1 Cost minimization

Consider a firm which uses only two inputs in production, capital (K) and labor (L). The firm has production function $q(K, L) = L^{\beta} K^{\alpha}$. Let w be the wage rate for L and r be the rental rate of capital. Suppose the firm wishes to produce 1080 units of the good. Let $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$, and let $w = \$4$ and $r = \$27$.

1. Find the marginal product of capital.

2. Find the marginal product of labor.

3. Find the marginal rate of technical substitution.

4. Find the cost-minimizing bundle of capital and labor for this firm, as well as the associated total cost amount.

   Suppose that the price of capital increases, so that now $r = \$64$. The firm still wishes to produce 1080 units.

5. Find the cost-minimizing bundle of capital and labor for this firm given the increase in r, as well as the associated total cost amount.

2 Production

Consider the following production functions, where q is the quantity produced of the good, K is the quantity of capital used, and L is the quantity of labor used:

Production function 1

$$q(K, L) = K^{\alpha} L^{\beta}$$

Production function 2

$$q(K, L) = K^{\alpha} + L^{\beta}$$

1. For production function 1, for what values of α and β will this production function exhibit (a) increasing, (b) constant, and (c) decreasing returns to scale?

2. For production function 2, for what values of α and β will this production function exhibit (a) increasing, (b) constant, and (c) decreasing returns to scale?

3 Monopoly

Suppose that a monopolist faces the following inverse demand curve, $P(Q) = 65 - 5Q$. The monopolist's total cost function is: $TC = 2.4Q^3 - 19Q^2 + 66.5Q + 40$.

1. Find the monopolist's marginal revenue function.
2. Find the monopolist's marginal cost function.
3. Find the monopolist's profit-maximizng price and quantity in this market, as well as the monopolist's profit at this price and quantity.

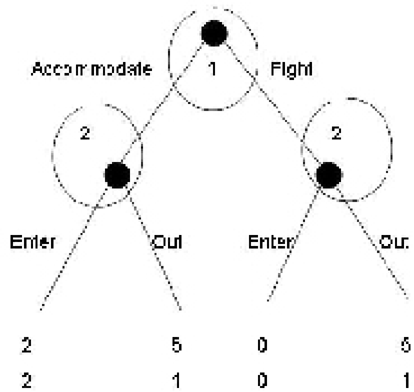
4 Cournot with Different MC

Assume the following: there are two firms competing in a Cournot (quantity) game. The firms face the following inverse demand function: $P(Q) = a - bQ = 15000 - 50Q$. Firm 1 has a cost structure such that $TC_1 = c_1 * q_1$, so that Firm 1's marginal cost is $MC_1 = c_1$. Firm 2 has a cost structure such that $TC_2 = c_2 * q_2$, so that Firm 2's marginal cost is $MC_2 = c_2$. Let $c_1 = 50$ and $c_2 = 100$, so that we have $c_1 < c_2$.

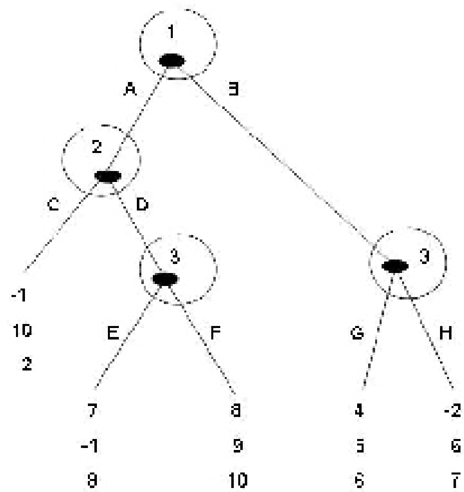
1. Find the best-response functions for Firm 1 and Firm 2.
2. Find the Nash equilibrium for this game.
3. Find the market price and resulting firm profits at the Nash equilibrium.

5 Game Trees

Here are 2 extensive form games. Answer the questions below.



Game tree 1



Game tree 2

1. In Game tree 1, find the subgame perfect NE (SPNE). Also find a pure strategy Nash Equilibrium that is not subgame perfect.

2. In Game tree 2, find the subgame perfect NE (SPNE). Explain why the set of strategies A, D, F, G (which would yield the efficient payoff) will NOT constitute a NE (so not only is that set of strategies not SPNE, it is not even NE).