Test 2, ECON 6090-92, Summer 2013

Instructions: Answer all questions as completely as possible. If you cannot solve the problem, explaining how you would solve the problem may earn you some points. Point totals are in parentheses.

1. Short answer questions

a There are two cigarette manufacturers in an industry. They both choose advertising levels simultaneously. The payoff matrix for their choices is shown below:

	Firm 2		
		Advertise	Don't Advertise
Firm 1	Advertise	$\boxed{27}, \boxed{27}$	$\boxed{61}, 21$
	Don't Advertise	$21, \boxed{61}$	$45, 45$

A (5 points) Find the Nash Equilibrium to this game.

Answer:

The Nash equilibrium is that Firm 1 chooses Advertise and Firm 2 chooses Advertise.

B (5 points) The federal government in the late 1960s and early 1970s passed legislation that prohibited cigarette companies from many forms of advertising. Based on this game, explain how the government's legislation can increase the profit to both firms.

Answer:

By prohibiting advertisement, it is as if the government is causing the firms to credibly commit to choosing Don't Advertise, and they will both receive 45 if they choose Don't Advertise as opposed to the 27 they receive for choosing Advertise.

b (5 points) Explain why the typical shape of the **short-run** average total cost function is U-shaped.

Answer:

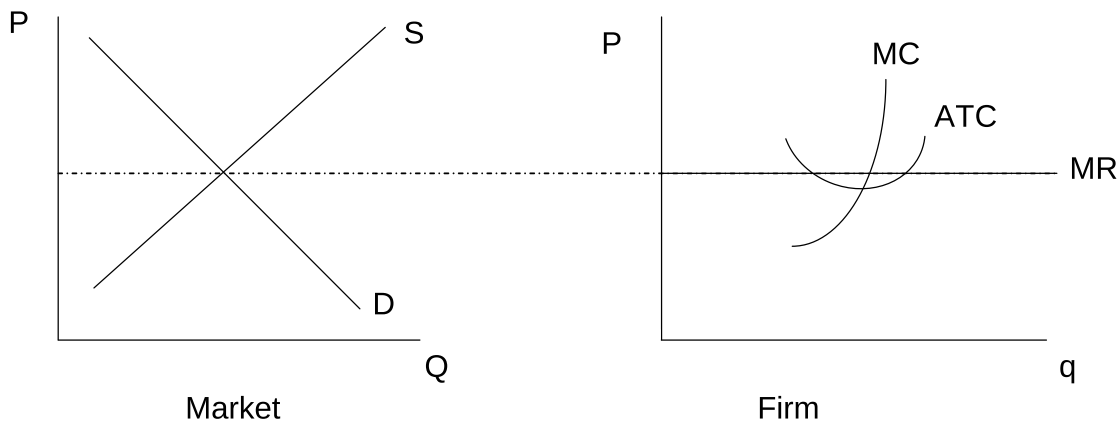
The short-run ATC is U-shaped because for low quantities of production there is a high average fixed cost and for high quantities of production there is a high marginal cost (due to diminishing marginal returns). Thus, the short-run ATC begins at a high level and then declines as AFC declines, but then turns upward as marginal cost begins to increase.

c (5 points) Explain the shutdown rule for a firm referencing specific cost and revenue terms.

Answer:

There are basically 3 states for the firm: continue operation (if making nonnegative economic profit), go out of business (if making an economic loss and does not expect to make a profit in the future), and shutdown (currently making an economic loss but expects to make nonnegative economic profit in the future). If a firm is currently making an economic loss but expects to make a nonnegative economic profit in the future, then it must decide whether to continue operation or to shutdown. A firm will continue operation if it can cover its total variable costs and will shutdown otherwise. Thus, the shutdown rule is if $TR < TVC$ then shutdown, and if $TR \geq TVC$ continue to operate. The reason is that the firm will incur its fixed costs regardless of this decision, but if $TR \geq TVC$ then the firm can take some of its revenue in excess of total variable cost and apply it to the fixed costs.

d (5 points) Consider the following picture.



Is this perfectly competitive market in long run equilibrium? **Explain why or why not**.

Answer:

No, this perfectly competitive market is not in long run equilibrium. The representative firm is making positive economic profits, which should attract other firms to the market. The perfectly competitive market would be in long run equilibrium if the firm was making zero economic profit.

2. (25 points) Consider a simultaneous quantity choice game between 2 firms. Each firm chooses a quantity, q_1 and q_2 respectively. The inverse market demand function is given by $P(Q) = 1506 - 10 * Q$, where $Q = q_1 + q_2$. Firm 1 has total cost function $TC(q_1) = 6q_1 + 1000$ and Firm 2 has total cost function $TC(q_2) = 6q_2 + 1000$.

a (10 points) Find the best response functions for Firms 1 and 2.

Answer:

To find the best response functions we need to take the partial derivative of each firm's profit function, set the result each to zero, and solve for each firm's quantity.

For Firm 1 we have:

$$\begin{aligned}
\Pi_1 &= (1506 - 10q_1 - 10q_2)\, q_1 - 6q_1 - 1000 \\
\frac{\partial \Pi_1}{\partial q_1} &= 1506 - 20q_1 - 10q_2 - 6 \\
0 &= 1500 - 20q_1 - 10q_2 \\
20q_1 &= 1500 - 10q_2 \\
q_1 &= \frac{1500 - 10q_2}{20} \\
q_1 &= \frac{150 - q_2}{2}
\end{aligned}$$

For Firm 2 we have:

$$\begin{aligned}
\Pi_2 &= (1506 - 10q_1 - 10q_2)\, q_2 - 6q_2 - 1000 \\
\frac{\partial \Pi_2}{\partial q_2} &= 1506 - 10q_1 - 20q_2 - 6 \\
0 &= 1500 - 10q_1 - 20q_2 \\
20q_2 &= 1500 - 10q_1 \\
q_2 &= \frac{1500 - 10q_1}{20} \\
q_2 &= \frac{150 - q_1}{2}
\end{aligned}$$

Technically the best response functions for Firm 1 and Firm 2 are $q_1 = Max\left[0, \frac{150-q_2}{2}\right]$ and $q_2 = Max\left[0, \frac{150-q_1}{2}\right]$ because the firm's will not produce a quantity less than zero.

b (10 points) Find the Nash equilibrium to this game.

Answer:
For this we need to solve the best response functions simultaneously, so we have:

$$\begin{aligned} q_1 &= \frac{150-q_2}{2} \\ q_2 &= \frac{150-q_1}{2} \end{aligned}$$

Substituting the first equation into the second we have:

$$\begin{aligned} q_2 &= \frac{150-\left(\frac{150-q_2}{2}\right)}{2} \\ 2q_2 &= 150-\left(\frac{150-q_2}{2}\right) \\ 4q_2 &= 300-150+q_2 \\ 3q_2 &= 150 \\ q_2 &= 50 \end{aligned}$$

Now using Firm 1's best response function and that $q_2 = 50$ we have:

$$\begin{aligned} q_1 &= \frac{150-50}{2} \\ q_1 &= 50 \end{aligned}$$

So the Nash equilibrium is $q_1 = q_2 = 50$.

c (5 points) Find the total market quantity (Q), market price, and profit for each firm.

Answer:
The total market quantity is $Q = q_1+q_2 = 50+50 = 100$. The market price is $P(Q) = 1506-10*100 = 506$. Each firm's profit is:

$$\begin{aligned} \Pi_1 &= P*q_1 - TC(q_1) \\ \Pi_1 &= 506*50-(6*50+1000) \\ \Pi_1 &= 25300-1300 \\ \Pi_1 &= 24000 \\ & \text{and} \\ \Pi_2 &= P*q_2 - TC(q_2) \\ \Pi_2 &= 506*50-(6*50+1000) \\ \Pi_2 &= 24000 \end{aligned}$$

3. (20 points) Consider the constant elasticity of substitution production function:

$$q(K,L) = (L^\rho + K^\rho)^{\frac{1}{\rho}}$$

Let $\rho = \frac{1}{2}$, and let $r = 5$ be the price of capital and $w = 3$ be the price of labor.

a (10 points) Find the marginal rate of technical substitution.

Answer:

First find the marginal product of labor:

$$\begin{aligned}\frac{\partial q}{\partial L} &= \frac{1}{\rho}\left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}\rho L^{\rho-1}\\ \frac{\partial q}{\partial L} &= \left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}L^{\rho-1}\end{aligned}$$

Now find the marginal product of capital:

$$\begin{aligned}\frac{\partial q}{\partial K} &= \frac{1}{\rho}\left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}\rho K^{\rho-1}\\ \frac{\partial q}{\partial K} &= \left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}K^{\rho-1}\end{aligned}$$

The marginal rate of technical substitution is simply the negative of the ratio of those two marginal products:

$$\begin{aligned}MRTS &= -\frac{MP_L}{MP_K}\\ MRTS &= -\frac{\left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}L^{\rho-1}}{\left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}-1}K^{\rho-1}}\\ MRTS &= -\frac{L^{\rho-1}}{K^{\rho-1}}\end{aligned}$$

Substituting in for ρ we have:

$$\begin{aligned}MRTS &= -\left(\frac{L}{K}\right)^{-1/2}\\ MRTS &= -\left(\frac{K}{L}\right)^{1/2}\end{aligned}$$

b (10 points)Find the cost-minimizing bundle of inputs if the firm wishes to produce 320 units.

Answer:

You could set it up as an optimization problem, but we know that at the optimal bundle (interior) that:

$$MRTS = -\frac{w}{r}$$

so

$$-\frac{L^{\rho-1}}{K^{\rho-1}} = -\frac{w}{r}.$$

Now we can find L in terms of K:

$$L = \left(\frac{w}{r}\right)^{\frac{1}{\rho-1}}K.$$

Putting this back into the production function and substituting $q = 500$ we have:

$$
\begin{aligned}
q &= \left(\left(\left(\frac{w}{r}\right)^{\frac{1}{\rho-1}} K\right)^{\rho} + K^{\rho}\right)^{\frac{1}{\rho}} \\
q &= \left(\left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}} K^{\rho} + K^{\rho}\right)^{\frac{1}{\rho}} \\
q &= \left(K^{\rho}\left(\left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}} + 1\right)\right)^{\frac{1}{\rho}} \\
q &= K\left(\left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}} + 1\right)^{\frac{1}{\rho}} \\
K &= \frac{q}{\left(\left(\frac{w}{r}\right)^{\frac{\rho}{\rho-1}} + 1\right)^{\frac{1}{\rho}}}
\end{aligned}
$$

Substituting in we find:

$$
\begin{aligned}
K &= \frac{320}{\left(\left(\frac{3}{5}\right)^{\frac{1/2}{-1/2}} + 1\right)^{1/(1/2)}} \\
K &= \frac{320}{\left(\left(\frac{3}{5}\right)^{-1} + 1\right)^{2}} \\
K &= \frac{320}{\left(\frac{5}{3} + 1\right)^{2}} \\
K &= \frac{320}{\left(\frac{8}{3}\right)^{2}} = \frac{320}{\frac{64}{9}} = 45
\end{aligned}
$$

You could simplify if you want but no need to. Now just plug K into the equation for L:

$$
\begin{aligned}
L &= K\left(\frac{w}{r}\right)^{\frac{1}{\rho-1}} \\
L &= 45\left(\frac{3}{5}\right)^{\frac{1}{(1/2)-1}} \\
L &= 45\left(\frac{3}{5}\right)^{\frac{1}{-1/2}} \\
L &= 45\left(\frac{3}{5}\right)^{-2} \\
L &= 45\left(\frac{5}{3}\right)^{2} \\
L &= 45 * \frac{25}{9} = 125
\end{aligned}
$$

So the cost minimizing bundle, if the solution is interior, is $K = 45$ and $L = 125$, which leads to a $TC = 45 * 5 + 125 * 3 = 600$. If a corner solution existed it would involve using all capital or all labor, and the production function would be either $q(K, L) = L$ or $q(K, L) = K$. If the firm uses only L it needs 320 units, and this costs 960. If the firm uses only capital it needs 320 units, and this costs 1600. Neither of those is less expensive than using $(K, L) = (45, 125)$.

4. (30 points) Bates Gill is the sole developer of underwater operating systems. His firm is protected by considerable barriers to entry. Gill faces the following inverse demand function for his underwater operating systems:

$$P(Q) = 4320 - 12Q$$

His total cost function is:

$$TC = 4Q^2 + 200,000$$

a (5 points) Find Gill's marginal revenue function solely as a function of quantity.

Answer:
Marginal revenue is simply the derivative of TR with respect to Q:

$$\begin{aligned} TR &= (4320 - 12Q)\,Q \\ \frac{\partial TR}{\partial Q} &= 4320 - 24Q \\ MR &= 4320 - 24Q \end{aligned}$$

b (10 points) Find Gill's profit-maximizing

- Quantity

Answer:
Just set $MR = MC$. We know that $MC = \frac{\partial TC}{\partial Q}$:

$$MC = 8Q$$

So that:

$$\begin{aligned} MR &= MC \\ 4320 - 24Q &= 8Q \\ 4320 &= 32Q \\ 135 &= Q \end{aligned}$$

- Price

Answer:
The price is given by:

$$\begin{aligned} P(Q) &= 4320 - 12Q \\ P(Q) &= 4320 - 12 * 135 \\ P(Q) &= 2700 \end{aligned}$$

- Profit at the profit-maximizing price and quantity

Answer:
Profit is simply $TR - TC$ at $Q = 135$, so:

$$\begin{aligned} \Pi &= (4320 - 12Q)\,Q - \left(4Q^2 + 200000\right) \\ \Pi &= (4320 - 12 * 135) * 135 - \left(4 * 135^2 + 200000\right) \\ \Pi &= 364500 - 272900 \\ \Pi &= 91600 \end{aligned}$$

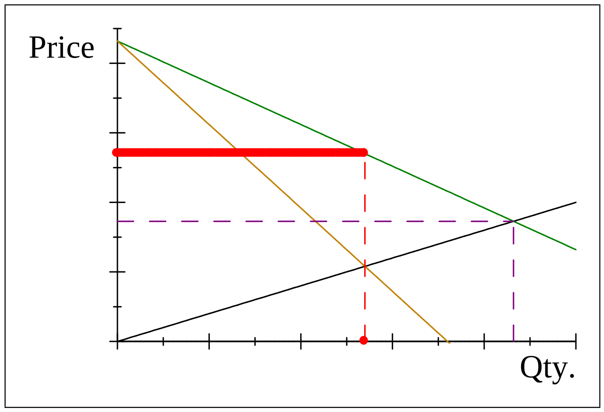
Adding the government

The government realizes that Gill is a monopolist and that "considerable" deadweight loss is being created in the underwater operating systems market. Use the functions above to answer these questions.

a (5 points) Draw a picture (it does not have to be drawn to scale) that illustrates

- Gill's profit-maximizing price and quantity
- The quantity and the price that would be charged if the market were to operate efficiently (with no deadweight loss)
- The deadweight loss due to this monopoly (shade it in using a pencil or pen or marker)

Answer:



The profit-maximizing price and quantity are given by the red dashed lines. The efficient price and quantity are given by the purple (lower price, higher quantity) dashed lines. The green line is the inverse demand function, the brown line is the marginal revenue function, and the black line is the marginal cost function. Deadweight loss is the triangle outlined by the inverse demand function (green line) between the red dashed line and the intersection of inverse demand and MC, the MC function (black line) between the red dashed line and the intersection of the MC and inverse demand function, and the red dashed line between the inverse demand function and MC.

b (5 points) For the completely efficient outcome (no deadweight loss), find the

- quantity

Answer:

The completely efficient outcome would be where $P = MC$. So:

$$\begin{aligned} 4320 - 12Q &= 8Q \\ 4320 &= 20Q \\ 216 &= Q \end{aligned}$$

- price

Answer:

To find the price simply sub in $Q = 216$ into $P(Q)$:

$$P(216) = 4320 - 12 * 216$$

and we should find that $P = 1728$.

(Note that these should be actual numbers based on the inverse demand and cost function given at the beginning of this problem.)

c (5 points) Suppose that Gill had not yet entered into this market, but was merely planning to enter into the market. If the government were to tell Gill ahead of time that they would regulate his price at the efficient level, would Gill enter this market? Explain.

Answer:
Gill's profit in this market at the efficient quantity of $Q = 216$ is:

$$\begin{aligned}
\Pi &= (4320 - 12 * 216) * 216 - \left(4 * 216^2 + 200000\right) \\
\Pi &= 373248 - 386624 \\
\Pi &= -13376
\end{aligned}$$

Because Gill would be earning a negative profit he should NOT enter the market if he will be regulated.