# 1 Supply and Demand

Consider the following supply and demand functions for Ramen noodles. The variables are defined in the table below. Constant values are given for the last 2 variables.

| **Variable** | **Meaning** | **Constant value** |
|---|---|---|
| $Q_D$ | Quantity demanded of Ramen | |
| $Q_S$ | Quantity supplied of Ramen | |
| $P_{Ramen}$ | Price of Ramen | |
| $P_{Kraft}$ | Price of Kraft Mac and Cheese | $0.99 |
| $Y$ | Consumer income | $11,500 |

$$\begin{gathered} Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + (100,000)\,P_{Kraft} - (20)\,Y \\ Q_S = -100,021 + (680,000)\,P_{Ramen} \end{gathered} \tag{1}$$

1. Write down the inverse demand function for Ramen noodles.

   **Answer**:

   To find the inverse demand function simply isolate $P_{Ramen}$ on the left-hand side of the equation. The inverse demand function is:

   $$P_{Ramen} = 0.425 - 0.000000373Q_D + 0.0373P_{Kraft} - 0.00000745Y$$

   Some of you substituted in for $P_{Kraft}$ and $Y$, which you did not have to do. I believe this altered the intercept to approximately 0.376, with the slope of the inverse demand function remaining the same.

2. Find the equilibrium price and quantity in this market.

   **Answer**:

   Plug in the constant values for income and the price of Kraft Macaroni and Cheese to find that:

   $$Q_D = 1,141,000 - (2,683,700)\,P_{Ramen} + 99,000 - 230,000$$

   We now have supply and demand functions:

   $$\begin{gathered} Q_D = 1,010,000 - 2,683,700P_{Ramen} \\ Q_S = -100,021 + (680,000)\,P_{Ramen} \end{gathered}$$

   Now, set $Q_D = Q_S$ and then set the 2 equations equations equal to one another so that:

   $$1,010,000 - 2,683,700P_{Ramen} = -100,021 + (680,000)\,P_{Ramen}$$

   Now, solve for $P_{Ramen}$.

$$\frac{1,110,021}{3,363,700} = P_{Ramen}$$

We find that the price of Ramen noodles is \$0.33. To calculate the equilibrium quantity just plug 0.33 into either the supply or demand function. You should find that the equilibrium quantity is $124,379$.

3. Suppose that $P_{Kraft}$ increases to \$1.33. Recalculate the equilibrium price and quantity given this change.

   **Answer**:

   Since nothing has changed in the supply function from the original problem it is still the same:

$$Q_S = -100,021 + (680,000)\, P_{Ramen}$$

   However, the change in the price of Kraft will cause a shift in the demand curve. To find the new demand function, plug in the new price of Kraft (as well as the old income value) to get:

$$Q_D = 1,141,000 - (2,683,700)\, P_{Ramen} + (100,000)\,(1.33) - (20)\,(11,500)$$

   Simplifying,

$$Q_D = 1,141,000 - (2,683,700)\, P_{Ramen} + 133,000 - 230,000$$

$$Q_D = 1,044,000 - (2,683,700)\, P_{Ramen}$$

   Now just set $Q_D = Q_S$ and solve for price.

$$-100,021 + (680,000)\, P_{Ramen} = 1,044,000 - (2,683,700)\, P_{Ramen}$$

$$3,363,700 P_{Ramen} = 1,144,021$$

   We should find that $P_{Ramen} \approx 0.34$. Plugging this back into the demand function we find that $Q_D = 131,252.3835$ or a quantity of about 131,252.

   NOTE: If you try to plug 0.34 in as the equilibrium price you will get different numbers for $Q_D$ and $Q_S$. You would have found $Q_D = 131,542$ and $Q_S = 131,179$.

4. Calculate the own-price elasticity of demand. Use the equilibrium price and quantity as your initial price and quantity. Is demand elastic or inelastic at the equilibrium price and quantity?

   **Answer**:

   The own-price elasticity of demand is given by the following formula, where $PED$ stands for price elasticity of demand:

$$PED = \frac{\Delta Q_D}{\Delta P_{own}} * \frac{P_{own}}{Q_D}$$

   Recall that the first term, $\frac{\Delta Q_D}{\Delta P_{own}}$, is simply the coefficient on the own-price of the good in the demand function. So it is $-2,683,700$. We know that $P_{own} = 0.33$ and $Q_D = 124,379$. Plugging these numbers in gives us:

$$PED = -2,683,700 * \frac{0.33}{124,379} = -7.120342$$

   So we find that the demand for Ramen noodles is elastic, since the PED is greater than 1 in absolute value.

5. Calculate the cross-price elasticity for a 1% increase in the price of Kraft Macaroni and Cheese. Are Ramen noodles and Kraft Macaroni and Cheese substitutes or complements? Explain how you know whether they are substitutes or complements.

   **Answer**:

   The formula for the cross-price elasticity of demand is:

$$X - price = \frac{\Delta Q_D^A}{\Delta P_B} * \frac{P_B}{Q_D^A}$$

   The term $\frac{\Delta Q_D^A}{\Delta P_B}$ is simply the coefficient on the price of good B (in our case, Kraft Macaroni and Cheese). We know that $P_B = 0.99$ and that $Q_D^A = 124,379$. Plugging in these values gives us:

$$X - price = 100000 * \frac{0.99}{124379} = 0.7959543$$

   Ramen noodles and Kraft Macaroni and Cheese are substitutes. We can tell because the cross-price elasticity is positive between the quantity demanded of Ramen and the price of Kraft.

6. Calculate the income elasticity for Ramen noodles. Use the equilibrium price and the constant value for income. Are Ramen noodles a normal good or an inferior good? How do you know? If it is a normal good, is it a necessity or a luxury?

**Answer**:

The formula for income elasticity is:

$$IE = \frac{\Delta Q_D}{\Delta Y} * \frac{Y}{Q_D}$$

The term $\frac{\Delta Q_D}{\Delta Y}$ is simply the coefficient on income in the demand function. We know that $Y = \$11,500$ and that $Q_D = 124,379$. Plugging in these values gives us:

$$IE = -20 * \frac{11500}{124379} = -1.849187$$

Ramen noodles are an inferior good because the income elasticity for Ramen noodles is negative.

# 2 Optimization problem

Rob's utility function over goods $a$, $b$, and $c$ is given by:

$$U(a, b, c) = 12a^2b^4\sqrt{c}$$

Rob has an income of $Y = 5200$ and the prices of goods a, b, and c are $p_a = 2$, $p_b = 8$, and $p_c = 4$ respectively. Find Rob's optimal bundle of goods $a$, $b$, and $c$.

**Answer**:

Note that we have an interior solution so that $a > 0$, $b > 0$, and $c > 0$ at the optimal bundle. To see this look at the utility function:

$$U(a, b, c) = 12a^2b^4\sqrt{c}$$

Note that if either $a = 0$, $b = 0$, or $c = 0$ then Rob's utility would equal zero, and he could easily increase his utility by consuming some small amount of the other one or two goods he is not consuming. Setting up the Lagrangian:

$$\max_{a,b,c} \mathcal{L}(a, b, c, \lambda) = U(a, b, c) + \lambda (Y - p_A a - p_B b - p_C c)$$

The resulting conditions are

$$\begin{aligned}
1 &: \frac{\partial \mathcal{L}}{\partial a} = 24ab^4c^{1/2} - \lambda p_A = 0 \\
2 &: \frac{\partial \mathcal{L}}{\partial b} = 48a^2b^3c^{1/2} - \lambda p_B = 0 \\
3 &: \frac{\partial \mathcal{L}}{\partial c} = 6a^2b^4c^{-1/2} - \lambda p_C = 0 \\
4 &: \frac{\partial \mathcal{L}}{\partial \lambda} = Y - p_A a - p_B b - p_C c = 0
\end{aligned}$$

I've set all 4 equal to zero because we know that $a > 0$, $b > 0$, $c > 0$, and $\lambda > 0$. I'm going to move the $\lambda p_X$ terms to the right hand side of the equations so we have:

$$\begin{aligned} 24ab^4c^{1/2} &= \lambda p_A \\ 48a^2b^3c^{1/2} &= \lambda p_B \\ 6a^2b^4c^{-1/2} &= \lambda p_C \\ Y - p_A a - p_B b - p_C c &= 0 \end{aligned}$$

If we take the ratio of the $2^{nd}$ equation to the first one we get:

$$\begin{aligned} \frac{48a^2b^3c^{1/2}}{24ab^4c^{1/2}} &= \frac{\lambda p_B}{\lambda p_A} \\ \frac{2a}{b} &= \frac{p_B}{p_A} \end{aligned}$$

Note that this is just $MRS_{BA} = MRT_{BA}$. Now take the ratio of the $3^{rd}$ equation to the $1^{st}$ one to get:

$$\begin{aligned} \frac{6a^2b^4c^{-1/2}}{24ab^4c^{1/2}} &= \frac{\lambda p_C}{\lambda p_A} \\ \frac{a}{4c} &= \frac{p_C}{p_A} \end{aligned}$$

This is just $MRS_{CA} = MRT_{CA}$. If we add in the budget constraint we have:

$$\begin{aligned} \frac{2a}{b} &= \frac{p_B}{p_A} \\ \frac{a}{4c} &= \frac{p_C}{p_A} \\ Y &= p_A a + p_B b + p_C c \end{aligned}$$

I won't go through the plugging and chugging, but we should get that $a = 800$, $b = 400$, and $c = 100$. To complete the problem I'll find $\lambda$, and it is:

$$\begin{aligned} 24ab^4c^{1/2} &= \lambda p_A \\ \frac{24ab^4c^{1/2}}{p_A} &= \lambda \\ \frac{24 * 800 * 400^4 * 100^{1/2}}{2} &= \lambda \\ 12 * 400 * 400^3 * 200 * 5 &= \lambda \end{aligned}$$

So $\lambda > 0$, but we already knew that from the equation $24ab^4c^{1/2} = \lambda p_A$ since all of the terms $(a, b, c, p_A)$ are positive.

## 3 Another optimization problem

Consider the linear utility function $u(x_1,x_2) = \alpha x_1 + \beta x_2$, where $\alpha$ and $\beta$ are constants and $\alpha > 0$ and $\beta > 0$. The consumer faces budget constraint $y - p_1 x_1 - p_2 x_2 > 0$ with $y > 0$, $p_1 > 0$, and $p_2 > 0$.

**a** Calculate the marginal utility for good $x_1$ and good $x_2$.

**Answer**: The marginal utility for $x_1$ and $x_2$ are

$$\begin{aligned} \frac{\partial u(x_1,x_2)}{\partial x_1} &= \alpha \\ \frac{\partial u(x_1,x_2)}{\partial x_2} &= \beta \end{aligned}$$

**b** Find the Walrasian demand for goods $x_1$ and $x_2$.

1. **Answer**: This is a little more difficult than it appears. If $\frac{\alpha}{\beta} > \frac{p_1}{p_2}$, then the consumer will consume only $x_1$. If $\frac{\alpha}{\beta} < \frac{p_1}{p_2}$, then the consumer will consume only $x_2$. If $\frac{\alpha}{\beta} = \frac{p_1}{p_2}$, then we have a demand correspondence as any point along the budget constraint is a solution to the maximization problem. Formally, we could write:

$$x_1(p,w) = \begin{cases} 0 & \text{if } \frac{\alpha}{\beta} < \frac{p_1}{p_2} \\ \frac{w}{p_1} & \text{if } \frac{\alpha}{\beta} > \frac{p_1}{p_2} \end{cases}$$

and

$$x_2(p,w) = \begin{cases} \frac{w}{p_2} & \text{if } \frac{\alpha}{\beta} < \frac{p_1}{p_2} \\ 0 & \text{if } \frac{\alpha}{\beta} > \frac{p_1}{p_2} \end{cases}$$

and

$$\left\{x_1, x_2 \in \mathbb{R}^2_+ : p_1 x_1 + p_2 x_2 = w\right\} \text{ if } \frac{\alpha}{\beta} = \frac{p_1}{p_2}.$$

where this last piece is a correspondence equal to the budget line. Many people set up the Lagrangian and found first order conditions. If you did this, you would have:

$$\max_{x_1,x_2} \mathcal{L} = \alpha x_1 + \beta x_2 + \lambda\left[w - p_1 x_1 - p_2 x_2\right]$$

With Kuhn-Tucker conditions:

$$\begin{array}{rcc} \frac{\partial \mathcal{L}}{\partial x_1} = \alpha - \lambda p_1 \leq 0 & x_1 \geq 0 & x_1 * \frac{\partial \mathcal{L}}{\partial x_1} = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} = \beta - \lambda p_2 \leq 0 & x_2 \geq 0 & x_2 * \frac{\partial \mathcal{L}}{\partial x_2} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = w - p_1 x_1 - p_2 x_2 \geq 0 & \lambda \geq 0 & \lambda * \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \end{array}$$

The budget constraint will hold with equality, so we know that $\lambda > 0$. It is easy to check that in this problem, because if $\lambda = 0$, then we have $\alpha \leq 0$ from $\frac{\partial \mathcal{L}}{\partial x_1}$, which violates our assumption that $\alpha > 0$. Now, if $x_1$ and $x_2$

are BOTH greater than zero, then $\frac{\partial \mathcal{L}}{\partial x_1} = 0$ and $\frac{\partial \mathcal{L}}{\partial x_2} = 0$. If this is true, then:

$$\begin{aligned} \alpha &= \lambda p_1 \\ \beta &= \lambda p_2 \end{aligned}$$

which means that

$$\text{if } x_1 > 0 \text{ and } x_2 > 0 \text{, then } \frac{\alpha}{\beta} = \frac{p_1}{p_2}$$

or alternatively

$$\text{if } \frac{\alpha}{\beta} \neq \frac{p_1}{p_2} \text{, then either } x_1 = 0 \text{ or } x_2 = 0.$$

Note that the negation of "if $x_1 > 0$ and $x_2 > 0$" for our problem is that one of them equals zero – it is not both because the consumer has positive $w$ that he must spend, and it is not less than or equal to zero because we assume that the consumer must consumer 0 or more. So if the ratio of prices is not equal to the ratio of marginal utilities (after all, that is what $\alpha$ and $\beta$ are, the marginal utilities of $x_1$ and $x_2$ – this is from part a), then the consumer will end up at a corner solution consuming either all of $x_1$ or all of $x_2$. Note that if $\frac{\alpha}{\beta} = \frac{p_1}{p_2}$, then the budget constraint and the indifference curves have the same slope, and thus overlap to form a correspondence.

**c.** Let $\alpha = 2$, $\beta = 6$, $w = 120$, $p_1 = 1$, and $p_2 = 4$. What is the optimal consumption bundle for our consumer for these parameters? What is the utility at this optimal consumption bundle?

**Answer**: Since we have

$$\frac{\alpha}{\beta} = \frac{1}{3} > \frac{p_1}{p_2} = \frac{1}{4}$$

then the consumer will choose to consume only $x_1$ given these parameters. So the consumer chooses $(x_1, x_2) = \left(\frac{w}{p_1}, 0\right) = (120, 0)$, and the consumer's utility is 240.

**d.** Now suppose that $p_2 = 3$ and the other parameters are as in part **c**. Find an optimal consumption bundle for the new set of parameters. What is the utility at the optimal consumption bundle you found?

**Answer**: We have

$$\frac{\alpha}{\beta} = \frac{1}{3} = \frac{p_1}{p_2} = \frac{1}{3},$$

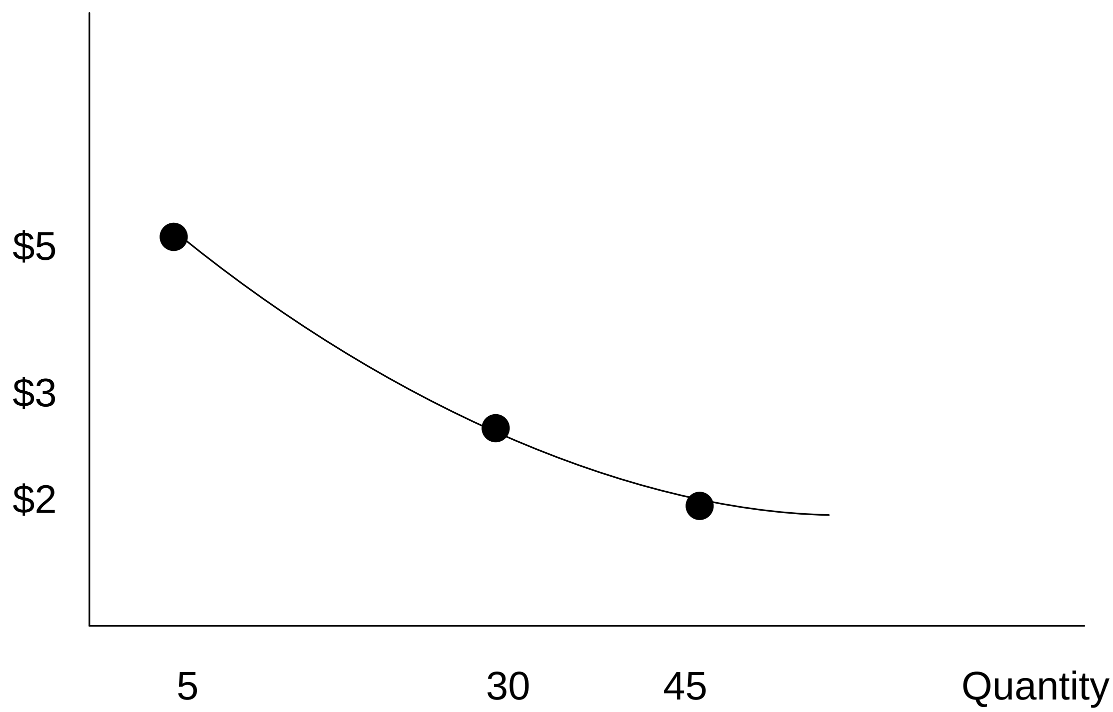
so that any point on the budget line will yield the same utility. To see this consider the points where the consumer chooses only $x_1$ and only $x_2$. His utility from $(x_1, x_2) = \left(\frac{w}{p_1}, 0\right) = (120, 0)$ is 240, as in part c. His utility from $(x_1, x_2) = \left(0, \frac{w}{p_2}\right) = (0, 40)$ is also 240. You can choose any point on the budget line, and the consumer's utility will be the same.

## 4 Income and Substitution (35 points)

The picture above shows a set of indifference curves for a consumer, as well as some budget constraints. We will assume that the price of good A and the income of the consumer are fixed. Suppose that the budget constraints in the picture correspond to prices of \$5, \$3, and \$2 for good B. Note that the faint budget constraint (or the one that does not pivot at the same point as the others) is to be used in determining income and substitution effects. It is in a sense a "hypothetical" budget constraint.

1. Derive the consumer's demand curve for good B using these 3 prices.

   **Answer**:




2. Calculate the total effect given a price decrease from \$5 to \$3.

   **Answer**:

   The total effect is $30 - 5 = 25$.

3. Calculate the substitution effect given a price decrease from \$5 to \$3.

   **Answer**:

   The substitution effect is $15 - 5 = 10$.

4. Calculate the income effect given a price decrease from \$5 to \$3.

   **Answer**:

   The income effect is $30 - 15 = 15$.

5. Is this good a normal good or an inferior good? How do you know?

   **Answer**:

   The good is a normal good because the income effect is positive.