we make certain that maximizing the Lagrangian is the same as maximizing the utility function.

b (5) Is the price elasticity of demand for a good likely to be more elastic in the short term or the long term? Explain.

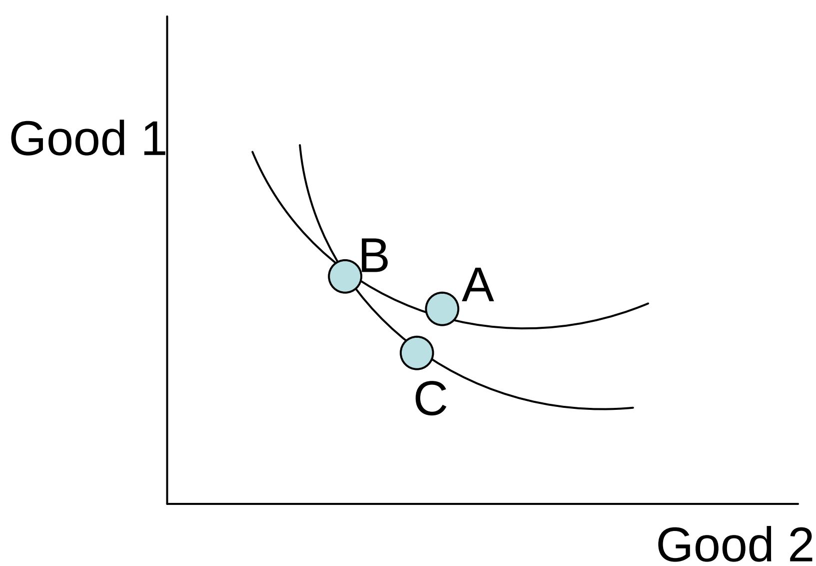
Answer:

The long term, because the longer amount of time a person has, the more possible substitutes for the good a person should be able to find. Because the consumer likely has less options in the short term, their quantity purchases will not be that responsive to price.

c (5) Explain why indifference curves cannot cross, citing specific properties of consumer preferences.

Answer:

Indifference curves cannot cross because they would violate either transitivity or the more is better property. Consider the following picture:

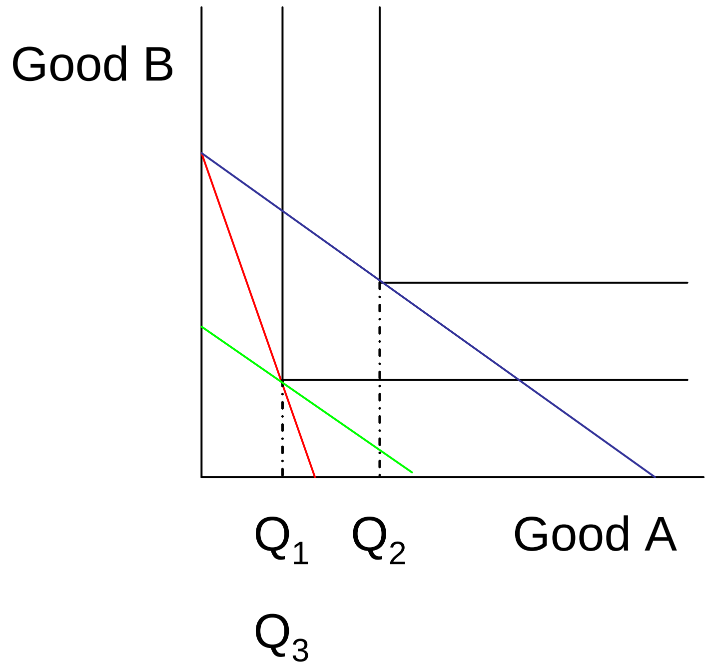


We have $C \succsim B$ (since they are on the same indifference curve), and $B \succsim A$ (since they are on the same indifference curve), but Bundle A is on a higher indifference curve than Bundle C, so $A \succ C$ (which means C cannot be at least as good as A). Thus, transitivity is violated. Also, we have $A \succsim B \succsim C$, but bundle A has strictly more of both goods, so we need $A \succ C$, which it cannot be if $C \succsim A$.

d (5) Consider the utility function for perfect complements, $U(A,B) = \min(A,B)$. Suppose that the individual consumes 20 units of both A and B, and after the price of A decreases now consumes 30 units of both A and B. How much of this 10 unit increase in consumption of Good A is due to the substitution effect and how much is due to the income effect?

Answer:

The entire effect (10) is an income effect. There is no substitution effect with this utility function because the goods are perfect complements. To see this, look at the picture:



The red line is the initial budget constraint, the blue line is the new budget constraint, and the green line is the "hypothetical" budget constraint (the one with the same slope as the new line. Note that the total effect is given by $Q_2 - Q_1 = 10$. The income effect is given by $Q_2 - Q_3 = 10$, and the substitution effect is given by $Q_3 - Q_1 = 0$ because Q_3 and Q_1 are the same point.

2. (30) Maurice has the following utility function:

$$U(A,B) = 20A + 80B - A^2 - 2B^2.$$

The price of A is \$1, the price of B is \$2, and Maurice's income is \$41. You may assume that the solution is an interior solution.

a (10) Find the marginal utility for good A and the marginal utility for good B.

Answer:

The marginal utility for any generic utility function and good X is:

$$MU_X = \frac{\partial U}{\partial X}.$$

So the marginal utility function for Good A using the given utility function is:

$$\frac{\partial U(A,B)}{\partial A} = 20 - 2A.$$

The marginal utility function for Good B using the given utility function is:

$$\frac{\partial U(A,B)}{\partial B} = 80 - 4B$$

b (15) Find Maurice's optimal bundle of Good A and Good B.

Answer:

Begin by setting up the Lagrangian:

$$\mathcal{L}(A, B, \lambda) = U(A, B) + \lambda [y - p_A A - p_B B].$$

The set of conditions we need is:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial A} &= \frac{\partial U(A, B)}{\partial A} - p_A \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial B} &= \frac{\partial U(A, B)}{\partial B} - p_B \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= y - p_A A - p_B B = 0
\end{aligned}$$

I have set the first two equations equal to zero because we are assuming it is an interior solution and the last equation because the consumer will spend all his income. Now, $\frac{\partial U(A,B)}{\partial A} = 20 - 2A$ and $\frac{\partial U(A,B)}{\partial B} = 80 - 4B$, so just plugging these in we get:

$$\begin{aligned}
20 - 2A - p_A \lambda &= 0 \\
80 - 4B - p_B \lambda &= 0 \\
y - p_A A - p_B B &= 0
\end{aligned}$$

Now just solve for A, B, and λ. From the first 2 equations we have:

$$\frac{20 - 2A}{p_A} = \frac{80 - 4B}{p_B}$$

so that:

$$\begin{aligned}
A &= \left(\frac{-1}{2}\right)\left(\frac{80 - 4B}{p_B}\right) p_A - \left(\frac{-1}{2}\right) 20 \\
A &= \frac{(2B - 40)\, p_A}{p_B} + 10 \\
A &= \frac{(2B - 40)\, p_A + 10 p_B}{p_B}
\end{aligned}$$

Plugging this into the budget constraint we get:

$$\begin{aligned}
y - p_A \left(\frac{(2B - 40)\, p_A + 10 p_B}{p_B}\right) - p_B B &= 0 \\
y p_B - p_A \left((2B - 40)\, p_A + 10 p_B\right) - p_B^2 B &= 0 \\
y p_B - p_A \left(2B p_A - 40 p_A + 10 p_B\right) - p_B^2 B &= 0 \\
y p_B - 2B p_A^2 + 40 p_A^2 - 10 p_A p_B - p_B^2 B &= 0 \\
y p_B + 40 p_A^2 - 10 p_A p_B &= p_B^2 B + 2B p_A^2 \\
\frac{y p_B + 40 p_A^2 - 10 p_A p_B}{p_B^2 + 2 p_A^2} &= B
\end{aligned}$$

Plugging in for y, p_B, and p_A we have:

$$\frac{41 * 2 + 40 * (1)^2 - 10 * 2 * 1}{2^2 + 2 * (1)^2} = 17.$$

So the quantity of good B is 17. Now using:

$$A = \frac{(2B - 40)\, p_A + 10 p_B}{p_B}$$

we can find A, which is:

$$A = \frac{(2*17-40)*1+10*2}{2} = 7.$$

So the consumer consumes the bundle with 7 units of good A and 17 units of good B.

c (5) Show that the utility at the solution you found is greater than that of either corner solution, thus "proving" that the interior solution is the optimal solution.

Answer:

If only good A is consumed then the consumer purchases 41 units of good A and $U(A,B)$ is:

$$20*41+80*0-41^2-2*0^2=-861.$$

If only good B is consumed then the consumer purchases 20.5 units of good B and $U(A,B)$ is:

$$20*0+80*20.5-0^2-2*20.5^2=799.5.$$

At the optimal bundle where the consumer chooses a quantity of 7 for good A and 17 for good B, $U(A,B)$ is:

$$20*7+80*17-7^2-2*17^2=873.$$

Thus, the consumer is better off at the interior solution than at either corner solution.

3. (25) Following are the supply and demand function for Nike shoes. Assume that the constant values for income and the price of basketballs are $Y=2004$ and $P_{Basketballs}=30$ respectively.

$$\begin{aligned} Q_D^{Nike} &= 500-10P_{Nike}+\frac{1}{2}Y-20P_{Basketballs} \\ Q_S^{Nike} &= 20+4P_{Nike} \end{aligned}$$

a (10) Find the equilibrium price and quantity for this market.

Answer:

Plugging in for Y and $P_{Basketballs}$ and setting $Q_D^{Nike}=Q_S^{Nike}$ we have:

$$\begin{aligned} 500-10P_{Nike}+\frac{1}{2}*2004-20*30 &= 20+4P_{Nike} \\ 902-10P_{Nike} &= 20+4P_{Nike} \\ 882 &= 14P_{Nike} \\ 63 &= P_{Nike} \end{aligned}$$

Plugging back into the supply function we have:

$$\begin{aligned} Q_S^{Nike} &= 20+4P_{Nike} \\ Q_S^{Nike} &= 20+4*63 \\ Q_S^{Nike} &= 272 \end{aligned}$$

So the equilibrium price and quantity in this market is $Q^*=272$, $P^*=63$.

b (5) Which measure of elasticity would you use to determine if basketballs and Nike shoes are complements or substitutes based on this demand function? Calculate the elasticity you suggest at the equilibrium price and quantity. Are Nike shoes and basketballs complements or substitutes?

Answer:

The elasticity measure that should be used is cross-price elasticity. Use the coefficient on $P_{Basketballs}$ and multiply that coefficient by the ratio of $\frac{P_{Basketballs}}{Q_{shoes}}$. You should find that the cross-price elasticity is $-20*\left(\frac{30}{272}\right) = -\frac{75}{34}$. Since the cross-price elasticity of the goods is negative, these goods are complements.

c (5) Which measure of elasticity would you use to determine if Nike shoes are a normal good or an inferior good? Calculate the elasticity you suggest at the equilibrium price and quantity. Are Nike shoes a normal good or an inferior good?

Answer:

The elasticity measure that should be used is income elasticity. Use the coefficient on Y and multiply that coefficient by the ratio of $\frac{Y}{Q_{Shoes}}$. You should find that the income elasticity is $\frac{1}{2} * \frac{2004}{272} = \frac{501}{136}$. Since the income elasticity is positive, Nike shoes are a normal good.

d (5) Suppose that a price floor is imposed at $P_{Nike} = 75$. How will this alter the price and quantity of Nike shoes in this market?

Answer:

If a price floor is imposed at $P_{Nike} = 75$, then suppliers would wish to supply 320 units and consumers would wish to purchase 152 units. Thus, the market price would increase to \$75 and the quantity traded in the market would decrease to 152.

4. (25) There is a popular game show called "Deal or No Deal". We will consider a modified version of the game. Suppose that there are 4 suitcases, each one with a different amount of money inside the suitcase. The amounts of money randomly distributed in the 4 suitcases are \$4, \$25, \$900, and \$1,600. In the game the contestant picks one suitcase (suppose it is Suitcase A) without looking inside to see the amount, then must choose to eliminate the other 3 suitcases (suppose Suitcases B, C, and D) one at a time. After each remaining suitcase is eliminated, a banker makes an offer of a certain amount of dollars to the contestant. The certain amount is an attempt to buy Suitcase A from the contestant. The contestant can accept the certain amount or continue eliminating suitcases. If the contestant eliminates the three remaining suitcases without accepting the banker's offer at any stage, then the contestant receives the amount inside Suitcase A. If the contestant accepts the banker's offer, then the contestant receives that amount of money and is asked to look inside Suitcase A to see which amount of money is inside.

a (5) Calculate the expected *value* of playing the game.

Answer:

The expected value is simply the weighted average, where the weights in this case are all $\frac{1}{4}$. So $E[\cdot] = \frac{1}{4} * 4 + \frac{1}{4} * 25 + \frac{1}{4} * 900 + \frac{1}{4} * 1600 = \frac{2529}{4} = \$632.25$

b (10) Suppose that the contestant has a utility function $u(x) = \sqrt{x}$ over amounts of money x. What is the certain amount of money that would make the contestant indifferent between playing the game and accepting the certain amount of money?

Answer:

The expected utility of the game is the weighted average of the utilities, which is: $\frac{1}{4} * \sqrt{4} + \frac{1}{4} * \sqrt{25} + \frac{1}{4} * \sqrt{900} + \frac{1}{4} * \sqrt{16000}$, which is $\frac{1}{4} * 2 + \frac{1}{4} * 5 + \frac{1}{4} * 30 + \frac{1}{4} * 40 = \frac{77}{4} = 19.25$. Now, what amount of money gives the consumer an expected utility of 19.25? It is $19.25 = \sqrt{x}$, or $x = \$370.56$ (technically \$370.5625).

c (10) Suppose that the contestant has opened two suitcases and they had the \$4 and \$900 amounts in them. This leaves the \$25 and \$1,600 amounts in the remaining two suitcases. The banker offers the contestant \$800. Based on the contestant's utility function in part **b**, should the contestant accept the banker's offer? Explain. Should the contestant accept the banker's offer if his utility function is $u(x) = x$? Explain.

Answer:

The contestant's expected utility with only the \$25 and \$1600 suitcases remaining is $\frac{1}{2} * \sqrt{25} + \frac{1}{2} * \sqrt{1600} = \frac{1}{2} * 5 + \frac{1}{2} * 40 = \frac{45}{2} = 22.5$. The certain amount of money that gives the contestant that utility is $22.5 = \sqrt{x}$, so $x = \$506.25$. Because $\$800 > \$506.25$, the contestant should accept the offer if his utility function is $u(x) = \sqrt{x}$. Alternatively, the expected utility of \$800 is $\sqrt{800} \approx 28.28 > 22.5$.

If $u(x) = x$, then the contestant's expected utility is $\frac{1}{2} * 25 + \frac{1}{2} * 1600 = \frac{825}{2} = 812.5$. The expected utility from \$800 is $u(800) = 800$, so the contestant should NOT accept the offer because $812.5 > 800$.