Revenues, Costs, and Profit Maximization

August 25, 2022

The last general model we will discuss is profit maximization by firms or organizations. While this discussion may seem tangential to policy, it is important to understand how policy changes affect the incentives of firms. Additionally, all organizations, whether they are for-profit, non-profit, government agencies, etc. have revenue and costs. Revenue can come from many sources: selling goods and services, donations, taxes redistributed by governments to the organizations, etc. For costs, at a minimum someone is likely being paid to run the organization, regardless of its status.[1]

In this section we will discuss firm revenues, costs, profit maximization, and market structure (competition vs. monopoly).

1 Revenues

We will consider revenues in a very simplistic manner – simply put, total revenue (TR) is the product of the number of units sold (Q) and the price of the units (P), so $TR = P * Q$. We will assume all units sell for the same price as this assumption makes the math easier but does not change the general results of interest.

Marginal revenue (MR) will be important for decision-making. Marginal revenue is the additional revenue earned from the production of an extra unit of the good. In our analysis, MR will either be a straight line or a decreasing line. If the producer does not have to decrease price in order to sell more (which is an assumption that we make when a firm is operating in a perfectly competitive market), then marginal revenue will be a straight line at whatever the price is. If the price is \$5, the producer receives a total revenue of \$5 if one unit is sold, \$10 if two units are sold, \$15 if three units are sold, etc. Thus, when going from 0 units sold to 1 unit sold the producer goes from earning \$0 in total revenue to \$5, so $MR = \$5 - \$0 = \$5$. When moving from 1 unit sold to 2 units sold the producer goes from earning \$5 in total revenue to earning \$10, so $MR = \$10 - \$5 = \$5$. Another way to think about this situation is that the price is fixed and does not depend on the producer's production decision. In our total revenue function of $TR = P * Q$, the price P is not a function of the quantity Q.

Alternatively, it is possible that the price P does depend upon the quantity Q, so that $TR = P(Q) * Q$, where $P(Q)$ means that price is a function of quantity. In this case, marginal revenue will be decreasing if we assume that firms need to reduce price in order to sell more. As an example, assume that $P(Q) = 10 - Q$, so that $TR = (10 - Q) * Q$, or $TR = 10Q - Q^2$. Table 1 shows P, TR, and MR for selected quantity choices for the TR function $10Q - Q^2$. Note that the price decreases from \$10 to \$4 as the quantity increases from 0 to 6. The TR is increasing up to $Q = 5$, then it begins to decrease. The MR is decreasing from \$9 to $-\$1$ as the quantity increases from 0 to 6. Other than when $Q = 1$, the $TR > MR$ for each quantity level. Why is that true?

When $Q = 2$, the price is \$8, but the $MR = \$7$. While the producer receives an additional \$8 from selling the second unit at \$8, at the same time the producer loses \$1 because the producer has to reduce the price of the first unit from \$9 to \$8. Those two changes lead to the additional revenue generated from the 2^{nd} unit to be \$7. We can use the same reasoning to explain the MR for each quantity level.

[1] In 2016, about 12.3 million jobs, or 10.2% of the U.S. workforce, worked in nonprofits. https://www.bls.gov/opub/ted/2018/nonprofits-account-for-12-3-million-jobs-10-2-percent-of-private-sector-employment-in-2016.htm. I do not believe these 12.3 million workers worked for free.

Q	**P**	**TR**	**MR**
0	10	0	–
1	9	9	9
2	8	16	7
3	7	21	5
4	6	24	3
5	5	25	1
6	4	24	−1

Table 1: P, TR, and MR for the TR function 10Q-Qš

2 Costs

Costs are more involved than revenues because there are multiple types of costs to examine. There are total costs, average costs, and marginal costs. The different total costs are important when determining profit levels, but the average and marginal costs are the key costs for decision-making about how much to produce.

Total fixed cost (TFC): A fixed cost is one that is paid regardless of how much output is produced. For example, if a building was rented, then the monthly rental payment would be a fixed cost because regardless of whether zero units or one million units were produced that rental payment would need to be made. The total fixed cost is the sum of all the fixed costs.

Average fixed cost (AFC): Divide TFC by the number of units produced (Q) to find the average fixed cost, so $AFC = \frac{TFC}{Q}$. The AFC will be a decreasing curve because as more is produced, the fixed cost is spread over more units.

Total variable cost (TVC): A variable cost is one that is paid depending upon the number of units produced. Labor and materials are typical costs that vary with production.

Average variable cost (AVC): Divide TVC by the number of units produced (Q) to find the average variable cost, so $AVC = \frac{TVC}{Q}$. The AVC will generally begin at a high level, decrease, and then increase again.

Total cost (TC): Total cost is the sum of the total fixed cost (TFC) and total variable cost (TVC).

Average total cost (ATC): Divide TC by the number of units produced (Q) to find the average total cost, so $ATC = \frac{TC}{Q}$. The ATC will generally begin at a high level, decrease, and then increase again. It typically has the U-shape because average fixed cost is high for a small amount of production and average variable cost is high for larger amounts of production.

Marginal cost (MC): Marginal cost is the change in total cost of producing one additional unit of the good. So $MC = \frac{\Delta TC}{\Delta Q}$, where Δ is the symbol for "change in." Alternatively, for those with a calculus background MC is the derivative of the total cost function with respect to quantity. MC typically has a U-shape, where it is high for low quantities of production, decreases up to some point, and then increases for larger quantities of production. The reason for this shape is due to the **Law of Diminishing Marginal Returns**, which states that, at some quantity level, adding more variable resources to a fixed resource will increase output, but at a decreasing rate. This law is relevant for the marginal cost curve because if more of a variable resource is needed to increase output by an additional unit, the marginal cost curve will be upward sloping.

Figure 1 shows the AFC, AVC, ATC, and MC for the total cost function: $TC = 3Q^3 - 4Q^2 + 6Q + 3$. The ATC is in black, the AVC is in red, the AFC is in green, and the MC is in purple.[2][3] There are a few important relationships to note. First, the AVC and AFC will never lie above the ATC – the AVC and

[2] Unfortunately the TC needs to be a little messy in order to generate some of the interesting relationships. The associated functions for the ATC, AVC, AFC, and MC are:
$ATC = 3Q^2 - 4Q + 6 + \frac{3}{Q}$
$AVC = 3Q^2 - 4Q + 6$
$AFC = \frac{3}{Q}$
$MC = 9Q^2 - 8Q + 6$

[3] Note that for $TC = 3Q^3 - 4Q^2 + 6Q + 3$, the $TFC = 3$ and the $TVC = 3Q^3 - 4Q^2 + 6Q$. These just follow from the definitions for those costs – the TFC is the part of the total cost that does not vary with quantity and the TVC is the part of the total cost that does vary with quantity.

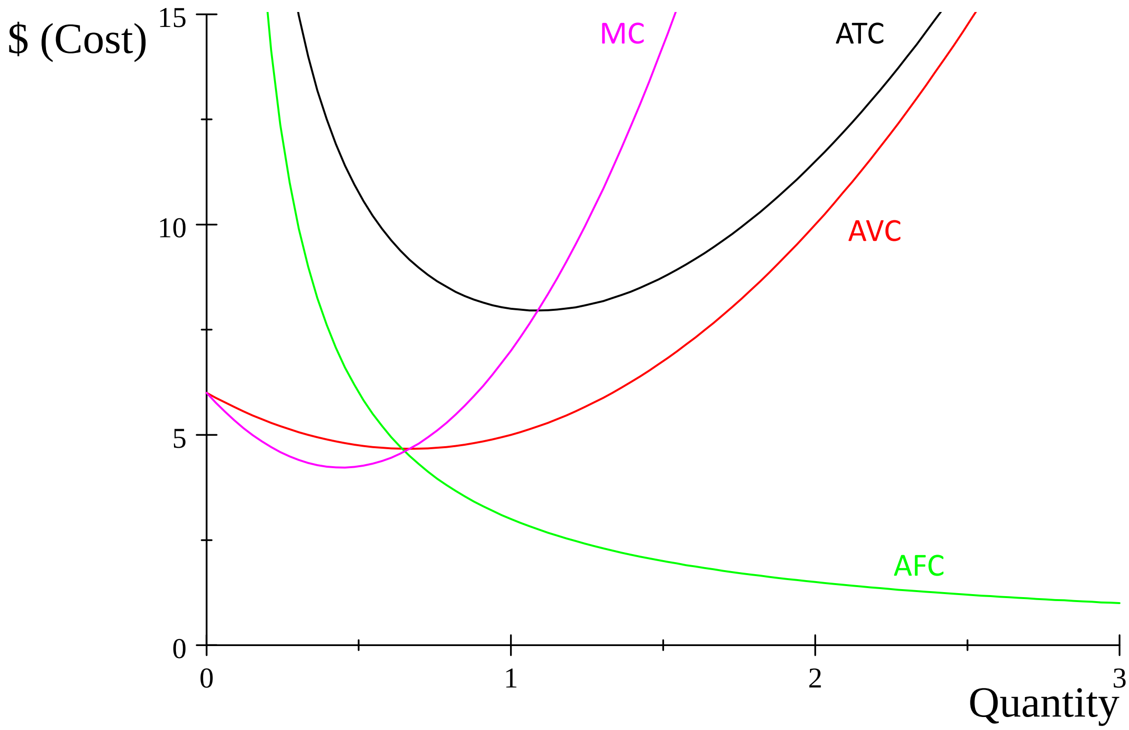

Figure 1: The ATC, AVC, AFC, and MC for the cost function: $TC = 3Q^3 - 4Q^2 + 6Q + 3$. The ATC is in black, the AVC is in red, the AFC is in green, and the MC is in purple.

AFC are, after all, just portions of the ATC. More importantly, the MC always intersects the ATC and AVC at their respective minimum points. The reason that is true is because if the marginal cost is below the ATC or AVC, then it is pulling those curves down; if it is above the ATC or AVC it is pulling those curves up.[4] Consider a student's test grades – if the next test grade is higher than the student's current test average it will pull the average up; if it is less than the student's current average it will pull the average down.

3 Profit Maximization

The typical stated goal for any firm is profit maximization, though there are other potential goals. One could attempt to maximize sales or set a sales/profit target. However, those goals may not be sustainable if the costs of the goals outweigh the revenues. Additionally, if the goals are attained, and profit is not being maximized, then the owner of the firm may realize that even more revenue might be able to be attained, and the cost to do so is less than the revenue that would be attained.

Profit (Π) is simply revenue minus cost, or, using the notation we have developed, $\Pi = TR - TC$. The TR and TC will be a function of quantity (Q), and so we would just need to find the quantity level that would maximize profit. We could create a table and list each quantity choice level and its associated profit – that is a time intensive method (less time intensive if one uses a spreadsheet) but straightforward. For those who have had calculus, a (likely) quicker way would be to take the derivative of the profit function, set it equal to zero, and solve for Q. But I am not concerned with the technical details for this particular course – my interest lies in establishing key results.

[4] Fixed costs play no role in the MC because they do not change as quantity changes, so the AFC is not subject to the same result.

	$TR = 29Q$					$TR = (97-2Q)\,Q$				
Q	TR	MR	TC	MC	Π	TR	MR	TC	MC	Π
8	232	29	36	10	196	648	67	36	10	612
9	261	29	48	12	213	711	63	48	12	663
10	290	29	62	14	228	770	59	62	14	708
⋮										
15	435	29	162	24	273	1005	39	162	24	843
16	464	29	188	26	276	1040	35	188	26	852
17	*493*	*29*	*216*	*28*	*277*	*1071*	*31*	*216*	*28*	*855*
18	522	**29**	246	**30**	276	1098	**27**	246	**30**	852
19	551	29	278	32	273	1121	23	278	32	843

Table 2: TR, MR, TC, MC, and Profit for selected quantities of two total revenue functions whenTC=Qš-5Q+12

As two different ways were used to determine price (one in which it was held constant, the other in which it depended on quantity), we will examine two different profit functions, one for each case. For the case where price is constant, we will use $TR = 29Q$; for the case in which price depends on quantity, we will use $TR = (97 - 2Q)\,Q$. We will use the same total cost function, $TC = Q^2 - 5Q + 12$, in both cases. Table 2 shows the TR, MR, TC, MC, and Π for selected quantities under both total revenue functions. Note that when $Q = 17$, profit is maximized under both total revenue functions.[5] More importantly, notice that profit is increasing until we reach the quantity level (18 in these examples) where $MC > MR$. Once we reach an output level for which $MC > MR$, we are reducing profit because the additional unit costs more than the revenue it generates. From Table 2 we can see that revenues are higher when $Q = 18$ than when $Q = 17$, and profit is still positive when $Q = 18$, but it is not as large as when $Q = 17$.

If we were to work out the math with the calculus,[6] we would see, in both examples, that profit is also maximized when $Q = 17$. The MR and MC numbers at $Q = 17$ would be a little different than what they are in the table, and would actually be equal.[7] And that is the general rule for profit maximization: firms produce the output level where $MR = MC$. The intuition is that production should occur right up until the additional cost of production (MC) is equal to the additional benefit created (MR). In general, if you take away one thing from this course (hopefully you will take away more), it is that economists use rules of equating marginal benefits of an activity with marginal costs of that activity to determine the optimal level of that activity.

Cost Minimization As an alternative to the profit maximization problem, one could choose to minimize costs for a desired output level. The same general results occur from solving this problem, but there is no guarantee that the firm is choosing the output level that maximizes profit. If you were to study the behavior of an organization that was being funded not from sales of a product but from some other source, the cost minimization approach could be used, provided the other source is not tied to some productivity measure.

4 Market Structure

We will study two types of market structures, perfect competition and monopoly. Most, if not nearly all, markets are unlikely to fall into either of these categories, but they are useful benchmarks for our analysis. On the one extreme, perfect competition (theoretically) yields the largest amount of economic surplus, and monopoly the least, as firms in perfect competition tend to produce the largest market quantity and the

[5] The choice of functions was deliberate on my part so that both of them ended up maximizing profit at the same quantity.

[6] I work out the math in the appendix, for both a specific example as well as in general.

[7] The reason for this difference is that in the table we calculate MR and MC as the additional revenue and cost, respectively, from an increase of one additional unit. With the calculus approach, we would be using a less discrete change as we would be looking at the instantaneous change in revenue and cost.

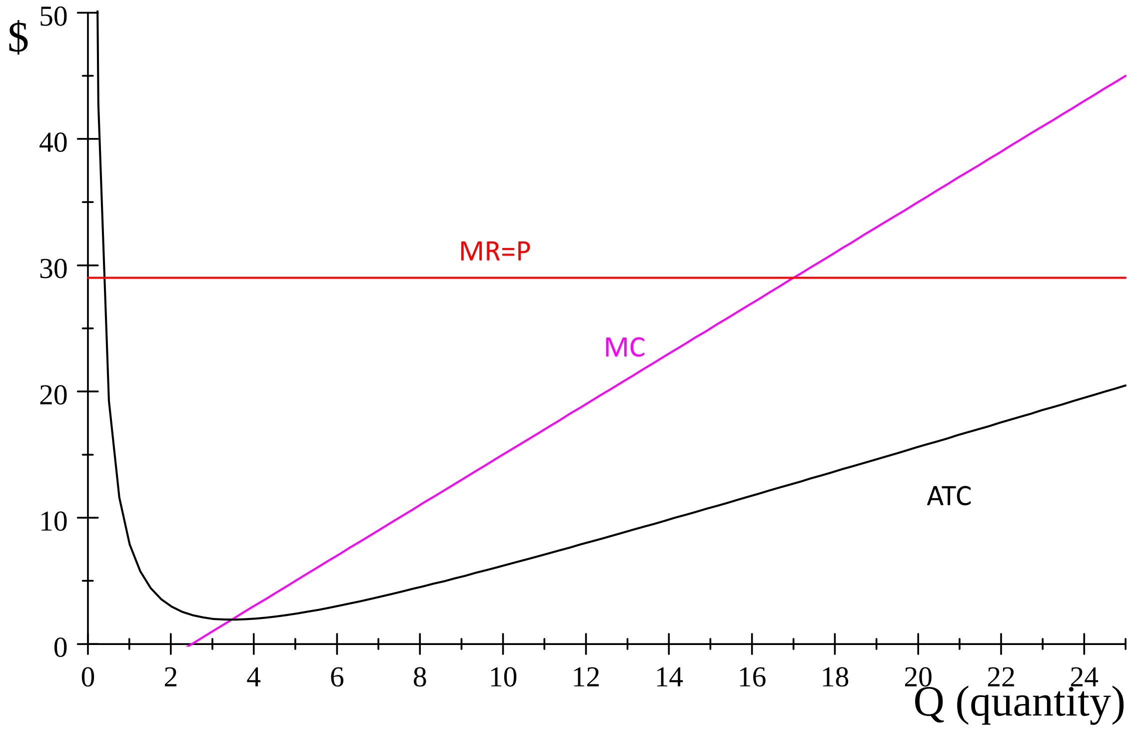

Figure 2: The MR, MC, and ATC for a perfectly competitive firm. The MR is in red, the MC is in purple, and the ATC is in black.

lowest price, while monopolies tend to produce a smaller market quantity at a higher price. Other market structures, such as monopolistic competition (or imperfect competition) and oligopoly yield results that lie between those of perfect competition and monopoly.

4.1 Perfect Competition

In the benchmark model of perfect competition, the main features are that there are a large number of firms, they sell an identical product, there are no entry or exit barriers in the market, there is perfect information on prices by both consumers and producers, firms take the market price as given, and none of the firms is large enough that a change in their production will affect the market price.[8] This structure is very idealistic and not very realistic, but the point is to provide a benchmark model so that we can make some predictions about the ideal case and then compare that to more realistic settings.

We have already discussed profit maximization with an example, but now we will work through the process graphically for an individual firm. Figure 2 shows the ATC (in black) and MC (in purple) from the total cost function $TC = Q^2 - 5Q + 12$, so $ATC = Q - 5 + \frac{12}{Q}$ and $MC = 2Q - 5$.[9] I have ignored the AFC and AVC because they are not relevant for this discussion. In a perfectly competitive market, firms take the market price as given, so it does not depend upon quantity produced. In an earlier example we had $TR = 29Q$, where the price was \$29. The MR for each additional unit was then \$29, so if we were to graph the price or MR they would be the same line and be a flat line at \$29, so we have $P = MR = 29$, which is

[8] The list of features for perfect competition varies from textbook to textbook. Some texts downplay the assumption on large numbers of firms, and others distinguish between assumptions and results. I have chosen the word "features" (rather than assumptions or results of the assumptions) to highlight key aspects of the perfectly competitive market.

[9] See the mathematical appendix later in these notes for a discussion on how I arrived at $MC = 2Q - 5$. It involves taking the derivative of the total cost function. You can take the MC function as given, as I will not ask you to derive it, but I think it is good for you all to at least be told how it is derived.

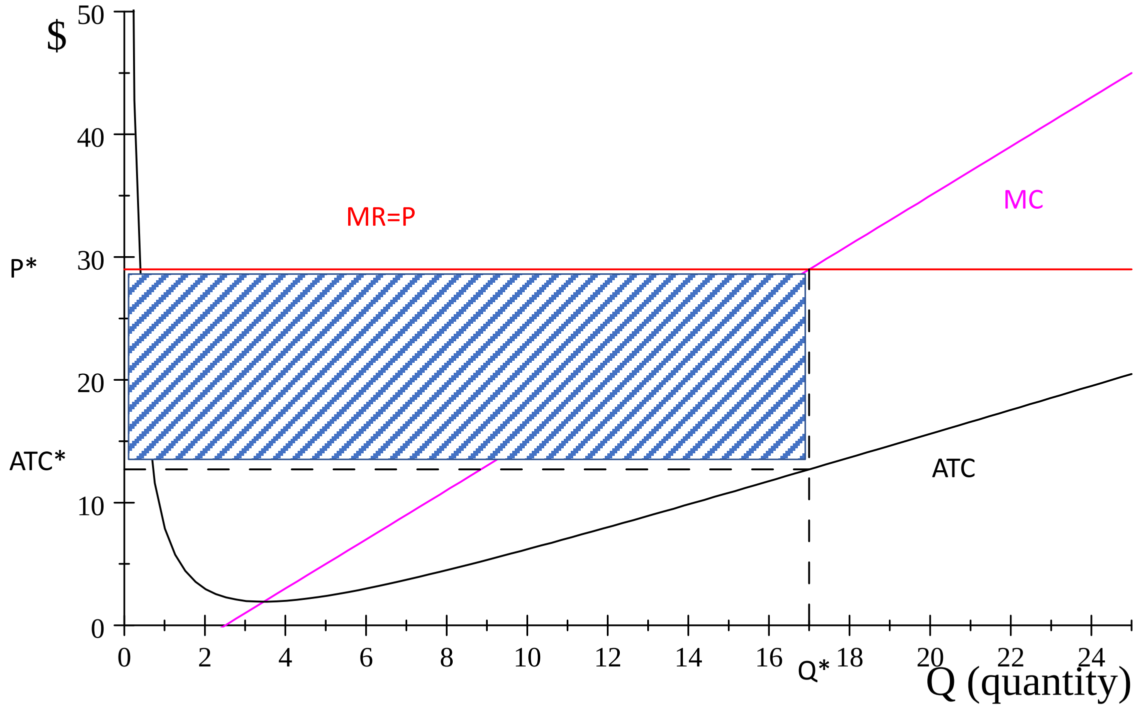

Figure 3: The profit-maximization picture for a perfectly competitive firm. The MR is in red, the MC is in purple, and the ATC is in black. The dashed vertical line shows the optimal quantity at which $MR = MC$. The dashed horizontal line shows the ATC at the optimal quantity. The shaded area is a graphical representation of the firm's profit.

shown in red.

We can find the profit maximizing quantity by finding the quantity where $MR = MC$. We know it to be $Q = 17$ from the earlier discussion. To identify the price, we find the price at $Q = 17$, which we know to be fixed at 29. If we want to determine profit, we need to know the total cost, which we can find by determining the ATC when $Q = 17$. We know that $ATC = \frac{TC}{Q}$, so if we know ATC and Q we know that $TC = ATC * Q$. From the graph itself the ATC is difficult to determine, but we know $ATC = Q - 5 + \frac{12}{Q}$ so $ATC = 17 - 5 + \frac{12}{17} = 12\frac{12}{17}$. Figure 3 shows the quantity, price, and ATC when the firm produces the profit-maximizing quantity. The quantity and ATC are represented by the dashed vertical and horizontal lines, respectively. The shaded area represents the firm's profit graphically. We know:

$$\begin{aligned} \Pi &= TR - TC \\ \Pi &= P * Q - ATC * Q \\ \Pi &= (P - ATC) * Q \end{aligned}$$

We can calculate that as $\Pi = \left(29 - 12\frac{12}{17}\right) * 17 = 277$, which matches our result from the table.

MC and profit maximization in perfect competition As a brief note, in perfect competition we can see that if we increase or decrease the price, the marginal cost will determine the firm's supply at that price. In discussing the model of consumer choice it was mentioned that we can create market demand curves from that model. Similarly, we can create market supply curves from this model. We would need this profit maximization picture for each firm, and then to vary price to determine how much each firm would produce at each price. We would then sum up the individual firm production at each price to create the market supply at each price. The point of this brief note is to connect the individual profit maximization decisions of the firm with the market supply and demand pictures from earlier.

A key result in the profit maximization problem is that $P = MC$ at the optimal quantity. With MC essentially being a supply curve, and the price being the firm's demand curve,[10] the profit maximizing quantity equilibrates supply and demand, at least for this individual firm. That means the marginal benefit to society (in the form of the demand curve) is equal to the marginal cost to society (in the form of the marginal cost curve). This result will be important when we discuss policy goals in a later set of notes.

4.1.1 Equilibrium

In Figure 3, we can tell that the firm is making a profit because the price is greater than the ATC at the production quantity. We also know that the firm is making the highest profit it can given the market conditions and its costs, so the firm would not have any incentive to change its decision. But is this picture a picture of equilibrium? One of the features of a perfectly competitive market is that there are no barriers to entry or exit. If there are profits, that should attract new firms into the industry. From our discussion of supply and demand, when new firms enter the industry that should cause the market supply to increase (shift to the right). Assuming the demand curve remains constant, that would lower the price in the market. A lower price in the market would lead to a lower price that each individual firm faces, changing their optimal quantity level.

When then does this process of new firms entering the market stop? When the profit for all individual firms equals zero. It may seem strange to think that equilibrium involves firms making zero profit − after all, firms that make no profit typically do not stay in business very long. However, there is a difference between accounting profit (which is what most people think of when they hear the word "profit") and economic profit (which is what economists think of when they hear the word "profit"). The difference between the two concepts is that economic profit takes into consideration opportunity costs, while accounting profit does not.

An opportunity cost is just the next best thing that a firm, person, agent, entity, etc. could do with the resources at their disposal. If there is a small business owner who is paying rent, utilities, wages for labor, a loan payment, etc., economists consider what the next best thing that the small business owner could do with those funds as a "cost." If the owner was paying \$20,000 a month, and the next best alternative was to invest that money elsewhere that would return 2% that month, that 2% return is considered the opportunity cost of those funds. In equilibrium, those opportunity costs plus any other accounting costs should equal the revenue the firm is generating. If the revenue is greater than the sum of those costs, other firms should enter to drive down the profit to zero. If the costs are greater than the revenue, the firm should then exit the market and pursue its alternative opportunity because it is "losing" money, even though it may be making a positive accounting profit.

Figure 4 shows this equilibrium outcome, using $TC = Q^2 - 5Q + 12$. Note that, in order for the firm to be making zero profit, the market price needs to be tangent to the minimum point of the ATC, which is also where the MC intersects the ATC. The market price in the original example was 29 and the firm had a profit of \$277. In this new example, the firm is producing $Q = 2\sqrt{3}$ and the market price is $P = 2\sqrt{12} - 5$.[11] A key result, that the price is equal to the minimum of the average total cost curve, means that the good is being produced at its lowest possible average cost; if the price were to decrease more, the firm would not find it profitable to produce the good, meaning that society would no longer benefit from having the good in circulation.[12] As when the firm was earning positive economic profit, we again see that $P = MC$ at the optimal quantity.

[10] Recall that the firm takes the market price as given, so the price and the marginal revenue are the same as the consumer demand for the individual firm's product.

[11] No firm is going to produce $Q = 2\sqrt{3}$ or set its price at $P = 2\sqrt{12} - 5$. But these are the results that come from the example cost function.

[12] A well-versed student in economics would recognize that this result only happens in the long run. In the short-run, if the market price is above the mininimum of the AVC, then the firm will continue to produce, and only when P is less than the minimum of AVC will the firm stop producing. That detail is important for a firm, but perhaps less important for policy analysts.

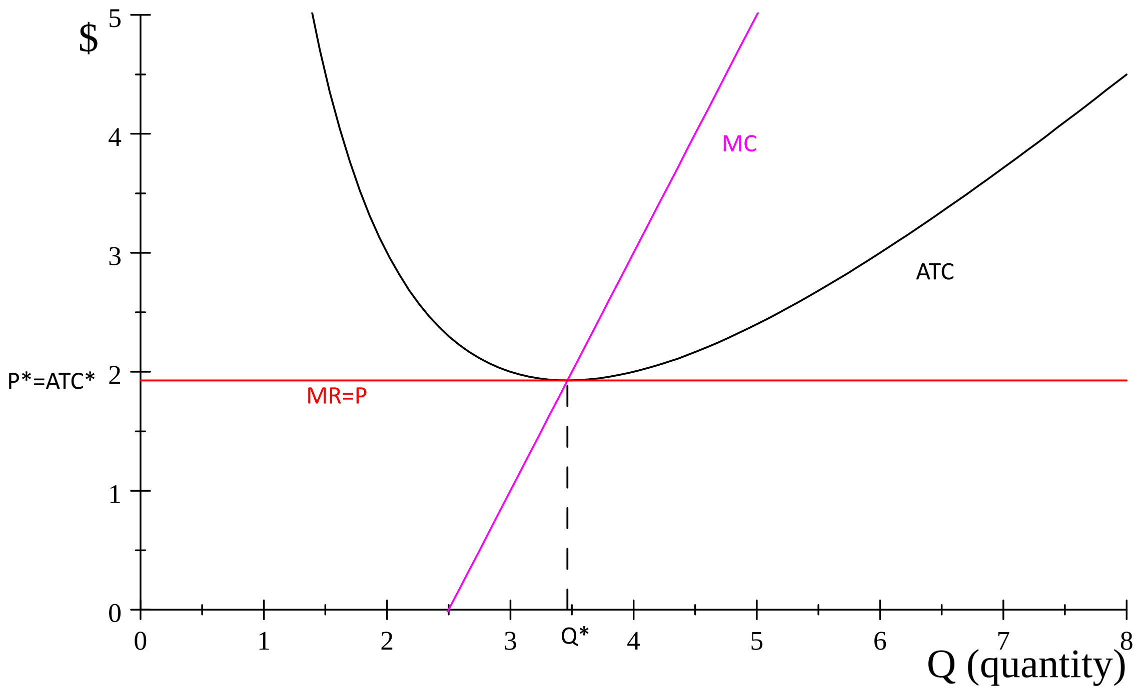

Figure 4: A firm in a perfectly competitive market earning zero economic profit. The ATC is in black, the MC is in purple, and the MR is in red.

4.2 Monopoly

If we were to create a scale of competitiveness for various market structures, perfect competition would be on the most competitive end and monopoly would be on the least competitive end. There is not much regulation imposed in perfectly competitive markets due to lack of competition, though there may be policy designed around those markets.[13] Monopolies, on the other hand, are the impetus for regulation regarding anti-competitive practices.[14]

A monopoly is defined as a single seller of a well-defined good or service for which there are no close substitutes. That is a very strict standard, and one which is not met in most industries. Nonetheless, examining the resulting price and output for a monopolist will be helpful in understanding why they are regulated, or at least more carefully watched, than firms in more competitive environments.

We will continue to use the same total cost function that we have been using, $TC = Q^2 - 5Q + 12$. Unlike the perfectly competitive firm, the monopolist's demand depends on the quantity produced. Under perfect competition it was stated that the production of a single firm would not affect the market price because there were many firms in the market; that cannot be true in the monopolist market because the monopolist is the only seller of the good. So we will use the total revenue function for which price depends on quantity, $TR = (97 - 2Q) * Q$. As a reminder, $P = 97 - 2Q$ is the demand for this good.

[13] Oftentimes agricultural markets are used as an example of perfectly competitive markets. While there is not much concern in those markets about a lack of competitiveness, policy does exist, perhaps in part due to too much competitiveness.
https://www.agriculture.com/news/business/record-high-ag-subsidies-to-supply-39-of-farm-income
https://www.gao.gov/farm-programs

[14] We will discuss the policy in greater detail later, but the Federal Trade Commission (FTC) and Department of Justice (DOJ) are tasked with enforcing antitrust laws.
https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/antitrust-laws
https://www.justice.gov/atr/antitrust-laws-and-you
If you are wondering why it is called "antitrust" regulation, when the regulation was originally proposed firms would join together (collude) to form a trust that would attempt to control many major industries.

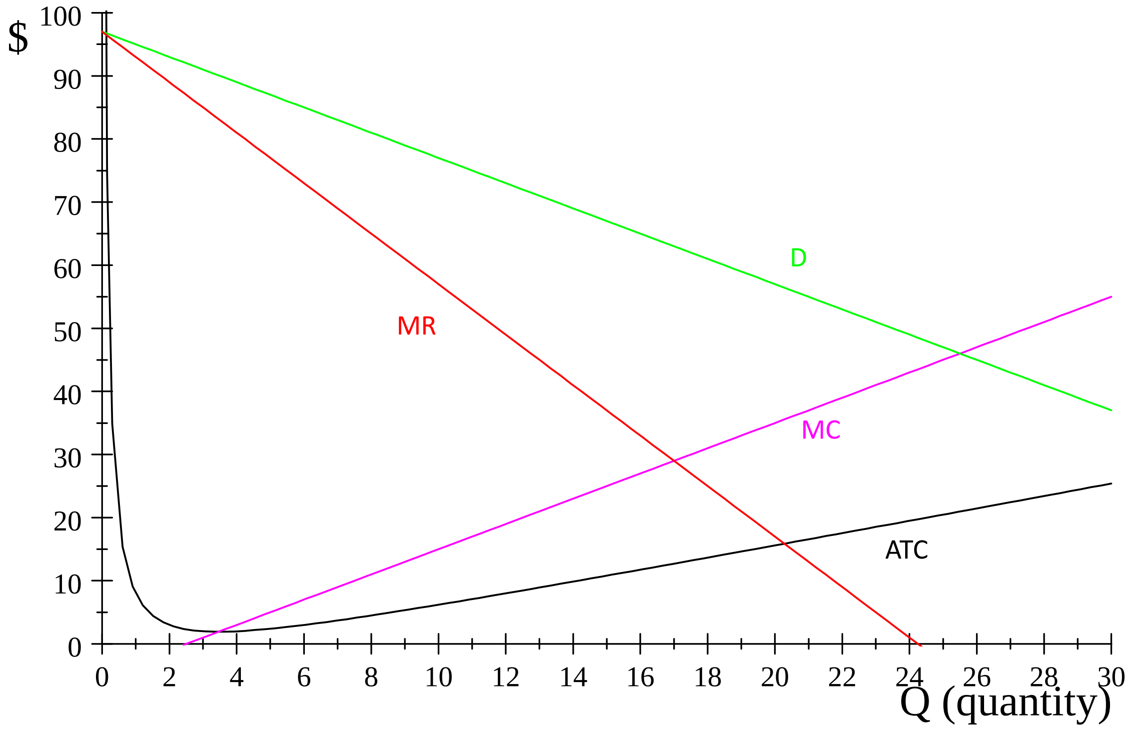

Figure 5: The demand (D), MR, MC, and ATC for a monopoly. The demand is in green, the MR is in red, the MC is in purple, and the ATC is in black.

Figure 5 shows the monopolist's ATC (in black), MC (in purple), demand (in green), and MR (in red). We have discussed graphical representation of demand curves, the ATC, and the MC, but we have not discussed a graphical representation of the MR curve when demand is downward sloping.[15] In Tables 1 and 2, we can see that the MR is downward sloping, and, in the former, less than or equal to the price for each quantity level. We discussed why that was the case but not the actual function. When the inverse demand function is linear, such as $P = a - bQ$, the MR will be $MR = a - 2bQ$, so if $P = 97 - 2Q$, $MR = 97 - 4Q$. That will be the case every time.[16]

How would we find the firm's profit-maximizing quantity, price, and profit using the graph?[17] The key is to recall that the firm chooses the quantity such that $MC = MR$ – so begin by finding the quantity at that intersection. To find the price we use the demand curve, because the demand curve tells us the price that consumers are willing to pay for a specific quantity of the good. Figure 6 shows the optimal quantity and price for the monopolist, as well as the ATC at the optimal quantity. The shaded area is profit.

4.2.1 Equilibrium

When firms in a competitive market are making positive economic profit, this profit attracts entrants into the market, which erodes the profit of the existing firms. However, in a monopoly, entrants are unable to enter the market, so in equilibrium profits can persist. Note that because there is only a single producer, the marginal cost curve for the monopolist IS the marginal cost curve for society.

[15] We have discussed the graphical representation of the MR curve when price was constant.

[16] In the mathematical appendix it is shown that this result occurs due to taking the derivative of the total revenue function when the inverse demand function is linear.

[17] From the example in Table 2, we already know that $Q = 17$ and $\Pi = 855$. We know that $P = 97 - 2Q$ so $P = 97 - 34 = 63$.

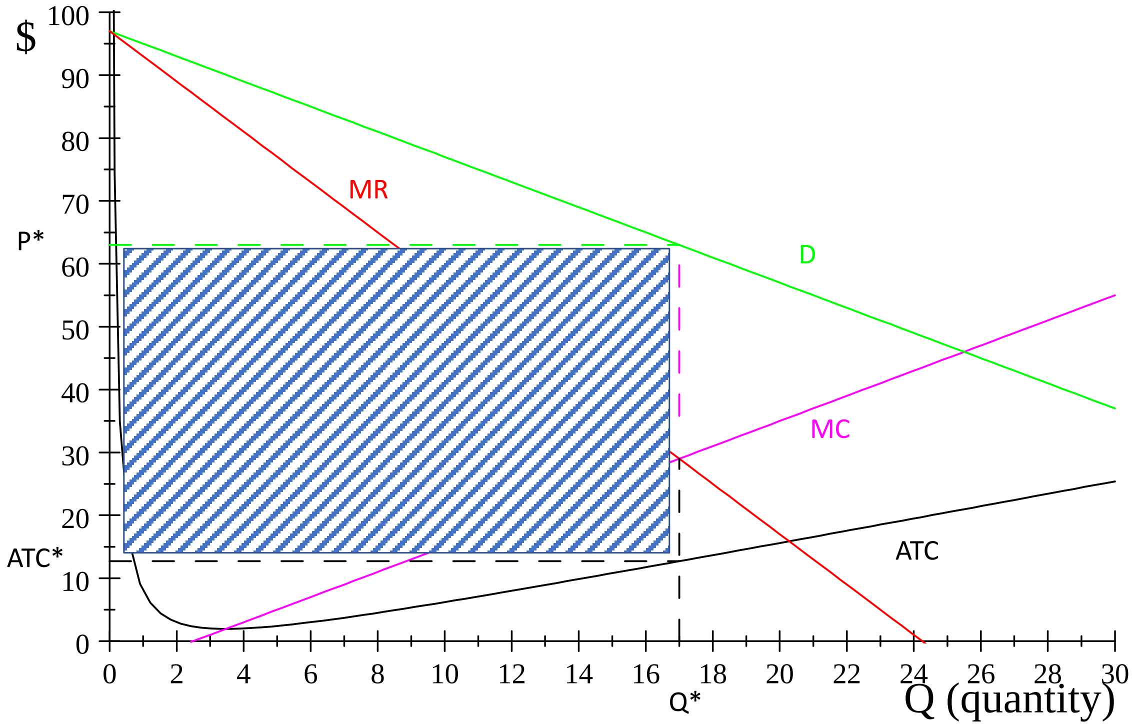

Figure 6: The profit maximization problem of the monopolist with the profit shaded.

4.3 Comparison of perfect competition and monopoly

When we compare the two market structures, we want to compare them when they are in equilibrium, so we begin with the perfectly competitive firm earning zero economic profit and the monopolist which may (and likely will) be earning positive economic profit. If we compare Figure 3 with Figure 4, a few differences, beyond the monopolist earning positive economic profit, should be noticeable. One is that the monopolist is not producing the quantity that minimizes average total cost. In this particular example that is not the main issue; the main issue is that the marginal cost for society is the MC curve, and the marginal benefit for society is the market demand. Unlike the perfectly competitive firm, when the monopolist is in equilibrium, the marginal cost for society and the marginal benefit for society are not equal at the profit maximizing quantity. This result occurs because the monopolist's marginal benefit curve (in the form of MR) is not equal to society's marginal benefit curve (in the form of demand). This difference will motivate some of our discussions about policy goals.

5 Criticisms

One major criticism of this approach is that it assumes that firms know total revenue and total cost and all the associated functions. While it is highly unlikely that any particular firm knows those functions with the degree of specificity used in the examples, they do attempt to estimate those functions. The estimation could be complicated (as in the firm is actually trying to estimate the functions) or much more simplistic (as in the firm is using a heuristic to forecast future demand in planning for its resource utilization), but reducing costs (which should increase profit) is generally part of any business' operating strategy. That leads to another criticism, that this approach leads to low quality goods because they are low cost, but none of the models we have discussed have mentioned quality levels. If the product is of too low a quality, such that consumers do not want to buy it, even though it is less expensive than the products of other producers, demand for the low quality product will be such that it is not profitable to produce the good.

Another major criticism is that firms may have motivations other than profit maximization and the profit maximization approach misses these other motivations. One such motivation could be to "go green" or use renewable energy sources for resource production. However, it is very likely that the firms are conducting some cost-benefit analysis when making these decisions to change production processes. We have been looking at a static problem – what does the firm do right now – and have not considered that firms may tradeoff lower profits today for higher profits in the future. Those higher profits could come by attracting more customers (who want to purchase goods produced using sustainable production processes) or by lowering future costs (from learning how to use the sustainable production technology more efficiently). And sometimes those two forces might work together – there are many products that have a statement such as "This packaging produced with 25% less plastic" which should (1) appeal to those consumers who are seeking out products that reduce waste and (2) also reduce cost because the firm is, after all, using 25% less plastic (though they may be using more of other resources). Also, as mentioned earlier when discussing organizations that have revenue streams that are not directly tied to a production amount, firms could choose a more sustainable (or cleaner) production technology that costs more than some alternative production technology that is not as sustainable, but conditional on choosing that production technology they would still like to minimize costs. The cost minimization problem is just the dual of the profit maximization problem.

6 Mathematical Appendix: Profit Maximization

I want to show the calculus method of profit maximization and some other results. I begin with the specific example where $TR = (97 - 2Q)\,Q$ and $TC = Q^2 - 5Q + 12$. We can set up the profit function:

$$\begin{aligned}
\Pi &= TR - TC \\
\Pi &= (97 - 2Q)\,Q - \left(Q^2 - 5Q + 12\right) \\
\Pi &= 97Q - 2Q^2 - Q^2 + 5Q - 12 \\
\Pi &= 102Q - 3Q^2 - 12
\end{aligned}$$

To find the quantity that maximizes profit, differentiate Π with respect to Q:

$$\begin{aligned}
\Pi &= 102Q - 3Q^2 - 12 \\
\frac{d\Pi}{dQ} &= 102 - 6Q
\end{aligned}$$

Now set that derivative equal to zero and solve for Q:

$$\begin{aligned}
102 - 6Q &= 0 \\
102 &= 6Q \\
17 &= Q
\end{aligned}$$

Now I want to take a step back and find the MR and MC curves separately. The MR curve is just the derivative of the TR curve, so to find the MR curve, differentiate TR with respect to Q:

$$\begin{aligned}
TR &= (97 - 2Q)\,Q \\
TR &= 97Q - 2Q^2 \\
\frac{dTR}{dQ} &= 97 - 4Q \\
MR &= 97 - 4Q
\end{aligned}$$

Note that when $Q = 17$, $MR = 29$.

The MC curve is just the derivative of the TC curve, so to find the MC curve, differentiate TC with respect to Q:

$$\begin{aligned} TC &= Q^2 - 5Q + 12 \\ \frac{dTC}{dQ} &= 2Q - 5 \\ MC &= 2Q - 5 \end{aligned}$$

Note that when $Q = 17$, $MC = 29$. So at that optimal quantity level of $Q = 17$, we have $MR = MC$.

If the profit function was written more generally and we "solved" for the optimal quantity, we would differentiate the profit function with respect to Q and set the derivative equal to zero:

$$\begin{aligned} \Pi &= TR - TC \\ \frac{d\Pi}{dQ} &= \frac{dTR}{dQ} - \frac{dTC}{dQ} \end{aligned}$$

But using our knowledge from earlier, we know that $\frac{dTR}{dQ}$ is MR and $\frac{dTC}{dQ}$ is MC, so substituting:

$$\begin{aligned} \frac{d\Pi}{dQ} &= \frac{dTR}{dQ} - \frac{dTC}{dQ} \\ \frac{d\Pi}{dQ} &= MR - MC \end{aligned}$$

Now setting that derivative equal to zero we have:

$$\begin{aligned} \frac{d\Pi}{dQ} &= MR - MC \\ 0 &= MR - MC \\ MC &= MR \end{aligned}$$

That is just the calculus derivation of the result.