Decision-making Under Risk and Uncertainty

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Thus far in the course we have considered situations where economic agents know everything with certainty. They know supply curves, demand curves, cost functions, revenue functions, market prices, etc. However, many times economic agents do not have certainty about an outcome they will face. This risk or uncertainty should not affect the results of our market supply and demand model because it should simply shift one of the curves to price the risk into the market. We can consider two labor markets which require similar skill, but one of which has a higher level of risk of injury. The market with a higher level of risk should see higher wages, because it should take more money to compensate individuals to perform the riskier task.[1] If we believe that market prices will reflect any inherent risk and uncertainty, that leaves the focus on individual behavior.

1 Expectations

There are different ways to think about risk and uncertainty. One could be a situation where an individual knows all the possible outcomes, knows the likelihood (probability) of each of those outcomes, and needs to make a decision that will ultimately lead to a (probabilistically determined) outcome. These situations are ones that economists call risky decisions. You can imagine someone rolling dice or playing cards, where that individual could objectively know the probability of each outcome, though in some cases there may be many outcomes. Formally, we call the list of outcomes and their associated probabilities of occurring a **lottery**.

It is also possible that the outcomes are known, but the probabilities of those outcomes are unknown. Economists would call these situations uncertain. Consider a salesperson who exerts effort to sell a product. The salesperson knows how many units can be sold, and may know how much effort effects sales. However, the salesperson may not know how all the other factors will affect sales, so while the salesperson may have subjective probabilities about the outcomes, the subjective probabilities of that salesperson may not be the same as the subjective probabilities of another salesperson.

Our focus will be on decision-making under risk so that we can discuss how risk averse or risk loving individuals may make different decisions when faced with the same set of outcomes and objective probabilities. While the case of unknown but subjective probabilities is likely a more realistic setting, the risky decision-making construct will provide us some framework for understanding decision-making under uncertainty. Typically individuals prefer known probabilities to unknown probabilities[2] and individuals will need to assign subjective probabilities if they do not know the objective probabilities and make decisions. We hope that they make decisions similarly in the case where the objective probability over two outcomes is 50/50 and where they subjectively believe the probability over those same two outcomes is 50/50.

As a starting point, consider the following pairs of lotteries:

Lottery A which pays $1 million with an 11% chance, and $0 with an 89% chance, and Lottery B which pays $5 million with a 10% chance and $0 with a 90% chance.

Would you prefer Lottery A or Lottery B?

[1] I would guess that drilling offshore is much more dangerous than drilling onshore, at least in the sense that being on an oil rig in water is more dangerous than being on dry land. Assuming the skill levels necessary for the two tasks are equal (which they may not be), I would assume that workers doing offshore drilling are paid more than those doing onshore drilling due to the increased risk.

[2] At times you may see the term "ambiguity aversion." This term refers to the case where individuals prefer known probabilities to unknown probabilities – they want clarity, and want to avoid ambiguity.

You might also hear ambiguity called "Knightian uncertainty" after Frank Knight.

Lottery C which pays \$1 million with a 100% chance, and Lottery D which pays \$5 million with a 10% chance, \$1 million with an 89% chance, and \$0 with a 1% chance.

Would you prefer Lottery C or Lottery D?

We will return to these lotteries later in the notes. Make a note of which you prefer – there are no right or wrong answers as your own choices depend on your preferences.

1.1 Expected Value

We begin by establishing the concept of expected value, which is the weighted average of the outcomes, where the weights are the probabilities of occurrence for each respective outcome. We will only consider outcomes that are numerical (specifically, dollars) so that we can make some calculations.[3] As a few examples, we could have a lottery where the outcomes are \$0, \$4, \$8, \$20 and each has a 25% probability of occurring. The expected value of that lottery would be:

$$\begin{aligned} EV &= \$0 * .25 + \$4 * .25 + \$8 * .25 + \$20 * .25 \\ & \text{or} \\ EV &= \$0 * \frac{1}{4} + \$4 * \frac{1}{4} + \$8 * \frac{1}{4} + \$20 * \frac{1}{4} \\ EV &= \$8 \end{aligned}$$

A degenerate lottery is defined as a certain amount of money with 100% probability, such as if someone is guaranteed \$8 with certainty. Note that there is only one outcome in a degenerate lottery, but those lotteries will play an important role in our discussion of risk. Finally, it is important to make sure that the probabilities sum up to 100% (or 1). One of the outcomes must occur.[4]

1.2 Expected Utility

Expected utility is very similar to expected value, except now we take the weighted average of the utilities of the dollar amounts instead of the weighted average of the dollar amounts themselves. In a sense we are rescaling the outcomes. It may seem odd to rescale the numbers but there is a point to that process.

Suppose that someone offers you the following: You can have either \$5 with certainty (100% probability) or you can have \$0 with a 50% chance and \$10 with a 50% chance. When looking at this offer from the perspective of expected value, an individual should not care. The first lottery has an expected value of $\$5 * 1 = \$5$, while the second lottery has an expected value of $\$0 * \frac{1}{2} + \$10 * \frac{1}{2} = \$5$. So on average someone will receive \$5 regardless of which lottery is chosen, though they will receive \$0 or \$10 (and never actually \$5) from the second lottery. If the lotteries were all fairly low stakes many (most?) people would likely be indifferent between the two.

However, now consider that someone offers the following: You can have either \$5 million with certainty (100% probability) or you can have \$0 with a 50% chance and \$10 million with a 50% chance. The expected value of both of those lotteries is \$5 million, but I would guess that whereas many people would not care much about whether they received \$0, \$5, or \$10,[5] many people would care about the difference between

[3] One can imagine that there are outcomes that are not numerical: sunny, partly cloudy, cloudy, etc. and that each of those outcomes has a probability of occurring. In order to determine what the expected weather is for the day we would need to somehow convert those words into a scale, compute the expectation, and then convert that scale back to words.

[4] We can define expected value more formally. Assume there are n outcomes. If we let x_i represent an individual outcome and p_i be the probability of that outcome, then the expected value is:

$$EV = \sum_{i=1}^{n} x_i p_i$$

where:

$$\sum_{i=1}^{n} p_i = 1.$$

That second equation just ensures that the probabilities sum to 1.

[5] None of those amounts are likely to put an individual on a completely different life path.

receiving \$0, \$5 million, and \$10 million.[6] Even though the expected value for both lotteries is \$5 million, people likely prefer one to the other. Because people have different preference rankings for these lotteries even though they have the same expected value, we need another scale by which we can determine an individual's preference ranking of the two lotteries.

Expected utility is one method of providing such a preference ordering. As mentioned earlier, when calculating expected utility we will take the weighted average of the utilities of the dollar amounts, with the weights being the objective probabilities. To calculate expected utility we need to know a individual's utility function.[7] We will call $u(x)$ the individual's utility function, where $u(\bullet)$ is the utility function and x is the sure amount of money. We will look at three example utility functions. To make comparisons a little easier, let L_1 represent the lottery when the individual receives \$5 with certainty, and let L_2 represent the lottery where the individual receives \$0 with a 50% chance and \$10 with a 50% chance.

Case a Suppose that $u(x) = x$. In that case the expected utility of L_1 is: $u(5) * 1 = 5 * 1 = 5$, and the expected utility of L_2 is $u(0) * \frac{1}{2} + u(10) * \frac{1}{2} = 0 * \frac{1}{2} + 10 * \frac{1}{2} = 5$. Note that both lotteries have an expected utility of 5, so an individual with this utility function is still indifferent between the two lotteries.

Case b Suppose that $u(x) = \sqrt{x}$. In that case the expected utility of L_1 is: $u(5) * 1 = \sqrt{5} * 1 = \sqrt{5}$, and the expected utility of L_2 is $u(0) * \frac{1}{2} + u(10) * \frac{1}{2} = \sqrt{0} * \frac{1}{2} + \sqrt{10} * \frac{1}{2} = \frac{\sqrt{10}}{2}$. Note that $\sqrt{5} \approx 2.23$ and $\frac{\sqrt{10}}{2} \approx 1.58$, so an individual with this utility function prefers L_1 to L_2.

Case c Suppose that $u(x) = x^2$. In that case the expected utility of L_1 is: $u(5) * 1 = 5^2 * 1 = 25$, and the expected utility of L_2 is $u(0) * \frac{1}{2} + u(10) * \frac{1}{2} = 0^2 * \frac{1}{2} + 10^2 * \frac{1}{2} = \frac{100}{2} = 50$. This individual prefers L_2 to L_1.

In our discussion of risk preferences, the relationship between the expected utility and the expected value of a lottery will be important.

2 Risk Preferences

Individuals take risks in various forms every day. We use the term risk preferences to capture an individual's attitude towards risk. An individual may be risk neutral, risk averse, or risk loving. As a reminder, our concept of expected utility has only been developed for sure amounts of money. While we can extend it to other areas or attempt to force other areas into this model, it is possible for someone to be risk averse over money, yet be risk loving in other areas (thrill-seeking behavior).

To determine an individual's risk preferences, we compare an individual's expected utility of the lottery to the expected utility of receiving the expected value of the lottery with certainty.[8] Using our example in Section 1.2, the individual is comparing a lottery over two outcomes with an expected value of \$5 to a degenerate lottery with a certain amount of \$5. If the individual is indifferent between the two, then the individual is risk neutral. In that case, which is shown by **Case a** in Section 1.2, risk plays no role in determining the individual's preferences. However, in **Case b**, the individual prefers the certain amount to the lottery that pays an expected value equal to the certain amount. Because the individual prefers the certain amount to the lottery, the individual is unwilling to take on risk, and we would label that individual's preferences as risk averse. In **Case c** the individual actually prefers the lottery to the certain amount. Because the individual prefers to take on risk, we would call this individual risk loving.

2.1 Certainty Equivalents

Suppose an individual is risk averse. We know that an individual prefers the certain amount of the expected value of the lottery to the lottery itself. However, a different question to ask is what certain amount is equivalent to the lottery. Using our example, if we have one lottery when the individual receives \$5 with certainty, and a second lottery where the individual receives \$0 with a 50% chance and \$10 with a 50%

[6]Receiving either \$5 million or \$10 million will likely put someone on a much different life path. The 50% probability of receiving \$0 would outweigh the benefits of the possibility of receiving an extra \$5 million should one get lucky and receive the \$10 million.

[7]Economic theorists will distinguish between a utility function over lotteries themselves and a utility function over sure amounts of money. We are focusing on the utility function over sure amounts of money. This distinction is not critical for public policy scholars.

[8]Recall that this type of lottery is know as a degenerate lottery because there is only a single outcome.

chance, we know that the risk averse individual prefers \$5 with certainty. But is the individual indifferent between \$4 for certain and the 50/50 lottery over \$0 and \$10? \$3 for certain? \$2 for certain?

Using $u(x) = \sqrt{x}$ as our risk averse utility function, we know that the expected utility of the lottery is:

$$\begin{aligned} EU &= u(0) * \frac{1}{2} + u(10) * \frac{1}{2} \\ EU &= \sqrt{0} * \frac{1}{2} + \sqrt{10} * \frac{1}{2} \\ EU &= \frac{\sqrt{10}}{2} \end{aligned}$$

To find the certainty equivalent, we want to find the certain amount x such that $u(x) = \frac{\sqrt{10}}{2}$. We know $u(x) = \sqrt{x}$, so:

$$\begin{aligned} u(x) &= \frac{\sqrt{10}}{2} \\ \sqrt{x} &= \frac{\sqrt{10}}{2} \\ \left(\sqrt{x}\right)^2 &= \left(\frac{\sqrt{10}}{2}\right)^2 \\ x &= \frac{10}{4} \\ x &= 2.5 \end{aligned}$$

In this case, the individual would be indifferent between a certain amount of \$2.5 and the lottery that pays \$0 with probability 50% and \$10 with probability 50%. That result is consistent with our notion of risk preferences as the expected value of that lottery is \$5, but the individual is willing to accept less, \$2.5, rather than play that lottery.

We can also view this relationship graphically. Figure 2.1

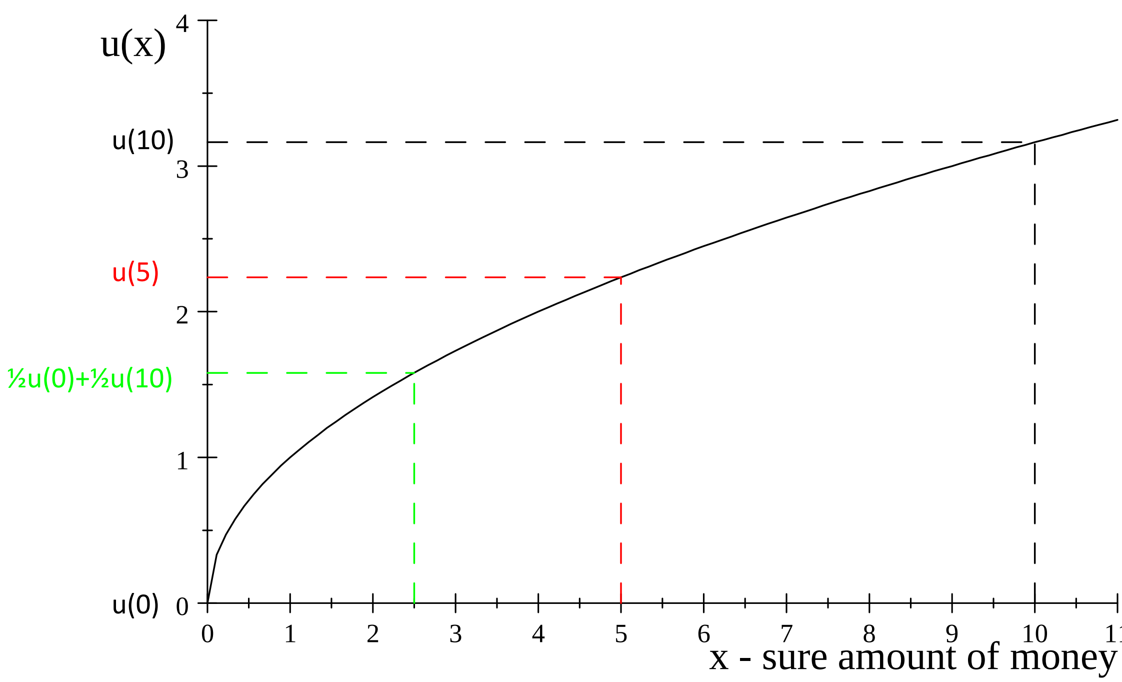

Figure 2.1: The certainty equivalent of the lottery $\frac{1}{2}u(0) + \frac{1}{2}u(10)$ when the individual had the utility function $u(x) = \sqrt{x}$.

shows the utility of the two amounts of money (0 and 10) that comprise the lottery, as well as the utility of the expected value of the lottery (5) in red, and the utility of the certainty equivalent of the lottery (2.5) in green. Because this individual is risk averse, the certainty equivalent of the lottery is less than the expected value of the lottery. Intuitively, the risk averse individual is giving up some expected payoff for a sure gain.

3 Criticisms

The criticisms of expected utility theory fill volumes. Whereas utility theory itself is very difficult to disprove because it is always possible there is some other good the individual is considering, expected utility theory makes very precise predictions about behavior under very precise circumstances. Many of these criticisms focus on violations of specific assumptions made when developing expected utility theory.[9] Schoemaker (1982) provides an overview of the concept and discusses its limitations. Machina (1987) provides a discussion of problems solved and unsolved regarding choice under uncertainty. A little earlier, Kahneman and Tversky (1979) questions some of the fundamentals of expected utility theory. Some work in behavioral economics was developed as a criticism of expected utility theory.

3.1 Allais Paradox

Return to the two pairs of lotteries in the opening.

Lottery A which pays \$1 million with an 11% chance, and \$0 with an 89% chance. and Lottery B which pays \$5 million with a 10% chance and \$0 with a 90% chance.

Lottery C which pays \$1 million with a 100% chance, and Lottery D which pays \$5 million with a 10% chance, \$1 million with an 89% chance, and \$0 with a 1% chance.

This example was created by Maurice Allais, who won the Nobel Prize in Economics in 1988. Many people, when choosing between Lottery A and Lottery B, will choose Lottery B. However, many people who choose Lottery B also choose Lottery C. That pair of choices is not "wrong," but it is inconsistent with expected utility theory. We want to remove the common element of Lottery A and B, and the common element of Lottery C and Lottery D.

Remove the 89% chance of receiving \$0 from Lottery A and Lottery B. Then it is a choice of \$1 million (Lottery A) or \$5 million with a $\frac{10}{11}$ chance and \$0 with a $\frac{1}{11}$ chance (Lottery B).

Now remove the 89% chance of \$1 million from Lottery C and Lottery D. Then it is a choice of \$1 million (Lottery C) or \$5 million with a $\frac{10}{11}$ chance and \$0 with a $\frac{1}{11}$ chance (Lottery D).

In essence, Lottery A and Lottery C are equivalent, and Lottery B and Lottery D are equivalent, so if someone chose Lottery B that same person should also choose Lottery D. As mentioned, many people choose Lottery B and Lottery C, which violates expected utility theory.[10] The conjecture is that the lure of certainty (\$1 million with certainty) causes people to choose Lottery C. Again, it is not that the people are making incorrect choices, just that the expected utility theory model fails to predict those choices and needs to be revised.

References

[1] Kahneman, Daniel and Amos Tversky (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica* 47:2, 263-292.

[2] Machina, Mark J. (1987). Choice Under Uncertainty: Problems Solved and Unsolved. *Journal of Economic Perspectives* 1:1, 121-154.

[3] Schoemaker, Paul J.H. (1982). The Expected Utility Model: Its Variants, Purposes, Evidence, and Limitations: *Journal of Economic Literature* 20:2, 529-563.

[9] We have not gone through these assumptions in great detail, and it is not necessary to do so for this course, but the interested reader can find background information here:
https://belkcollegeofbusiness.uncc.edu/azillant/wp-content/uploads/sites/846/2014/12/BPHD8100_chapter6notes.pdf
Fair warning: Those notes are much more mathematical than the discussion we have here.

[10] For those who are interested, formally those choices are a violation of the independence axiom.