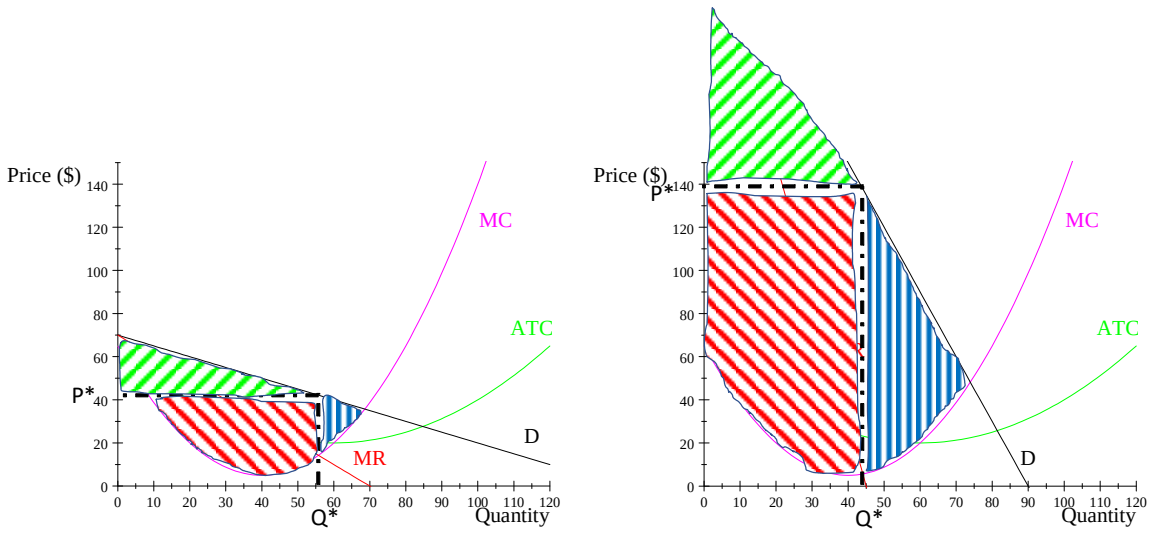
PPOL 8640 Assignment 2 Answers

Due: September 27, 2022 at beginning of class

1. (20 points) The pictures below show monopolists that face two different market demand curves and their associated marginal revenue curves. The monopolists have the same cost curves, and the average total cost and marginal cost are shown. The graphs are scaled identically so you can compare results by looking at the pictures.



Monopolist 1 on left; Monopolist 2 on right

a Identify the profit-maximizing quantity and price for each monopolist on their respective graphs. Which monopolist charges the higher price?

Answer:

Find the profit-maximizing quantity at the intersection of MC and MR. Use the demand curve to find the price at that quantity. The respective profit-maximizing price and quantity are shown by P^* and Q^* for each monopolist. Monopolist 2 charges a much higher price, almost $100 more.

b Identify the consumer surplus, producer surplus, and deadweight loss for each monopolist on their respective graphs. Under which monopolist is deadweight loss the greatest?

Answer:

The consumer surplus in each market is the green triangle above the price but below the demand curve. In the Monpolist 2 market I have approximated where the demand curve intersects the price axis. The producer surplus is given by the red shaded area below the price but above the marginal cost. The deadweight loss is given by the blue shaded area between the demand curve and MC for all quantities between Q^* and the quantity where MC intersects demand. The deadweight loss is greatest under Monopolist 2. While that means there are more gains from trade that are not captured in the market, that does not necessarily mean that the gains from trade that are captured in the market are lower

under Monopolist 2. Consumers value Monpolist 2's product more than Monopolist 1's, so there is a much larger combined consumer and producer surplus.

c The demand curve for Monopolist 2 is much steeper than the demand curve for Monopolist 1. Assuming the cost curves remain the same, as the demand for a monopolist's product shifts from nearly horizontal to nearly vertical, how would we expect the total deadweight loss in the market to change? Use your answer to part **b** as a guide.

Answer:

We should expect the deadweight loss to increase, though as I mention in part **b** that is partly a function of individuals valuing the good with the steeper demand curve more. Intuitively, as the demand curve becomes steeper, there is more distance between the demand curve and the MR curve. It is that distance between those two curves that cause deadweight loss. If the demand curve is perfectly horizontal, the demand curve and MR are the same, so there is no difference between the incentives for the firm and society. At the other extreme case, if the demand curve is nearly perfectly vertical,[1] there is a larger gap between marginal revenue and demand.

2. (25 points) Suppose there are two consumers, J and K, in an Edgeworth box economy. There are two goods, Good 1 and Good 2. There are 40 units of Good 1 and 20 units of Good 2. Consumer J begins with 25 units of Good 1 and 5 units of Good 2.

a How many units of Good 1 and Good 2 does consumer K have?

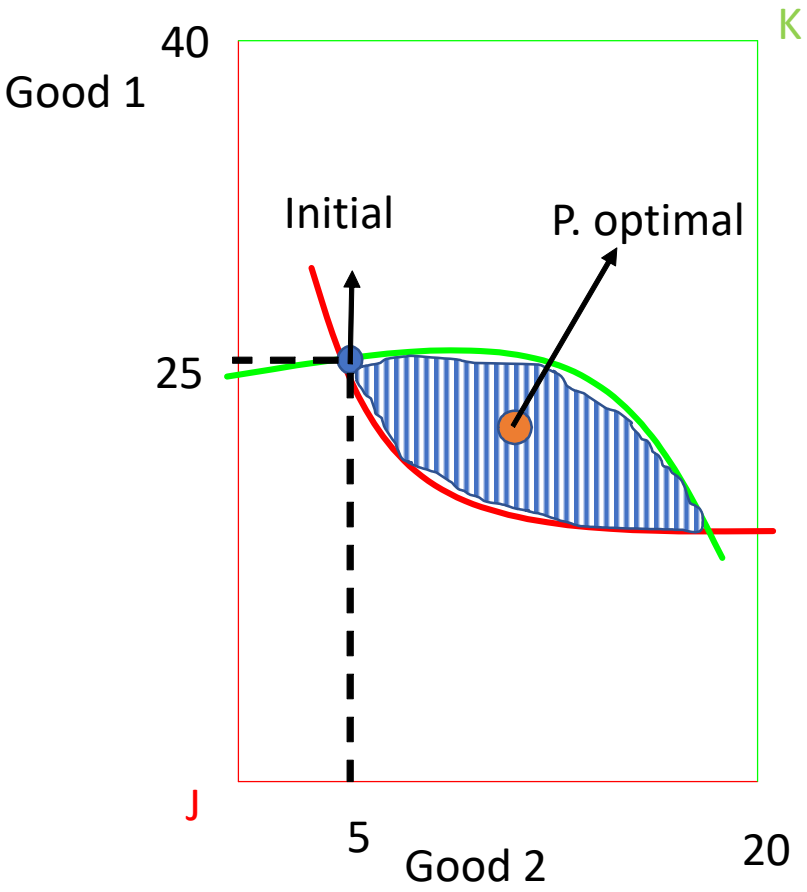
Answer:

Consumer K has $40 - 25 = 15$ units of Good 1 and $20 - 5 = 15$ units of Good 2.

b Draw an Edgeworth box for these two consumers. Assume the consumers have standard shaped indifference curves and that the initial endowment point is *not* a Pareto optimal point.

Answer:

[1] I have used nearly perfectly vertical demand curve rather than perfectly vertical demand curve because with a perfectly vertical demand curve we have a demand function like $Q = constant$ or $Q = 1000$ to choose as specific number. With a perfectly vertical demand curve price is not a function of quantity, so the derivation of MR breaks down.



This Edgeworth box shows the initial starting endowment for consumer J to be 25 of Good 1 and 5 of Good 2. The initial allocation to the consumers is not a Pareto optimal allocation because their indifference curves cross.

c Based on your Edgeworth box in part **b**, identify the set of Pareto improving points.

Answer:

In the figure, the blue shaded area is the set of Pareto improving points (the lens).

d Choose one of those Pareto improving points to be a Pareto optimal point. Explain what conditions need to hold at a Pareto optimal point.

Answer:

At a Pareto optimal point, we need a set of prices such that the marginal rate of substitution of consumer J is equal to the price ratio, as is the marginal rate of subsitution of consumer K, which means that $MRS_J = MRS_K$ at the Pareto optimal point. This result is just an extension of the consumer choice model, except now instead of having the marginal rate of substitution for that single individual to be equal to the price ratio, we have both MRS equal to the price ratio as well as each other.

3. (10 points) Explain why the key result for firm cost minimization, that the ratio of marginal products of inputs to their respective prices, or $\frac{MP_L}{p_L} = \frac{MP_K}{p_K}$ needs to hold at interior solutions. What similarities and differences are there between the process of firm cost minimization and consumer utility maximization?

Answer:

The cost minimization is very similar to the consumer choice problem, except with the cost minimization problem the target level of production is fixed and the goal is to find the lowest cost method of producing that level of output, whereas with the consumer choice problem the budget is fixed and the goal is to find the maximum utility. Graphical representations of these choices look similar. We need $\frac{MP_L}{p_L} = \frac{MP_K}{p_K}$ to hold at an interior solution for similar reasons as we need $\frac{MU_A}{p_A} = \frac{MU_B}{p_B}$ to hold in the consumer choice problem. Rearranging, we have $\frac{MP_L}{MP_K} = \frac{P_L}{P_K}$ in the cost minimization problem and $\frac{MU_A}{MU_B} = \frac{P_A}{P_B}$ in the consumer choice problem. If the ratio of marginal products in the cost minimization problem is not equal to the price ratio, then that means either (1) the output level cannot be produced at that low of a cost or (2) if the output level can be produced at that cost, the cost could be lower. Like in the consumer choice problem, in the production problem we need to find the tangency point between the expenditure line and the curve that tells us the different combinations of resources that will produce a particular quantity (the isoquant).

4. (25 points) Consider an individual who is faced with the lottery of receiving \$8,100 with 15% probability, \$900 with 25% probability, \$64 with 30% probability, or \$0 with 30% probability.

a Calculate the expected value of the lottery.

Answer:

The expected value of the lottery is the weighted average of the dollar amounts of the payoffs of the lottery, where the weights are the respective probabilities that each outcome occurs.

$$EV = 8100 * 0.15 + 900 * 0.25 + 64 * 0.30 + 0 * 0.30 = 1459.20$$

The expected value is $\$1,459.20$.

b Suppose that individual A has a utility function for sure amounts of money of $u_A(x) = \sqrt{x}$. Suppose that individual B has a utility function for sure amounts of money of $u_B(x) = 6 + \sqrt{x}$. What is the expected utility of the lottery for each individual? Are these individuals risk loving, risk neutral, or risk averse? Explain how you know.

Answer:

The expected utility of a lottery is the weighted average of the utilities of the payoffs of the lottery, where the weights are the respective probabilities that each outcome occurs. For individual A:

$$\begin{aligned} EU_A &= \sqrt{8100} * 0.15 + \sqrt{900} * 0.25 + \sqrt{64} * 0.30 + \sqrt{0} * 0.30 \\ EU_A &= 90 * 0.15 + 30 * 0.25 + 8 * 0.30 + 0 * 0.30 = 23.4 \end{aligned}$$

For individual B:

$$\begin{aligned} EU_B &= \left(\sqrt{8100} + 6\right) * 0.15 + \left(\sqrt{900} + 6\right) * 0.25 + \left(\sqrt{64} + 6\right) * 0.30 + \left(\sqrt{0} + 6\right) * 0.30 \\ EU_B &= (90 + 6) * 0.15 + (30 + 6) * 0.25 + (8 + 6) * 0.30 + (0 + 6) * 0.30 \\ EU_B &= 96 * 0.15 + 36 * 0.25 + 14 * 0.30 + 6 * 0.30 = 29.4 \end{aligned}$$

For individual A, the expected utility is 23.4; for individual B the expected utility is 29.4. Technically, they are risk averse because the utility functions are concave, though that is not something we discussed. But both individuals would prefer to receive a monetary amount equal to the expected value of the lottery rather than to face the lottery itself, which means they are willing to "pay" some amount (in terms of reduced payoff) in order to forgo risk. You can see this by comparing the expected utility of the expected value for each individual ($\sqrt{x} = \sqrt{1459.2} = 38.2$ for individual A and $\sqrt{x} + 6 = \sqrt{1459.2} + 6 = 44.2$) to the expected utility of the lottery for each individual (23.4 for individual A and 29.4 for individual B).The expected utility of the expected value is greater than the expected utility of the lottery, so they are risk averse.

c Find the certainty equivalent for this lottery for each individual.

Answer:

The certainty equivalent is the sure dollar amount that gives the individual the same expected utility as the lottery. Let CE_A represent the certainty equivalent for individual A and CE_B represent the certainty equivalent for individual B. For individual A:

$$\begin{aligned} U(CE_A) &= 23.4 \\ \sqrt{CE_A} &= 23.4 \\ \left(\sqrt{CE_A}\right)^2 &= (23.4)^2 \\ CE_A &= 547.56 \end{aligned}$$

For individual B:

$$\begin{aligned} U(CE_B) &= 29.4 \\ \sqrt{CE_B} + 6 &= 29.4 \\ \sqrt{CE_B} &= 23.4 \\ CE_B &= 547.56 \end{aligned}$$

The certainty equivalent for each of these individuals is the same, \$547.56.

d In our standard consumer choice model of utility maximization, in which individuals have utility over goods, individuals with utility functions of $u(x_1, x_2)$ or $u(x_1, x_2) + 6$ or $u(x_1, x_2) - 6$ will make the same choices assuming they have the same budget constraints and face the same prices for goods. Does that result of individuals making the same choices for a utility function that has ± 6 hold for individuals A and B in this example of *expected utility*? Explain referencing the certainty equivalents you found in part **c**.

Answer:

While individuals A and B have different expected utility functions, and different expected utilities for this particular lottery, they have the same certainty equivalents for this lottery. We can take any lottery and show that these individuals will have the same certainty equivalents for those lotteries, so they will have the same preferences over all lotteries even though they have different numerical values for their expected utility of each lottery. As with consumer preferences and utility functions over goods, expected utility is really just a placeholder concept that ranks alternatives; its particular value is only important in ranking lotteries and sure amounts of money.

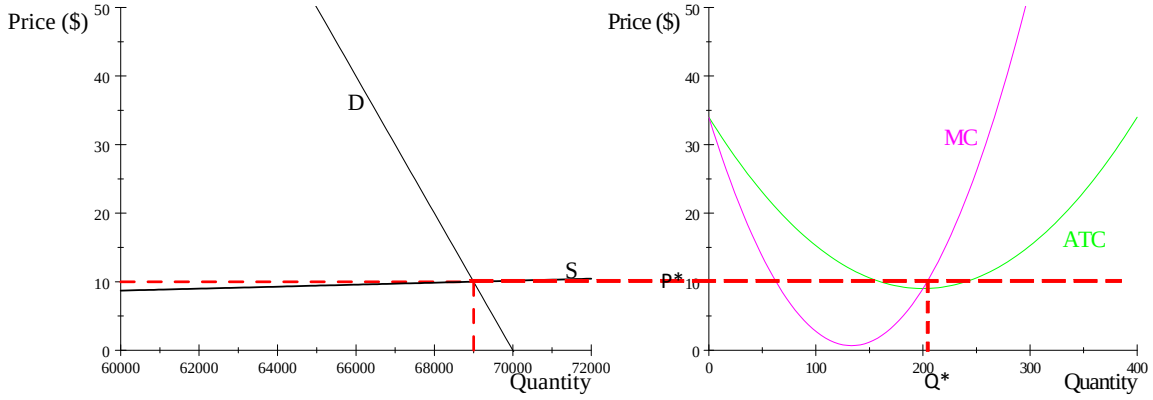
e Suppose there is a third individual, C, who has utility function $u_C(x) = x$. We know this individual is risk neutral, but it is also true that $u_C(x) = (u_A(x))^2$. Will individual C and individual A make the same choices? Explain.

Answer:

Individuals A and C will not make the same choices. The certainty equivalent for individual C is the expected value of $\$1,459.20$, which is much larger than the certainty equivalent for individual A (or, as we know from part **d**, individual B). So for any sure amount of money between \$547.56 and $\$1,459.19$, individual A will prefer the sure amount of money to the lottery, while individual C will prefer the lottery to the sure amount of money. Those are not the same preferences.

As a comparison, with our utility functions over goods, a utility function $(u(\cdot))$ and its square $(u(\cdot)^2)$ will lead to the same preferences, provided the original utility function adheres to our properties of consumer preferences.

5. (20 points) Consider the market supply and market demand model for a perfectly competitive industry below and a representative firm in the perfectly competitive industry. The equilibrium price and quantity for the market are \$10 and $69,000$ units sold. Note that the minimum of the firm's ATC is \$9.



Market on the left; representative firm on the right.

a What is the price that the representative firm will face? Explain.

Answer:

As this industry is perfectly competitive, the firm charges the market price. The market price is \$10, so that price is the firm's price.

b Identify the firm's profit maximizing quantity and price on the graph using the price you found in part **a**.

Answer:

See picture. The price (\$10) is given by the market. The profit-maximizing quantity is the quantity at which MC intersects MR (or the price or demand for the perfectly competitive firm).

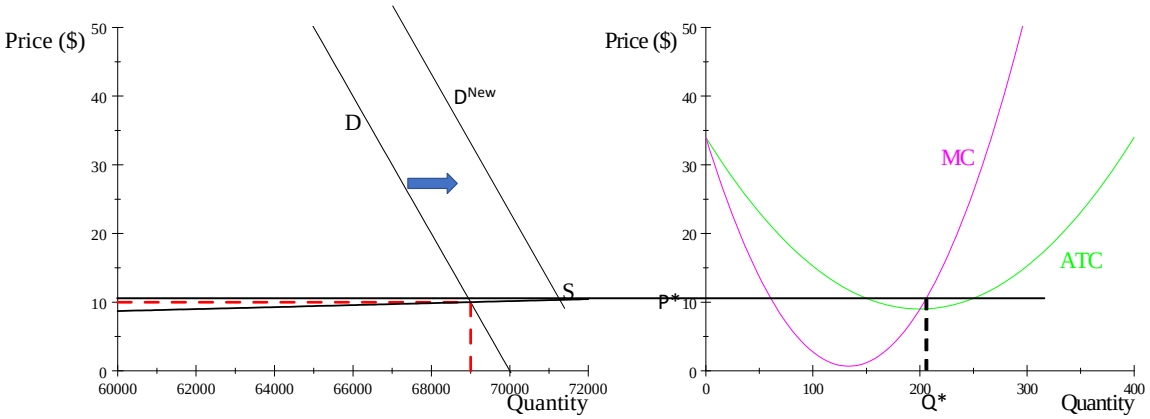
c Is the representative firm in long-run equilibrium? Explain how you know, and if the firm is not in long-run equilibrium explain what would happen in the market to push the firm towards being in long-run equilibrium.

Answer:

The representative firm is not in long-run equilibrium. In order for the representative firm to be in long-run equilibrium, the firm must be earning zero economic profit. However, the market price is \$10 and the minimum of the ATC is \$9, so the firm is earning positive profit (it may not be \$1 per unit sold, but it is some positive number).To push the firm towards long-run equilibrium, other firms would be attracted to this market by the economic profits (supply should increase), which should drive the price down towards \$9, which is the minimum of ATC. It is also possible that as more firms enter the market the demand for the resources needed to produce the good increases, which raises the price of the resources, which causes the cost curve to shift upwards. Those changes depend upon the particular nature of the industry, but ultimately the economic profit of a firm in a perfectly competitive industry should tend towards zero.

d Suppose that a policy change has increased the amount of spendable income consumers have. Explain how this change affects the representative firm's choice of profit-maximizing quantity and price, referencing both the market picture and the representative firm picture.

Answer:



If consumers have more spendable income, then the demand curve in the market should increase, as in the picture. This increase in market demand (assuming supply stays constant) should lead to a higher market price, which should lead to an increase in the profif-maximizing quantity by the firm (because the higher price now intersects the MC further to the right), leading to more economic profit for the firm (assuming costs remain unchanged). Ultimately then we would be back to part **c** of the question, where the increased profits attract more firms and drive economic profit towards zero.