These notes essentially correspond to chapter 6 of the text.

1 Firms and Production

We will now turn our attention to firms. We will assume that the underlying goal of firm behavior is to maximize profits. While there may be other goals of firms (get the largest market share, minimize risk, reach a pre-specified profit level), it is generally believed that in the long-run firms that stay in business are the ones that are maximizing profits, whether or not this is their specific goal. We will define profit as revenue minus costs, or $\Pi = R - C$, where revenue is defined as price times quantity, or $R = P * q$. The focus of chapter 6 is on HOW firms produce their goods, while chapter 7 discusses costs and some later chapters discuss profit maximization depending on the type of industry in which the firm operates.

You should also note that we are assuming that the firms are all owner-operated firms. If we discuss firms as being owned by one individual but run by another individual (a manager), then we have to discuss problems that may arise due to the fact that the manager's incentives may not be the same as the owner's incentives (which are to maximize profits).

2 Production

Firms use production processes to transform inputs (or resources or factors of production) into a final product. Three broad classes of inputs are:

1. Capital (K) – includes physical machinery and land

2. Labor (L) – "Human-power", both physical and mental

3. Materials – items combined by capital and labor to create the final product

We will usually perform analysis only with capital and labor. We assume that the firm has a production function, which is a function that tells us the greatest amount of final product that could be produced with various combinations of capital and labor. We will write the general production function as:

$$q = f(K, L)$$

If we choose a functional form for the production function then we will be able to determine how much output would be produced given various combination of inputs. For example, if we choose the form $f(K, L) = K + L$, then 1 unit of capital and 1 unit of labor will produce 2 units of final output. However, if we choose $f(K, L) = K * L$, then 1 unit of capital and 1 unit of labor will produce 1 unit of final output.

Short run vs. long run An important distinction is made between the short-run (SR) and the long-run (LR). We will define the short-run as an amount of time such that at least one factor of production CANNOT be varied (or is a fixed input). We will define the long-run as an amount of time such that all factors of production CAN be varied (or are variable inputs). Depending on the industry, the short-run and long-run can be different lengths. For example, it may be difficult for a business to change locations if the business is involved in leasing office space. Thus, the business would be in the short-run for the amount of time for which it has contracted to lease the office space. However, once that lease is up, and assuming that no other inputs are fixed, the business would be in the long-run.

2.1 SR Production

Once again, in the short-run we assume that one factor of production is fixed. Typically we assume that the fixed factor is capital, although this is more for convenience than anything else. Suppose that capital is fixed at a level $\bar{K}$. The production function is then:

$$q = f(\bar{K}, L)$$

This change in notation, although only a slight change, is an important one.

2.1.1 Defining terms

One of the necesseties of the discussion is defining terms. It's not fun, but it needs to be done.

1. Total product (TP) – Total product is just a fancy name for quantity or output. We could rewrite the production function as:

$$TP = f\left(\bar{K}, L\right)$$

2. Marginal product (MP) – Marginal product tells us how much total product increases when we increase one of our variable inputs by one unit, holding the other levels of the resources constant. We will typically want to know:

   - MP_L – the marginal product of labor. The marginal product of labor tells us how much total product will increase when we use one additional unit of labor, holding capital (and other resources) constant. We can write the marginal product of labor as:

$$MP_L = \frac{\partial q}{\partial L}$$

   - MP_K – the marginal product of capital. The marginal product of capital tells us how much total product will increase when we use one additional unit of capital, holding labor (and other resources) constant. We can write the marginal product of capital as:

$$MP_K = \frac{\partial q}{\partial K}$$

3. Average product (AP) – Average product tells us the amount of total product that is being produced per unit of resource. We can write average product as:

$$AP = \frac{q}{amount\ of\ resource}$$

   We will specify the average product of labor as $AP_L = \frac{q}{L}$ and the average product of capital as $AP_K = \frac{q}{K}$.

2.1.2 Relationship of the terms

If we were to graph each of those terms they would look something like the following, where TP is the highest inverted parabola, MP is the second highest inverted parabola and AP is the lowest inverted parabola. Note that there are two dashed lines: one of the dashed lines is where AP reaches its maximum (the one at quantity of inputs = 2) and the other dashed line is where TP reaches its maximum (the one at quantity of inputs = $\frac{8}{3}$).
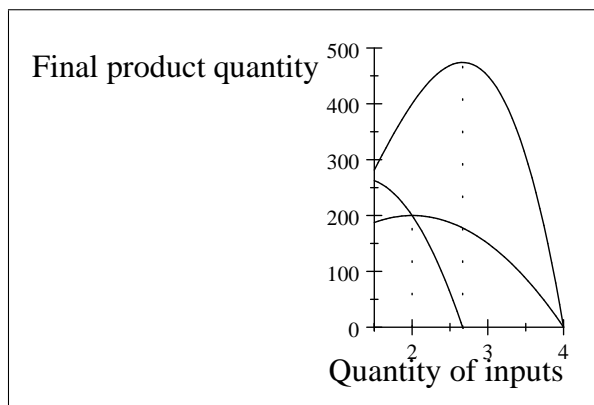
The key points of this graph are:

1. MP intersects AP at its maximum

2. When TP begins decreasing, $MP < 0$.

3. AP is upward-sloping when $MP > AP$; AP is downward-sloping when $MP < AP$.

2.1.3   Law of Diminishing Marginal Returns

The law of diminishing marginal returns states that at some point in the production process, when you add an additional unit of the variable resource to the production process, the additional unit of the variable resource will increase total output, but at a smaller rate than the previous additional unit of the variable input. What this means is that if we take a fixed input (say a piece of capital or land) and add more laborers to it, there will eventually be a time when adding another laborer (say the 5th laborer) will increase output, but by a smaller amount than the previous laborer (the 4th laborer).

Sizzler kitchen example   We will use the Sizzler kitchen as an example of diminishing marginal returns. The fixed input is the amount of space and appliances on the kitchen line (the capital). The variable input is the number of cooks (the labor). We will assume that the food (the materials) is infinite and magically appears. The table below shows how the number of variable inputs relates to total output.

Cooks (variable input)	Orders (TP)	MP	AP
0	0	–	–
1	7	7	7
2	18	11	9
3	33	15	11
4	40	7	10
5	45	5	9
6	42	-3	7

Thus, diminishing marginal returns sets in at the 3rd cook (technically, we say that diminishing marginal returns sets in at the maximum of the MP curve – this keeps us from having to say, "Dimishing marginal returns sets in at the 3.000000001^{th} cook", which is a bit cumbersome). The picture below shows the plot of TP, MP, and AP. Note that TP is the highest curve, MP is the middle curve, and AP is the lowest curve. The MP curve peaks at 3 cooks and a total output of 33.
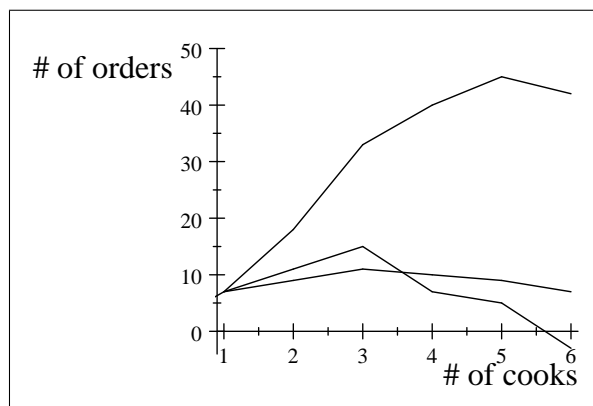

2.2   LR Production

In the long-run the firm is able to vary all of its inputs. In this case, we can start with a "Production Table", which will show the amount of total product (TP) that can be produced using various combinations of inputs. Technically, we would put the amounts into a production function and then see how much is produced and then place that number in the table. Suppose the production function is $q = 10 * \sqrt{K} * \sqrt{L}$ (or $q = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$). Then we have:

K/L	1	2	3	4	5	6
1	10	14	17	20	22	24
2	14	20	24	28	32	35
3	17	24	30	35	39	42
4	20	28	35	40	45	49
5	22	32	39	45	50	55
6	24	35	42	49	55	60

Notice that the combinations $(6,1)$, $(1,6)$, $(3,2)$, and $(2,3)$ all produce the same quantity level, 24. Also, the combinations $(6,2)$, $(2,6)$, $(4,3)$, and $(3,4)$ all produce a quantity level of 35. What we will want to do is find all of the combinations of inputs that produce the same quantity level and draw that curve. We will call this curve the isoquant (FYI: "iso" means "the same"). It is in the picture below.
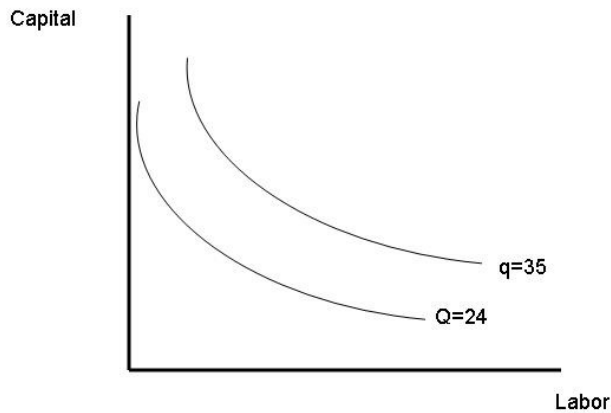
Notice that these isoquants look very similar to indifference curves. We will also have rules for isoquants which will be very similar to the rules for indifference curves.

1. The farther from the origin, the greater the level of output – if we use more of BOTH inputs, we must get a larger output.

2. Isoquants do not cross.

3. Isoquants are downward-sloping. Again, you have to have a little leeway with this one, especially considering the special cases we will consider below.

2.2.1 Special cases of isoquants

Suppose that our production function is:

$$q = K + L$$

In this case, capital and labor are perfect substitutes. If we want to produce $q = 1$, then we can use 1 unit of labor, 1 unit of capital, or any combination of the two that adds to 1. Thus, our isoquants for this production function will look like:

Capital

Labor

Q_1 Q_2
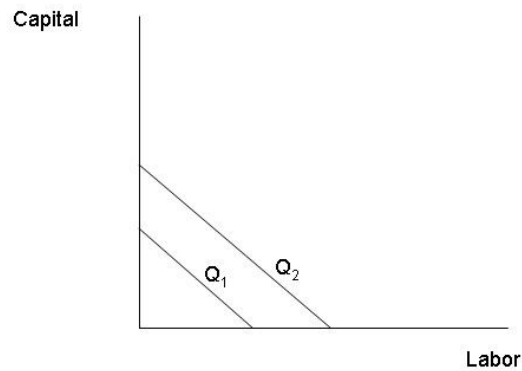
Now suppose that our production function is $q = \min(K, L)$. This means that we can only produce a quantity equal to the minimum of either K or L. In other words, if we want to increase the quantity produced we will need to increase BOTH K and L. This is the production function for perfect complements. Recall that indifference curves for perfect complements are right angles. The same is true for the isoquants of a good where the inputs are perfect complements.

Capital

Q_2

Q_1

Labor
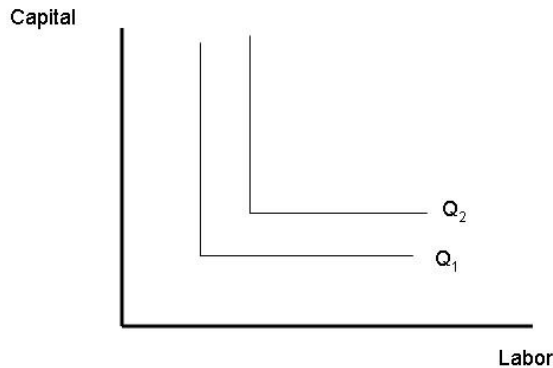
2.2.2   Slope of the isoquant

The slope of the isoquant is an important term. Recall that the slope of the indifference curve was the Marginal Rate of Substitution. The slope of the isoquant is very similar, called the Marginal Rate of Technical Substitution (MRTS). The MRTS tells us how many units of capital we can give up if we use one more unit of labor and we want to hold the quantity produced constant. We can write the MRTS as:

$$MRTS = \frac{\partial K}{\partial L}$$

There is also a useful relationship between the marginal product of labor (MP_L) and the marginal product of capital (MP_K). We know that:

$$MP_L = \frac{\partial q}{\partial L}$$
$$MP_K = \frac{\partial q}{\partial K}$$

If we take the ratio of the MP_L to the MP_K, we get:

$$\frac{MP_L}{MP_K} = \frac{\partial K}{\partial L}$$

So the MRTS is just equal to the ratio of the marginal products. This will be a useful relationship to store in your memory banks for later.

A quick example Consider the production function:

$$q = AK^\alpha L^\beta.$$

Note that:

$$
\begin{aligned}
MP_K &= A\alpha K^{\alpha-1}L^\beta \\
MP_L &= A\beta K^\alpha L^{\beta-1} \\
MRTS &= -\frac{MP_K}{MP_L} \\
MRTS &= -\frac{A\beta K^\alpha L^{\beta-1}}{A\alpha K^{\alpha-1}L^\beta} \\
MRTS &= -\frac{\beta K}{\alpha L}
\end{aligned}
$$

If we solve the production function in terms of capital, we get:

$$
\begin{aligned}
q &= AK^\alpha L^\beta \\
\frac{q}{AL^\beta} &= K^\alpha \\
\left(\frac{q}{AL^\beta}\right)^{1/\alpha} &= K
\end{aligned}
$$

Now find the partial derivative with respect to L (I will rewrite the production function first):

$$
\begin{aligned}
K &= \left(\frac{q}{A}\right)^{1/\alpha} L^{-\beta/\alpha} \\
\frac{\partial K}{\partial L} &= \left(\frac{q}{A}\right)^{1/\alpha}\left(\frac{-\beta}{\alpha}\right)L^{\frac{-\beta}{\alpha}-1}
\end{aligned}
$$

Note that this looks nothing like the MRTS that was obtained earlier, but if we substitute in $q = AK^\alpha L^\beta$, we have:

$$
\begin{aligned}
\frac{\partial K}{\partial L} &= \left(\frac{AK^\alpha L^\beta}{A}\right)^{1/\alpha}\left(\frac{-\beta}{\alpha}\right)L^{\frac{-\beta}{\alpha}-1} \\
\frac{\partial K}{\partial L} &= \left(K^\alpha L^\beta\right)^{1/\alpha}\left(\frac{-\beta}{\alpha}\right)L^{\frac{-\beta-\alpha}{\alpha}} \\
\frac{\partial K}{\partial L} &= KL^{\beta/\alpha}\left(\frac{-\beta}{\alpha}\right)L^{\frac{-\beta-\alpha}{\alpha}} \\
\frac{\partial K}{\partial L} &= \left(\frac{-\beta}{\alpha}\right)KL^{-1} \\
\frac{\partial K}{\partial L} &= \left(\frac{-\beta}{\alpha}\right)\frac{K}{L}
\end{aligned}
$$

So the two methods are the same.

3   Returns to scale

When discussing returns to scale we are trying to determine how much output will increase if we double (we use double but it could be triple or quadruple) all of our inputs. We can classify the production function as having one of three types of returns to scale.

Constant Returns to Scale   If we double our inputs, we EXACTLY double our output. Letting $f(K, L)$ be the production function, this means that: $f(2K, 2L) = 2 * f(K, L)$.

Increasing Returns to Scale   If we double our inputs, we get more than double the output. This means that: $f(2K, 2L) > 2 * f(K, L)$.

Decreasing Returns to Scale   If we double our inputs, we get less than double the output. This means that: $f(2K, 2L) < 2 * f(K, L)$.

There are a variety of reasons we may have increasing and decreasing returns to scale. For increasing returns to scale, it could be that building a bigger plant allows greater specialization among the workforce, which typically means they will be more productive. For decreasing returns to scale, it could be that the workforce has gotten so large that it is impossible to monitor people to make sure that they are working, or that you are hiring lower quality workers because you have already hired the best workers.

3.1   Cobb-Douglas production function

The Cobb-Douglas production function is one of the most widely used production functions. The functional form is:

$$q = A * K^\alpha * L^\beta$$

The terms $A, \alpha,$ and β are all positive constants. We will use the Cobb-Douglas production function to determine the restrictions that we need to place on α and β in order to determine the type of returns to scale the production function has.

Let $q_1 = f(K, L) = AK^\alpha L^\beta$. Let $q_2 = f(2K, 2L) = A(2K)^\alpha (2L)^\beta$. Now, do some math:

$$q_2 = A(2K)^\alpha (2L)^\beta$$

$$q_2 = A2^\alpha K^\alpha 2^\beta L^\beta$$

$$q_2 = 2^\alpha 2^\beta AK^\alpha L^\beta$$

$$q_2 = 2^{\alpha+\beta} AK^\alpha L^\beta$$

All of the preceding steps were simply math steps. The next step is a substitution step. We know that $AK^\alpha L^\beta = q_1$. Substituting:

$$q_2 = 2^{\alpha+\beta} q_1$$

What can we learn from this about returns to scale? We know that if $q_2 = 2q_1$ then we have constant returns to scale. If $q_2 > 2q_1$ then we have increasing returns to scale. If $q_2 < 2q_1$ then we have decreasing returns to scale. The deciding factor in the above equation will be the term $\alpha + \beta$.

$$q_2 = 2q_1 : if \ \alpha + \beta = 1$$
$$q_2 > 2q_1 : if \ \alpha + \beta > 1$$
$$q_2 < 2q_1 : if \ \alpha + \beta < 1$$

So if $\alpha + \beta = 1$ then we have constant returns to scale with a Cobb-Douglas production function (this may not be true for other production functions). If $\alpha + \beta > 1$, then we have increasing returns to scale for a Cobb-Douglas production function. If $\alpha + \beta < 1$, then we have decreasing returns to scale for a Cobb-Douglas production function.

3.1.1 The "A" term in the Cobb-Douglas production function

We call the A term the technical efficiency parameter in the Cobb-Douglas production function. The larger the A the more efficient the firm is at producing its good.