Taxes

October 25, 2022

Taxes are a primary source of revenue for governments. In the U.S., about 90% of federal revenue is tax revenue in one form or another.[1] At a more local level, over 75% of Mecklenburg County revenues are from property and sales taxes alone.[2] Given the large role that taxes play in revenue creation for governments, it is important to understand how taxes affect the incentives of those individuals and organizations being taxed. We will start with taxes on goods and services and then discuss income taxes.

1 Goods and Services Taxes

We begin by analyzing taxes on goods and services. We will discuss two types of taxes, a per-unit tax and a percentage (or sales) tax. Both have fairly similar properties. Our analysis is a partial equilibrium analysis of the tax as we will be examining the effect of the tax on a particular good or service and ignoring the effect on other markets. We also focus on taxes placed on the suppliers of the goods rather than the consumers of the goods, as governments typically collect taxes on goods and services from those who supply them.[3]

1.1 Per-unit Taxes

A per-unit tax is a tax that is collected on each "unit" of a good sold.[4] Per-unit taxes are usually less visible to the consumer than a percentage (or sales) tax because the tax is built into the base price of the good, whereas the percentage (or sales) tax is calculated after totaling the price of the goods purchased. Tobacco, tobacco products, alcohol, and gasoline are all examples of items that have per-unit taxes placed on them.

From our discussion of supply and demand models, if a tax is placed on a good that tax should cause the supply of the good to decrease or shift to the left. The effect on the equilibrium price and quantity should be an increase in price and a decrease in quantity. But we can examine the effects of that tax in greater detail.

Suppose a per-unit tax of τ is placed on the supplier of the good, which means that the supplier will have to pay τ for each unit sold. In our original discussion of supply and demand we had used $P = 5Q_S + 7$ as the inverse supply function. If the tax τ is placed on the good, the supplier then receives the price P for each unit sold but must pay the tax τ, so the inverse supply curve is now:

$$P - \tau = 5Q_S + 7$$

Rewriting that to isolate P we have:

$$P = 5Q_S + 7 + \tau$$

Essentially all a per-unit tax does is shift the intercept of the supply curve upwards, which is consistent with our notion of a decrease in supply. If we let $\tau = \$10$, Figure 1 shows how that tax affects the supply of the

[1] https://www.taxpolicycenter.org/briefing-book/what-are-sources-revenue-federal-government

[2] https://www.mecknc.gov/CountyManagersOffice/OMB/Documents/BudgetFY20-6.%20Financial%20Sources%20and%20Uses.pdf Jump to page 10 to see a table that breaks down the revenue sources.

[3] The results of the analysis do not depend on whether the tax is placed on the supplier or consumer. There are some differences in whether supply or demand is affected based on whether the tax is placed on the supplier or consumer.

[4] By unit means there is typically a standard measure of the item based on weight or volume upon which the tax is levied. This standardization removes the possibility of one retailer selling a "unit" that is four times the size of a standard unit in an attempt to reduce the taxes paid.

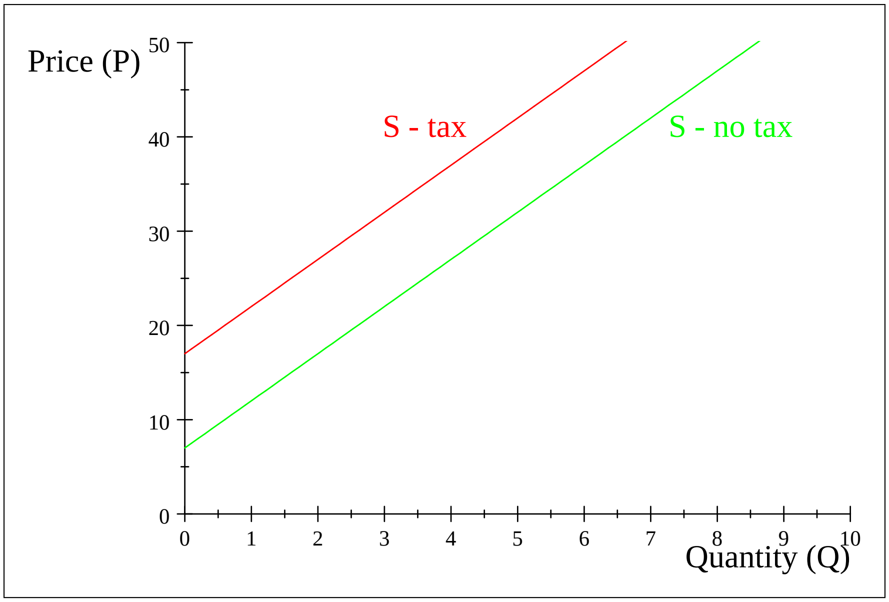

Figure 1: The effect of a tax on the supply of a good.

good.[5]

1.1.1 Equilibrium

In order to determine the effect on the equilibrium we need to add demand to the model. In our original discussion we used $P = 37 - 5Q_D$ as the inverse demand function, and combining that with the inverse supply of $P = 5Q_S + 7$ we had a price of \$22 with 3 units sold in the market. In our new analysis with the tax and inverse supply of $P = 5Q_S + 7 + \tau$, we have:

$$\begin{aligned} 5Q_S + 7 + \tau &= 37 - 5Q_D \\ 5Q + 7 + \tau &= 37 - 5Q \\ 10Q + \tau &= 30 \\ \text{If } \tau &= 10 \\ 10Q + 10 &= 30 \\ 10Q &= 20 \\ Q &= 2 \end{aligned}$$

When $Q = 2$, $P = \$27$. Figure 2 shows the resulting equilibrium outcome. At first glance it appears that not much is different from any other supply change – an underlying market condition has changed and led to a decrease in supply, which leads to an increase in equilibrium price and a decrease in equilibrium quantity. However, the supplier does not get to keep the entire \$27 that is collected from the consumer; the supplier must pay the tax, \$10 per unit sold, to the government. The per-unit tax drives a wedge between the price the supplier receives and the price the consumer pays. If we add the pre-tax inverse supply curve back to the picture we see additional effects of the tax. Figure 3 includes the pre-tax supply curve that shows that the price the supplier actually receives is \$17.

We can decompose the effect of the tax on the gains from trade into consumer surplus, producer surplus, deadweight loss, and, because this supply shift was caused by a tax, government revenue. Figure 4 shows the consumer surplus in green, the producer surplus in blue, the deadweight loss in red, and the government

[5] When creating examples I am using tax amounts that are likely to be high by most standards. The purpose of those high tax amounts is to show better separation between the no-tax and tax supply curves.

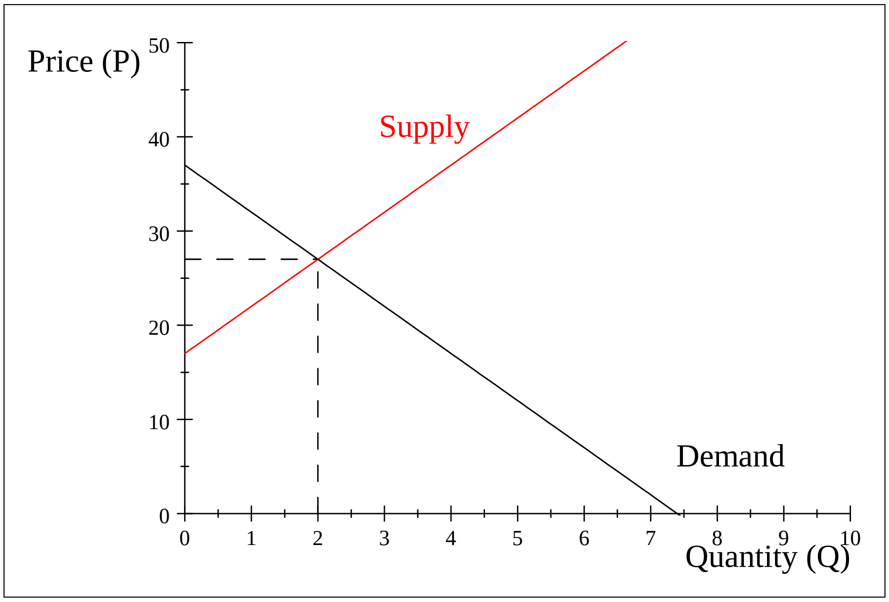

Figure 2: The resulting equilibrium after a per-unit tax is levied.

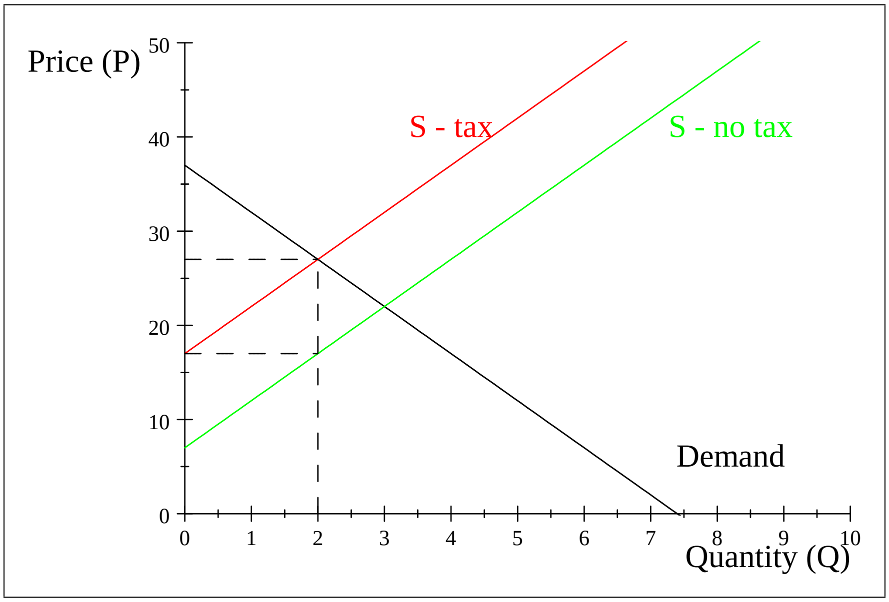

Figure 3: The full effect of a per-unit tax on a good.

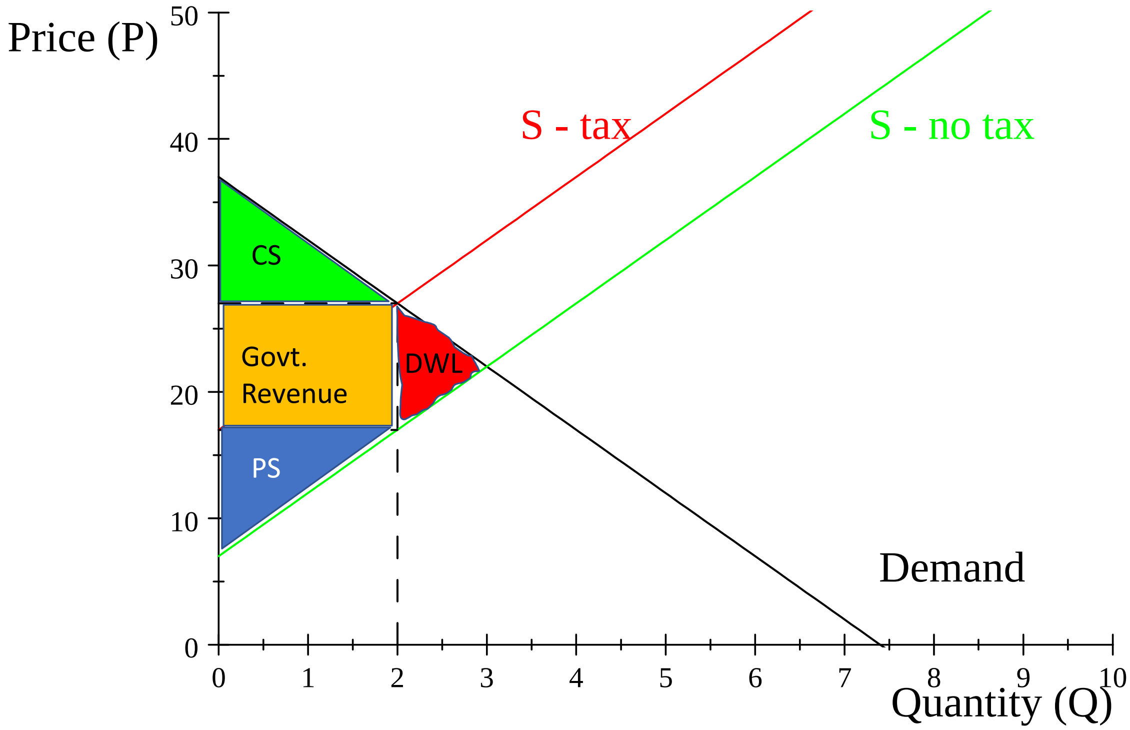

Figure 4: The decomposition of gains from trade after a per-unit tax is levied.

revenue in orange. It is straightforward to check that the orange rectangle is the government revenue, as there is a tax of \$10 levied on each unit sold and there are 2 units sold. The government revenue should be \$20, which it is because the area of the rectangle is given by the product of the difference (\$10) between the price the consumer pays (\$27) and the price the supplier receives (\$17) and the quantity sold (2).

Deadweight loss now appears in this market because the tax is preventing the market from producing at where society's marginal benefit equals society's marginal cost. There are arguments that could be made that the utility generated by the redistribution of the government revenues offsets the loss of utility from additional transactions in the market, so that the tax is welfare enhancing from society's perspective. There are also arguments that could be made that by increasing the cost of an item consumers substitute away from that item. Those arguments are typically used for levying taxes on goods (such as tobacco or alcohol) that might have private costs that are unrecognized by the consumer or on goods (such as gasoline) that might produce negative externalities.

1.2 Percentage Taxes

An alternative to per-unit taxes is to charge a tax that is a specified percentage of the sales price. We typically call this type of tax a sales tax. Many of the goods we purchase have a sales tax associated with them, with the tax percentage depending upon the specific type of good and the location of the purchase. These taxes tend to be more visible to the consumer as there is typically a line item on a receipt that informs the consumer of exactly how much tax has been paid.[6] As with the per-unit tax, our analysis will assume that the supplier is collecting the tax.[7]

Let ρ be the sales tax placed on a good. Let the pre-tax inverse supply curve be $P = 20 + \frac{1}{2}Q_S$. However, if there is a tax of $1+\rho$ on the price of the good, if the price P that the consumer pays includes the tax the seller will only receive $\frac{P}{1+\rho}$. As an example, if the tax is 10% of the sales price and the price the consumer pays is \$11, the seller will only receive $\frac{\$11}{1+.1} = \$10$.

As with a per-unit tax, the sales tax will decrease supply and shift the curve to the left. Unlike a per-unit tax, the shift of the supply curve will not be a parallel shift. Letting $P = 20 + \frac{1}{2}Q_S$ be our inverse supply function, if there is a sales tax then our new inverse supply function that incorporates the tax will be:

$$\begin{aligned}
\frac{P}{1+\rho} &= 20 + \frac{1}{2}Q_S \\
P &= \left(20 + \frac{1}{2}Q_S\right)(1+\rho) \\
P &= 20(1+\rho) + (1+\rho)\frac{1}{2}Q_S
\end{aligned}$$

We can see that the sales tax will affect both the intercept and the slope of the supply curve. Let the sales tax $\rho = \frac{2}{3} = 0.66667$. Figure 5 shows the supply curves with and without the tax. As the price increases, the dollar amount paid in taxes increases.

Suppose that our inverse demand function is $P = 90 - 2Q_D$. We can solve for the equilibrium price and quantity before and after the tax. Before the tax, we have:

$$\begin{aligned}
90 - 2Q_D &= 20 + \frac{1}{2}Q_S \\
90 - 2Q &= 20 + \frac{1}{2}Q \\
70 &= 2.5Q \\
28 &= Q
\end{aligned}$$

[6] There are instances in which these taxes may be collected but not as visible to the consumer. The purchase price of an item in a vending machine typically includes any applicable sales tax.

[7] I find it easier to calculate the supply curve under taxation than to calculate the demand curve under taxation. Multiplying the price of the supplier by $1 + \textit{tax rate}$ is conceptually easier than dividing the price of the purchaser by $1 + \textit{tax rate}$, and mathematically it will not matter whether the supplier or consumer is taxed. Also, the seller is typically the one who collects the tax in the U.S.

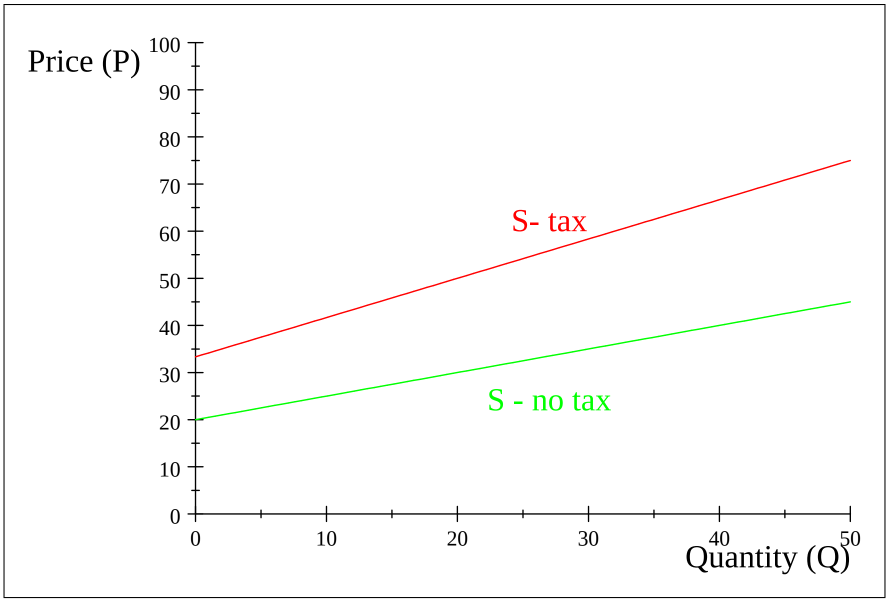

Figure 5: The supply curves before and after a sales tax is levied.

When $Q = 28$, $P = \$34$. When the tax is imposed we have:

$$\begin{aligned}
90 - 2Q_D &= \left(20 + \frac{1}{2}Q_S\right)\left(1\frac{2}{3}\right) \\
90 - 2Q &= \left(20 + \frac{1}{2}Q\right)\frac{5}{3} \\
270 - 6Q &= 100 + \frac{5}{2}Q \\
170 &= \frac{17}{2}Q \\
20 &= Q
\end{aligned}$$

When $Q = 20$, $P = \$50$. However, that \$50 is the price that the buyer pays. The seller receives $\frac{P}{1+\rho}$, which is $\frac{\$50}{1\frac{2}{3}} = \$30$. Given that the sales tax is $\frac{2}{3}$ of the sales price, the supplier is sending \$20 to the government. Figure 6 shows the full effect of the sales tax, including the before and after tax supply curves.

As with the per-unit tax, we can decompose the gains from trade in the sales tax market into consumer surplus, producer surplus, government revenue, and deadweight loss. Figure 7 shows the consumer surplus, producer surplus, government revenue, and deadweight loss in the market after a sales tax has been levied. While there are some differences between the imposition of a per-unit tax and a sales tax, from the perspective of economic efficiency the analysis is fairly similar. While the tax is a sales tax, we can calculate the government revenue the same as we would for a per-unit tax because the supplier is selling all units of the item at the same price (\$50). The supplier sells 20 units and sends $\frac{2}{3}$ of the sales price (\$20) on each unit to the government, so the government revenue is \$400.

1.3 Generating Government Revenue

Consider goods that tend to have high tax rates – they tend to be goods with negative externalities or goods that have some private cost that does not seem to be fully recognized by the consumer.[8] However, if the desire of the government is to generate revenue from the taxes, then how much the equilibrium quantity

[8] In a different time we might call these "sin" taxes because they were placed on goods that were considered sinful.

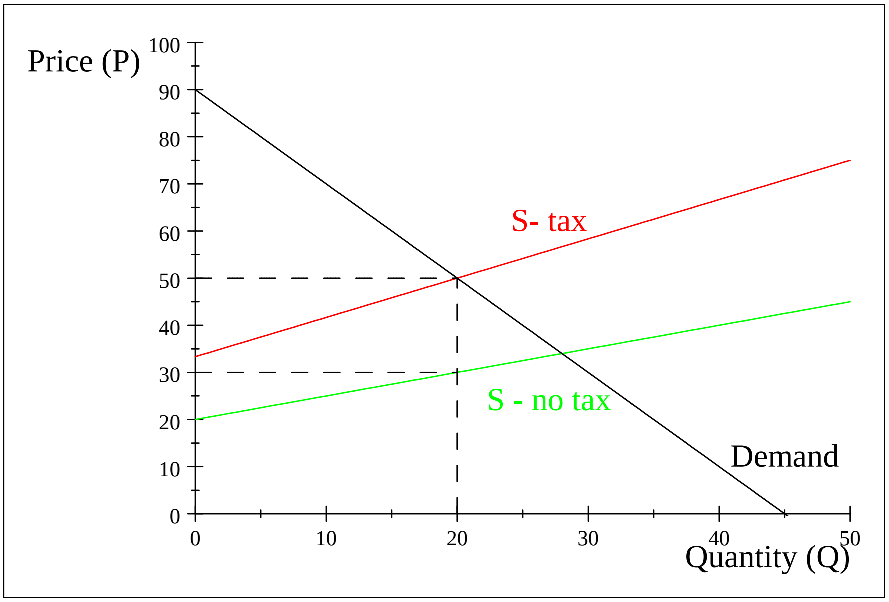

Figure 6: The full effect of a sales tax on a good.

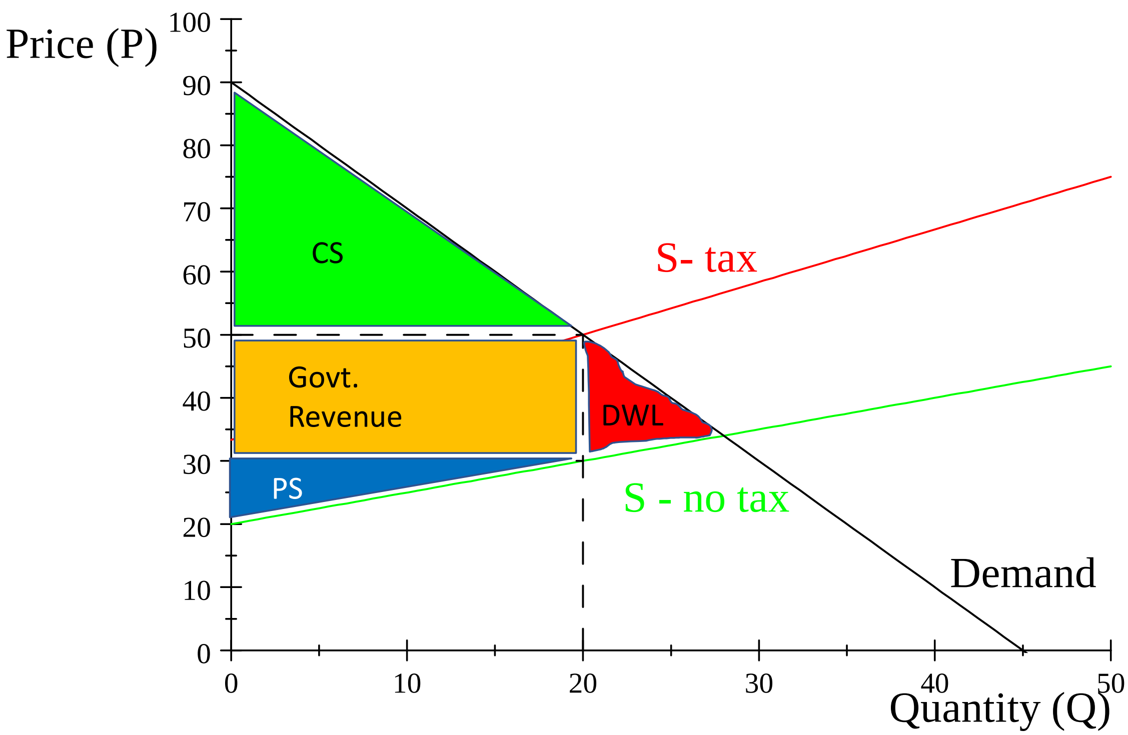

Figure 7: The decomposition of gains from trade after a sales tax is levied.

changes after a tax is imposed is important to consider. If a small tax is going to cause a dramatic reduction in the equilibrium quantity then that tax is unlikely to generate much government revenue; however, if a large tax only causes a small reduction in equilibrium quantity then that tax could generate substantial government revenue.

1.3.1 Defining Elasticity

Economists use the concept of price elasticity of demand (also called own-price elasticity) to determine how much a specific change in a good's price will affect quantity demanded for the good.[9] Price elasticity of demand is defined as the percentage change in the quantity of a good relative to the percentage change in the price of the good. Formally, we have:

$$PED = \frac{\%\Delta Q_D}{\%\Delta P_{own}}$$

A basic formula for percentage change is to subtract the initial amount from the new amount and divide by the initial amount,[10] so we have:

$$\%\Delta Q_D = \frac{Q_D^{new} - Q_D^{initial}}{Q_D^{initial}} \text{ and } \%\Delta P_{own} = \frac{P_{own}^{new} - P_{own}^{initial}}{P_{own}^{initial}}$$

Rewriting our elasticity formula:

$$\begin{aligned}
PED &= \frac{\frac{Q_D^{new} - Q_D^{initial}}{Q_D^{initial}}}{\frac{P_{own}^{new} - P_{own}^{initial}}{P_{own}^{initial}}} \\
PED &= \frac{Q_D^{new} - Q_D^{initial}}{Q_D^{initial}} \div \frac{P_{own}^{new} - P_{own}^{initial}}{P_{own}^{initial}} \\
PED &= \frac{Q_D^{new} - Q_D^{initial}}{Q_D^{initial}} * \frac{P_{own}^{initial}}{P_{own}^{new} - P_{own}^{initial}} \\
PED &= \frac{Q_D^{new} - Q_D^{initial}}{P_{own}^{new} - P_{own}^{initial}} * \frac{P_{own}^{initial}}{Q_D^{initial}}
\end{aligned}$$

Here we can recognize that $\frac{Q_D^{new} - Q_D^{initial}}{P_{own}^{new} - P_{own}^{initial}}$ is just a slope term, so that if we have a demand function (and in this case I specifically mean a demand function written as $Q_D = a - bP$), the price elasticity of demand is simply the product of the slope coefficient and the ratio of initial price to initial quantity, or $PED = -b * \frac{P_{own}^{initial}}{Q_D^{initial}}$. If we have an inverse demand function like $P = \frac{a}{b} - \frac{Q_D}{b}$, which is what we typically have been using, then we would have $PED = -\frac{1}{b} * \frac{P_{own}^{initial}}{Q_D^{initial}}$. It is important to note the difference.[11]

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[9] There are any number of measures of elasticity that economists use: income elasticity, cross-price elasticity, supply elasticity, etc.

[10] An alternative formula for percentage change could be to use the midpoint formula, which would be:

$$\%\Delta Q_D = \frac{Q_D^{new} - Q_D^{initial}}{\frac{Q_D^{new} + Q_D^{initial}}{2}}$$

The standard percentage change formula provides different answers depending upon whether you move from the initial to the new or if you reverse the process and move from the new to the initial. The midpoint formula provides the same answer regardless of whether you move from the initial to the new or the new to the initial.

[11] When reading empirical papers you will often see researchers mention that a coefficient estimate from a regression model is an elasticity. Briefly, if the researcher is estimating a model of the form:

$$\ln Q_D = \beta_0 + \beta_1 \ln P + \varepsilon$$

then the coefficient estimate β_1 is itself is the price elasticity of demand. That can be shown mathematically. I have used quantity and price as the dependent and independent variables, respectively, but elasticity is not a concept that is limited to just prices and quantities.

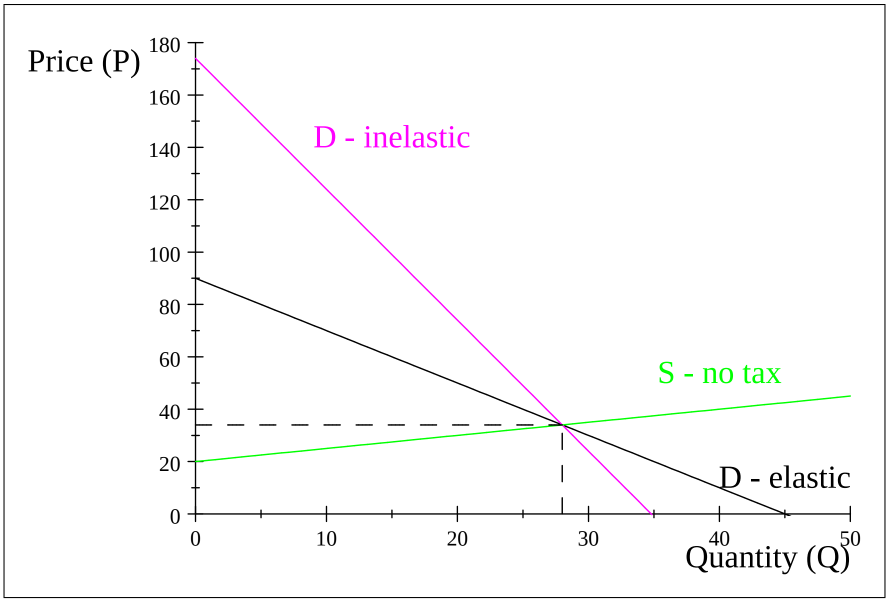

Figure 8: A relatively elastic (in black) and relatively inelastic (in magenta) demand curve.

We can tell from this definition that price elasticity of demand is always negative, and that result is due to the law of demand. Because price elasticity of demand is always negative, we drop the negative sign (technically we take the absolute value).[12] What is likely less clear is that elasticity is a unitless measure – it is just a number. As it is just a number, we need a means of interpreting the resulting number. Our initial goal was to determine how responsive quantity was to a change in price. If $|PED| > 1$, then we have $|\%\Delta Q_D| > |\%\Delta P_{own}|$ and we say that price elasticity of demand is elastic. If $|PED| < 1$, then we have $|\%\Delta Q_D| < |\%\Delta P_{own}|$ and we say that price elasticity of demand is inelastic.

When looking at a demand curve on a graph, curves with steeper slopes are relatively inelastic and curves with flatter slopes are relatively elastic.[13] At the extreme, a demand curve that is perfectly vertical (a specific quantity of the good will be purchased at any price) is called perfectly inelastic; a demand curve that is perfectly horizontal (which we have seen in our perfectly competitive market – any increase in price will cause demand to drop to zero) is called perfectly elastic.

A key determining factor of elasticity is the availability of close substitutes. If there are many close substitutes then demand is likely to be relatively elastic; if there are few close substitutes then demand is likely to be relatively inelastic.

1.3.2 Elasticity and Government Tax Revenue

If a goal of levying a tax in a market is to generate tax revenue for the government,[14] then, assuming similar sized markets, the market that should be taxed is the one with the more inelastic demand. Intuitively, if elasticity measures how responsive the quantity is to a price change, a government official would want to levy the tax in the market that will have a lesser change in quantity. In our sales tax example we had our inverse demand function as $P = 90 - 2Q_D$ and our before tax inverse supply function as $P = 20 + \frac{1}{2}Q_S$. Our equilibrium price and quantity before the tax was imposed was $Q = 28$, $P = \$34$. Suppose instead that our inverse demand function is $P = 174 - 5Q_D$. Figure 8 shows these two demand curves. Figure 8 also shows our original inverse supply function of $P = 20 + \frac{1}{2}Q_S$, and both markets have the same original equilibrium price and quantity of $Q = 28$ and $P = \$34$.

[12] For other measures of elasticity, particularly income elasticity and cross-price elasticity but possibly others as well, a negative sign is meaningful. Do not simply dismiss a negative sign for any type of elasticity.

[13] We use the term "relatively" because with linear demand functions the elasticity changes along the demand curve. We will not delve into those details.

[14] Again, we are only considering taxes levied in a market at this point. We will discuss income taxes soon.

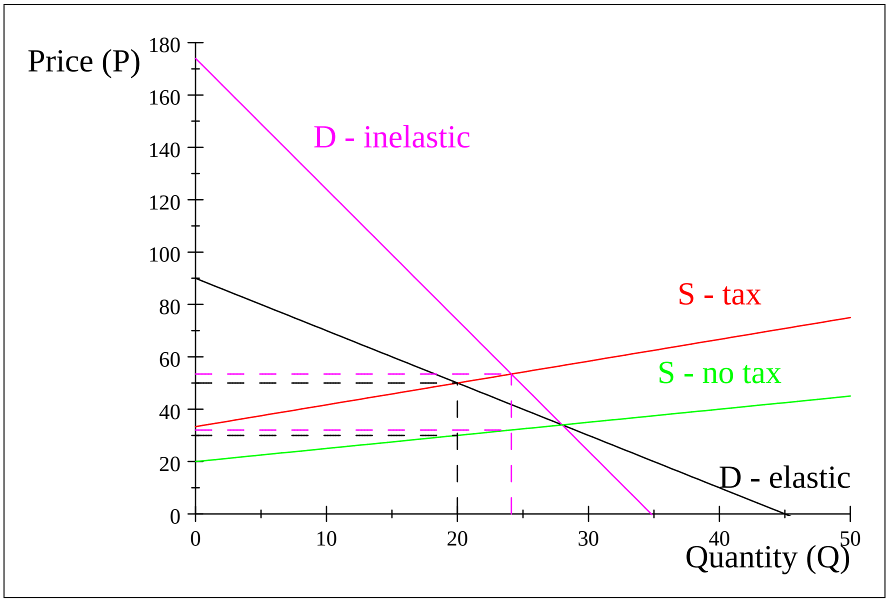

Figure 9: A relatively elastic (in black) and relatively inelastic (in magenta) demand curve.

Our tax rate in the original example was $\rho = \frac{2}{3}$. Figure 9 shows the after-tax equilibrium for both markets. Under relatively elastic demand, we have already seen that $Q = 20$, $P = \$50$ is the price the buyer pays, and $P = \$30$ is the price the seller collects. The seller sends \$20 per unit to the government and government tax revenue equals \$400. Under the relatively inelastic demand, we now have an after-tax equilibrium of:

$$\begin{aligned}
174 - 5Q_D &= \left(20 + \frac{1}{2}Q_S\right)\left(1\frac{2}{3}\right) \\
174 - 5Q &= \left(20 + \frac{1}{2}Q\right)\frac{5}{3} \\
522 - 15Q &= 100 + \frac{5}{2}Q \\
422 &= \frac{35}{2}Q \\
\frac{844}{35} &= Q
\end{aligned}$$

With $Q = \frac{844}{35} \approx 24.1$, we have the price that the buyer pays as $P = 174 - 5\left(\frac{844}{35}\right) = \frac{374}{7} \approx \$53.43$. The price the seller receives is $P = \frac{374}{7} / \frac{5}{3} = \frac{1122}{35} \approx \$32.06$. So more units are sold in the market with relatively inelastic demand, and the government collects more dollars per unit (about \$21.37) in the market with relatively inelastic demand, thus total tax collections are approximately \$515.07. Figure 9 shows a geometric comparison of the tax revenue generated under the relatively elastic and relatively inelastic demand curves. The orange rectangle shows tax collections that are common to both markets. The thin black rectangle below the orange rectangle shows tax revenue that is unique to the market with the relatively elastic demand curve. The areas shaded in magenta show tax revenue that is unique to the market with the relatively inelastic demand curve. While we have already seen the math that shows the relatively inelastic demand curve generates more tax revenue, I find it instructive to view the geometric representation as well.

Again, note that the comparison only holds for markets of similar size. It is possible to have a market with a relatively elastic demand curve that will generate more revenue than one with a relatively inelastic demand curve if the volume of transactions in the market with the elastic demand curve is much larger than that of the other market. The degree of elasticity of the demand curves relative to each other is also important in determining which market will generate more revenue. But in thinking generally about taxing

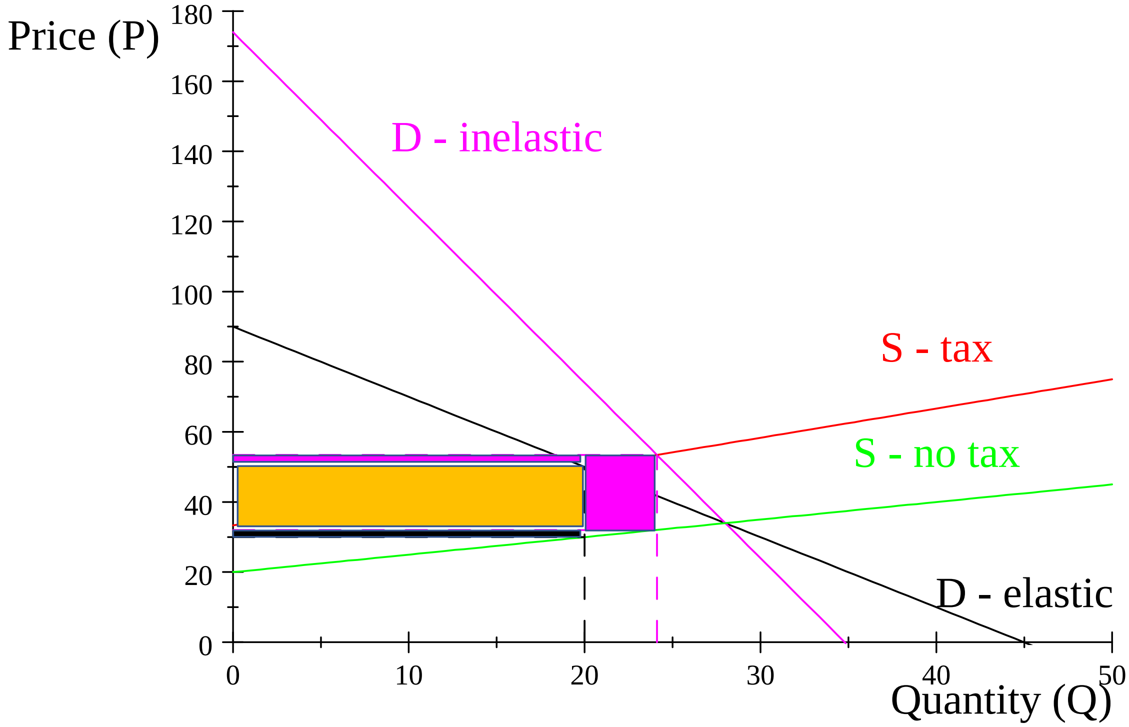

Figure 10: A geometric comparison of sales tax revenue generated under the relatively elastic and relatively inelastic demand curves. The orange rectangle is the common revenue generated. The thin black line (narrow rectangle) below the orange rectangle is revenue only collected under the relatively elastic demand curve; the magenta rectangles above and to the right of the orangle rectangles are tax revenue collected only under the relatively inelastic demand curve.

goods, those with inelastic demand should generate more revenue than those with elastic demand. Returning to some specific examples, tobacco, alcohol, gasoline, and similar type goods are fairly heavily taxed. They have two points in their favor for generating tax revenue, both of which are related to elasticity. The first is that people tend not to change their purchasing behavior much when the price of these items changes by small amounts. The second is that the way the taxes are defined is broad. Shell gasoline has any number of substitutes because Shell gasoline has competitors; gasoline itself has very few competitors. By taxing the broad class of goods the number of substitutes is limited. Someone who wants to drink a beer could purchase a homebrew kit, which may have lower taxes than purchasing a finished product. But homebrewing has costs (time, space, cost of materials to homebrew, etc.) which may outweigh the savings from avoiding the tax on the finished product.

2 Income Taxes

At the opening of these notes it is mentioned that taxes make up about 90% of federal government revenue in the U.S and about 50% of the federal government revenue is from individual income tax. Given the importance of the individual income tax as a revenue source for the federal government, it is important to understand how taxation effects decisions of individuals.

The relationship between individual income tax rates and tax revenue collected is not linear. At an income tax rate of 0% the government would collect $0 in income tax revenue. As the rate increases we would expect there to be a positive relationship between income tax rates and taxes collected, at least until a certain point. Consider the other extreme of an income tax rate of 100%. One argument would be that income taxes collected would return to $0 because no one would work in a position that had a taxable income because there is no incentive to work if one cannot retain some of the earnings. People would likely still "work," but the work would be in a position not subject to taxation – perhaps a return to self-sufficient family farms. At some tax rate between 0% and 100% individuals would shift from working hours in a position with taxable income, which generates tax revenue, to some other activity, which could be working hours in a position not subject to taxation or to leisure, which does not generate tax revenue.

The Laffer Curve depicts the relationship between the tax rate and tax collections.[15] While the Laffer Curve generally does not get good press, the underlying concept is sound.[16] In the model, there is an optimal tax rate such that government tax collections are maximized, and debates are generally about whether current tax rates are above or below that optimal rate. Figure 11 illustrates three different possible Laffer Curve relationships.[17] As Figure 11 shows, a tax rate increase or decrease could increase or decrease tax collections depending on whether the current tax rate is above or below the optimal rate. If the current tax rate is to the left of the optimal tax rate, then increasing the tax rate would generate more revenue; if the current tax rate is to the right of the optimal tax rate, then decreasing the tax rate should generate more revenue. As the concept is still used in debates about tax policies, it is useful to have an understanding of the hypothesized relationship between tax rates and tax collections and to understand the main issue of the debate. Also, it motivates a discussion about the tradeoffs an individual faces in the presence of proposed tax policies.

2.1 Tax Systems

The "why" of tax policy is fairly straightforward – taxes are in place to provide revenue to the governing authority. Which system to implement and how to implement that system is a much more complicated question.[18] There are two general methods for collecting income taxes. One is that the income tax could be

[15] This idea is named for Arthur Laffer, who popularized the idea, though it certainly predates him.

[16] There is current research that still attempts to estimate Laffer Curves:
Miravete, Seim, Thurk, 2018, *Econometrica* https://onlinelibrary.wiley.com/doi/full/10.3982/ECTA12307
Ferreira-Lopes, Martins, Espanhol, Bul, 2019, *Buletin of Economic Research*
https://onlinelibrary.wiley.com/doi/10.1111/boer.12211
Trabandt and Uhlig, 2011, *Journal of Monetary Economics*
https://www.sciencedirect.com/science/article/pii/S030439321100064X?via%3Dihub

[17] You may also see the Laffer Curve presented with the tax rate and government revenue axis switched.

[18] Mirrlees 1971 *Review of Economic Studies* article is generally considered a seminal paper in optimal taxation:
https://www.jstor.org/stable/2296779

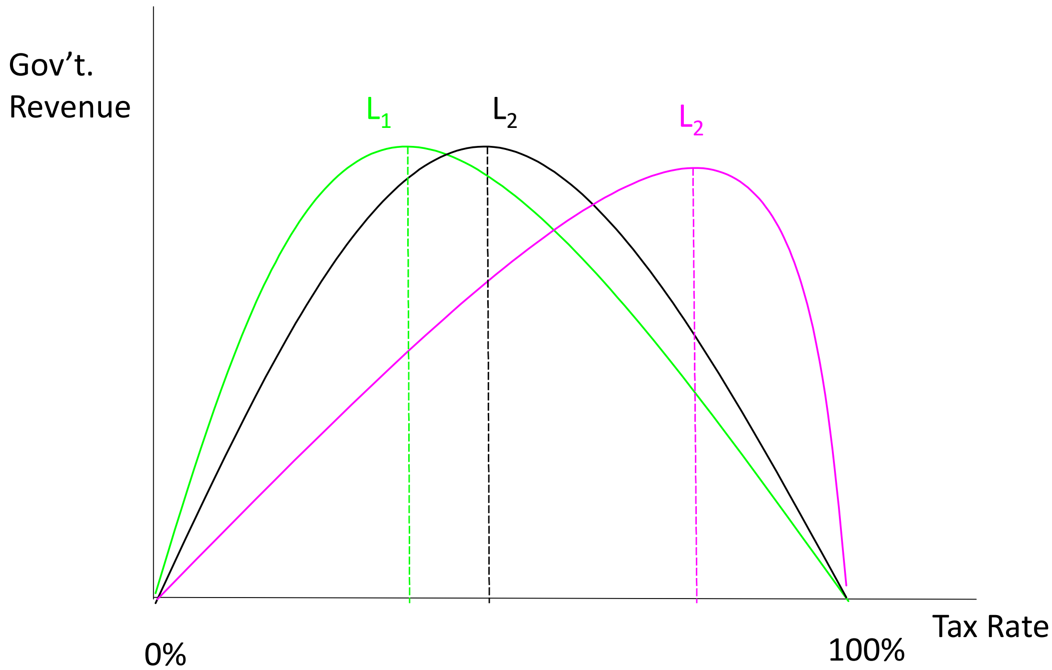

Figure 11: Three possible Laffer Curve relationships.

a lump sum and the other is that taxes could be collected on a percentage basis. We are abstracting from how to define income and what type of income is taxable and at what rates. The goal is to provide some general results of how taxes affect behavior.

2.1.1 Lump Sum Taxes

A lump sum tax is a tax of a specified amount that each individual pays regardless of the decisions made by that individual.[19] Lump sum taxes are nondistortionary, at least locally, because the only way to avoid the tax is to move out of the jurisdiction that is imposing the tax. When a lump sum tax is compared to a tax on a particular good, we can show that a lump sum tax can generate at least as much revenue for the government as a per-unit or percentage tax, and leave the individual no worse off than they would be with the tax on the good.

Figure 12 shows pre-tax and post-tax budget constraints for an individual in a two good economy when a tax is placed on Good 2. As a tax is effectively a price increase, the budget constraint swings inward as it would for any other price increase.

Figure 13 shows the indifference curves pre- and post-tax. We know that after the tax is imposed a consumer will choose a bundle on an indifference curve that provides lower utility to the consumer – the result can be seen by comparing the black indifference curve with the red indifference curve in Figure 13, and we cannot change that result. A question to consider is whether this particular tax system generates any excess burden of taxation[20] compared to other tax systems. But first we need to know how much tax is collected here. Under the pre-tax budget constraint, if the consumer were to purchase Q_2 units of Good 2,

Be forewarned that it uses mathematical concepts that we have not developed in this course.

Saez 2001 Review of Economic Studies article uses elasticities to derive optimal income tax rates: https://www.jstor.org/stable/2695925

If you are interested in this type of research, Dr. Musab Kurnaz, in the Department of Economics, has a research agenda focused on taxation.

[19] You might also see this type of tax referred to as a head tax, as each person (or "head") must pay a specified amount.

[20] When comparing losses between tax systems the term "excess burden" is used. It is a similar concept to deadweight loss, although the starting point for the loss calculation is typically the "efficient tax."

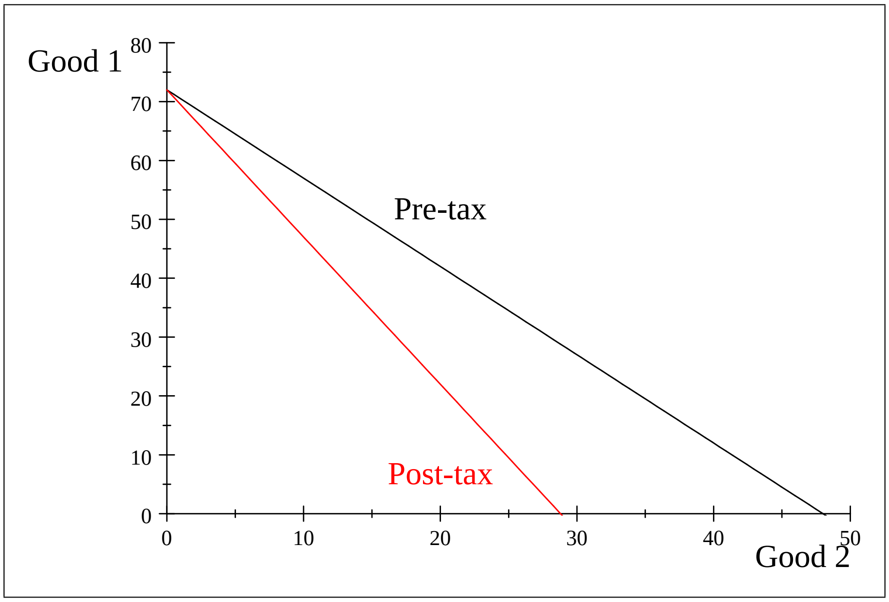

Figure 12: Pre- and post-tax budget constraints when there is a tax (effectively a price increase) on Good 2. The pre-tax budget constraint is in black; the post-tax budget constraint is in red.

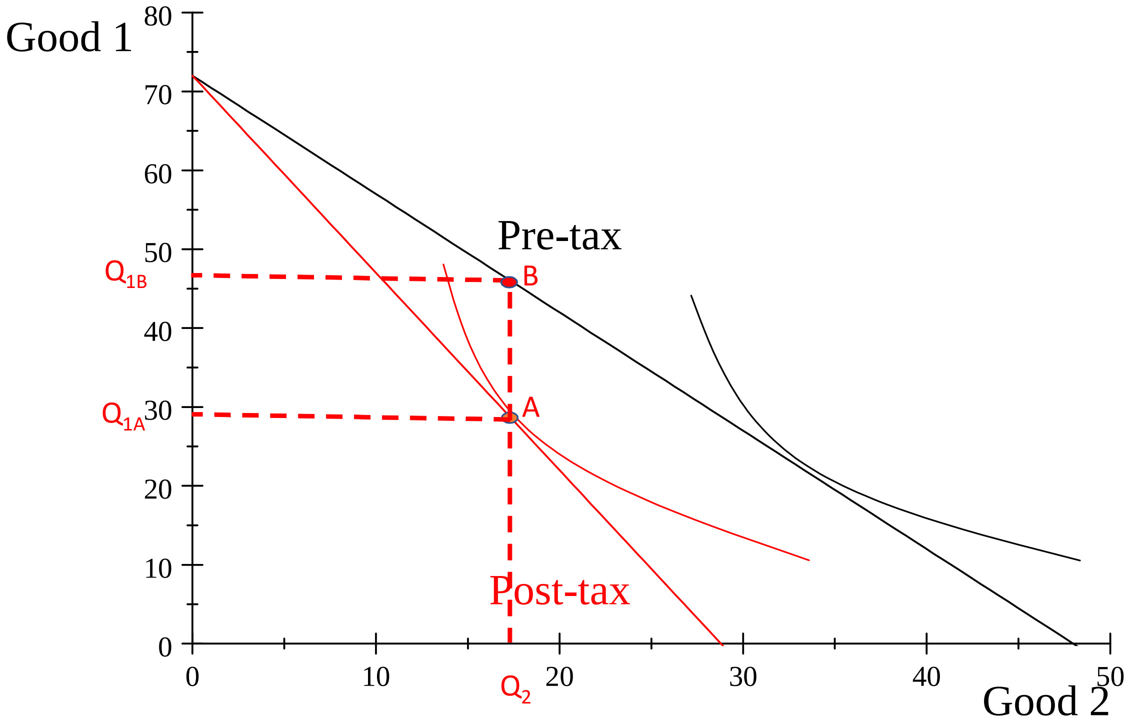

Figure 13: The tax collected, in units of Good 1, from levying a tax on Good 2.

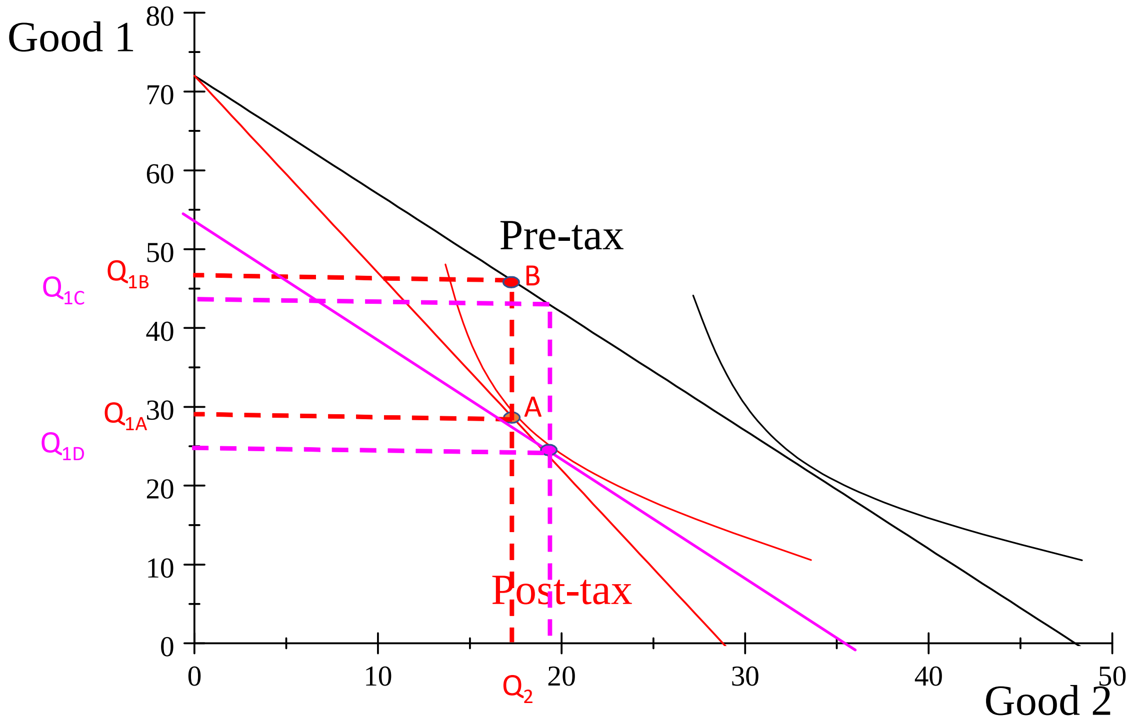

Figure 14: The magenta line shows the equivalent variation that leaves the consumer on the same post-tax indifference curve as the tax on Good 2.

the consumer could have purchased Q_{1B} units of Good 1. However, under the post-tax budget constraint, the consumer chooses the bundle that has Q_2 units of Good 2 but can only purchase Q_{1A} units of Good 1. That difference, $Q_{1B} - Q_{1A}$, is the tax collected by the government, at least in units of Good 1. If we so desired we could turn that amount into dollar value by multiplying the number of units of Good 1 given up by the price of Good 1, but it is more instructive to leave the tax collected in terms of Good 1 for now. It is important to note that the difference between the budget constraints, and hence the amount of tax collected, increases as the amount of Good 2 purchased increases.

Thus far the analysis has little to do with lump sum taxes. However, suppose that instead of taxing Good 2, the government took away an amount of income that would leave the consumer on the same post-tax indifference curve in Figure 13. When taking away a specific amount of income, the budget constraints should shift inward parallel to the pre-tax budget constraint. This idea should sound familiar because it is just the equivalent variation, which we discussed in the notes on cost-benefit analysis. Figure 14 shows the equivalent variation that leaves the consumer on the same post-tax indifference curve as the tax on Good 2. The amount of the tax collected under this lump sum collection method is more than the amount collected under the tax on Good 2 and the consumer has the same utility under both methods. While it may not be clear from the picture that $Q_{1C} - Q_{1D} > Q_{1B} - Q_{1A}$, it can be proved mathematically that at least as much revenue is collected under the lump sum method than by imposing a tax that changes the relative prices of the good, assuming that indifference curves have the standard shape.[21] You can also (barely – Figure 14 is not the best illustration) see it in Figure 14. The distance between Q_{1C} and Q_{1D} is the same as the distance between Q_{1B} and the corresponding point on the equivalent variation line at Q_2. That point is somewhere

[21] If indifference curves have non-standard shapes then the analysis may not hold. Intuitively, because the slope of the pre-tax budget constraint (and, by definition, equivalent variation) is flatter than the post-tax budget constaint, the optimal choice under the equivalent variation should be to the right of the optimal choice under the post-tax budget constraint. If that result occurs, then there is a guarantee that the equivalent variation yields more revenue than the tax on Good 2.

between Q_{1A} and Q_{1D}. The excess burden of the tax on Good 2 would be that small difference between Q_{1A} and the corresponding point on the equivalent variation line at Q_2.

The lump sum method is Pareto improving over the tax on Good 2 because the consumer is no worse off and the tax revenue collected is greater. Approaching the problem differently, if the goal of the government was to raise tax revenue of $Q_{1B} - Q_{1A}$, they could impose a lump sum tax of that amount and the consumer would be better off (on a higher indifference curve) than under the scenario where there is a tax on Good 2. Again, that would also be Pareto improving compared to taxing Good 2.

If lump sum taxes are efficient, why are they not readily used in practice? Politically they tend to be a non-starter because the politician is essentially saying that the richest people in the jurisdiction owe the same amount of taxes as the poorest people. They are equitable in that everyone pays the same amount but regressive in that lower income individuals are paying a larger (perhaps much larger) portion of their income in taxes. One could argue that the lump sum tax be income based for bands of income, but even then the tax would distort decisions around the boundaries of those bands. If someone making $\$50,000$ pays $\$5,000$ as a lump sum tax but someone making $\$49,999$ pays $\$2,500$ as a lump sum tax, that changes the decision-making process for those individuals who would make between $\$50,000$ to $\$52,500$. In the next section we will examine how income taxes may affect labor-leisure decisions.

2.1.2 Marginal Rates of Taxation and Labor-Leisure Decisions

The model that we will use for analyzing the effect of marginal rates of taxation is the consumer choice model applied to labor-leisure decisions. As mentioned at the beginning of the semester, we use these models to study markets for economic goods and if we have an economic bad we can typically reframe the bad as a good by taking the opposite. In the labor-leisure model, the consumer makes a decision about how many hours to work (or labor) each period (day, week, month, year, etc.). Working is generally viewed by the individual as an economic bad (the individual would rather be doing something else), so to turn "work" into an economic good we use the opposite of work: leisure. The tradeoff the individual then faces is between leisure hours (which do not generate income) and work hours (which do generate income). Note that any hours that do not generate income are counted as "leisure," even if those hours are sleep or cleaning the house or running errands.

Labor Supply Curve We begin by creating a two good economy where the goods are leisure and income. Leisure is a special good in that time periods are bounded – there are only 24 hours in a day, so no one can take more than 24 hours of leisure in a day. There are seven days in a week, so the maximum number of leisure hours per week is 168. The key is that maximum leisure time is fixed and we will denote the maximum amount of leisure that can be taken by any individual as T. We will let n denote the hours of leisure and h denote the hours of work, so that $h = T - n$.

Income is now created by working (taking non-leisure hours) rather than given to the individual. We will assume that there is an hourly wage, w, that the individual can earn for each hour of work.[22] The individual's income, Y, is determined by the product of hours worked and the wage rate, $Y = h * w$. We will have leisure hours on the x-axis and income earned on the y-axis of our consumer choice model. Figure 15 shows the budget constraints for a \$35 hourly wage (black line) and a \$15 hourly wage (red line) for a time period of a day (24 hours). The budget constraints can be created using the following relationships:

$$Y = wT - wn$$

Note that wT is a constant and represents the y-intercept; the slope of the line is $(-w)$, which is just the negative of the wage rate. For each hour of leisure taken, the individual gives up one hour of the wage.

Figure 16 shows the optimal labor-leisure decisions for this individual for the \$35 wage and \$15 wage. When the wage is \$35 the individual takes 14 hours of leisure per day, works 10 hours per day, and earns an income of \$350 per day. When the wage is \$15 the individual takes 16 hours of leisure per day, works 8 hours per day, and earns an income of \$120 per day. Recall that we can create demand curves for goods from this indifference curve analysis. For the labor-leisure decision, we know the price of an hour of labor (the wage) as well as how many hours of leisure the individual takes at that wage, so we could create a demand curve

[22] While many individuals have salaried positions, a salaried position complicates this analysis considerably.

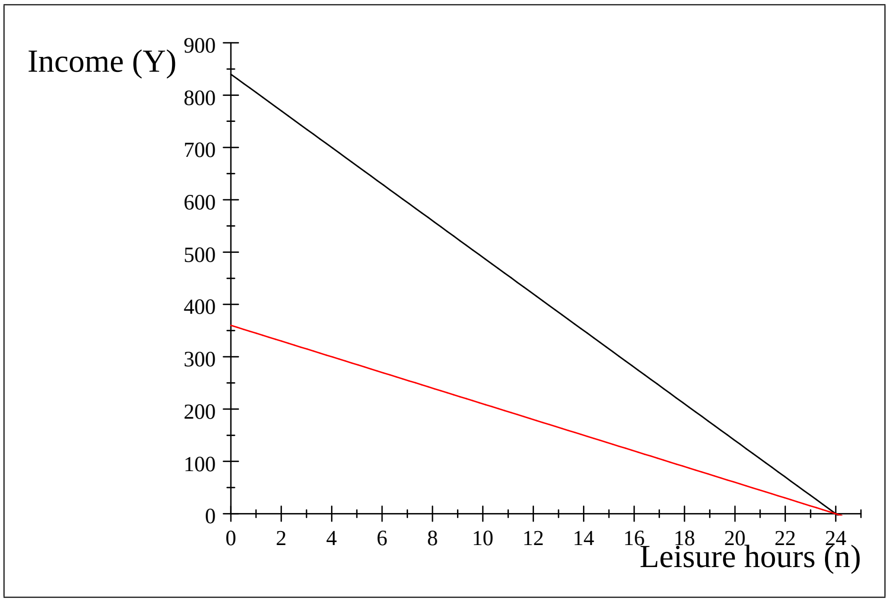

Figure 15: Budget constraints for a labor-leisure model. The black line is for a wage of \$35/hour; the red line is for a wage of \$15/hour.

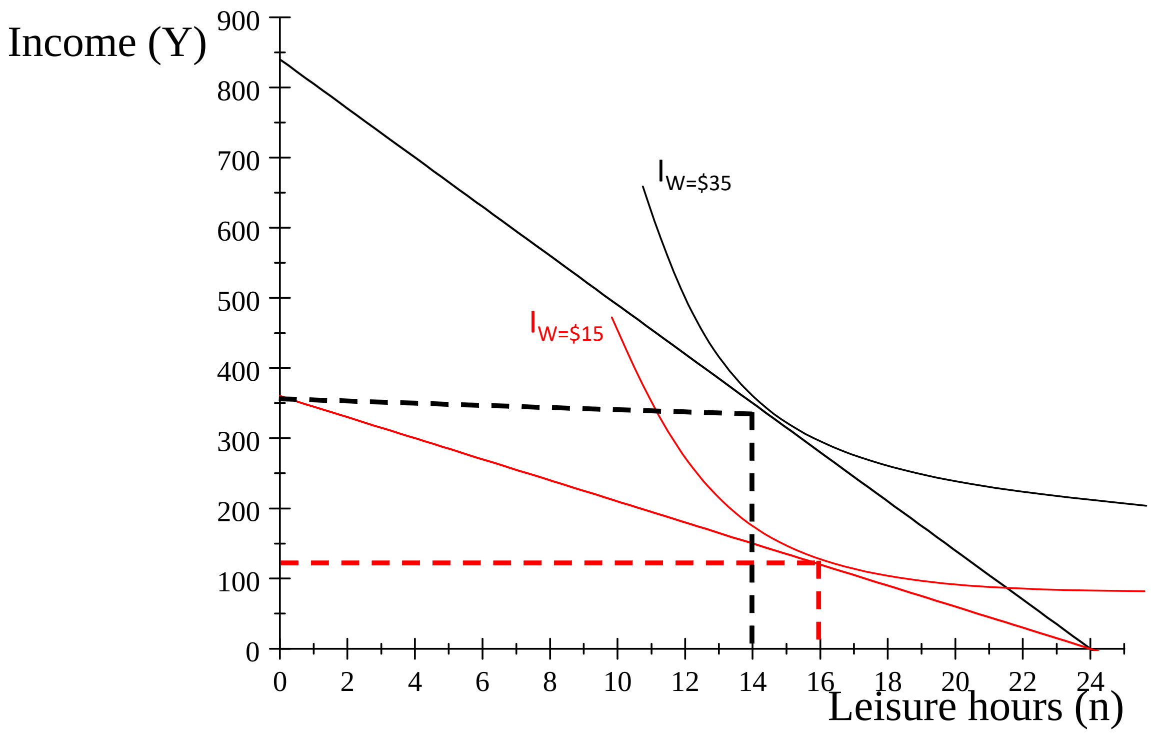

Figure 16: Optimal labor-leisure decision for this individual under the \$35 wage (black line) and \$15 wage (red line).

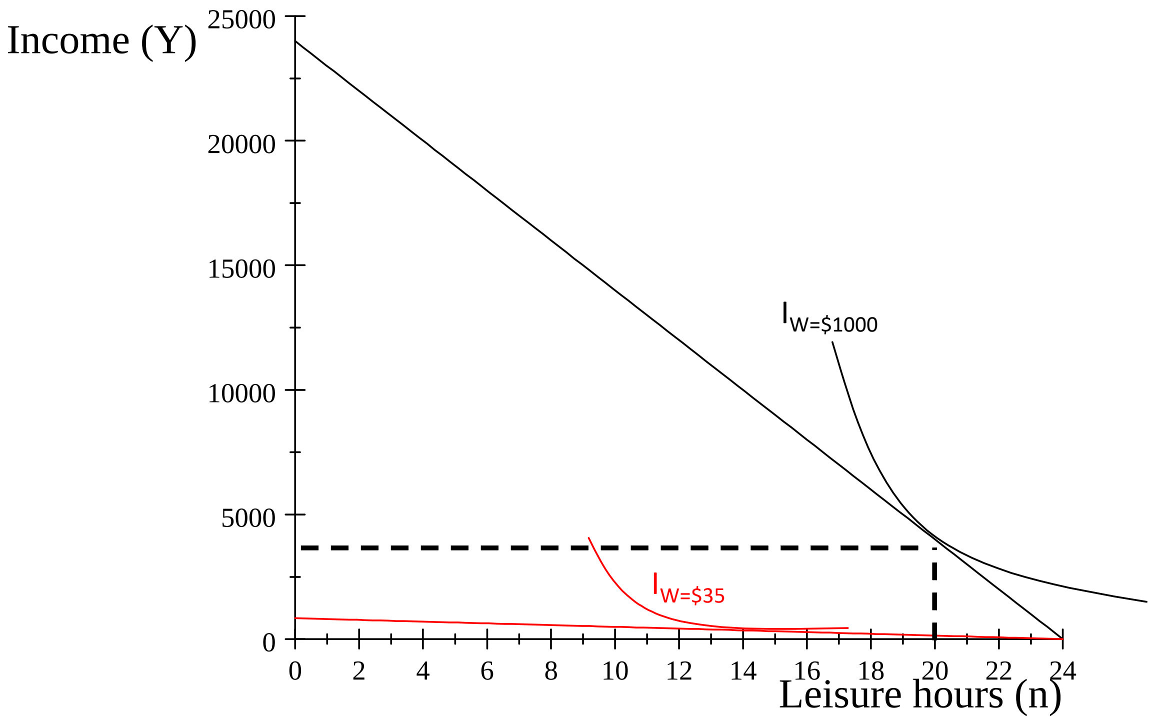

Figure 17: The optimal labor-leisure decision for an individual who faces a wage rate of $1000 per hour (black line) and $35 per hour (red line).

for leisure time. In this model, if time is not spent on leisure it is spent at work. As the wage increases the hours of work per day increase, so in essence we could create a labor supply curve using this analysis. When $w = \$15$, $h = 8$; when $w = \$35$, $h = 10$.

Now, suppose that the hourly wage rate is $w = \$1,000$. Figure 17 shows the budget constraints for that wage rate as well as the budget constraint for $w = \$35$, which is barely perceptible at the bottom of the graph as the red line. From Figure 16 we know the individual chooses 14 hours of leisure and 10 hours of work when the wage rate is $35; now, when the wage rate is $\$1,000$, the individual chooses to work only 4 hours per day and takes 20 hours of leisure. Thus, the wage rate has increased (alternatively, the opportunity cost of leisure has increased) and yet the individual works fewer hours! That result is not consistent with our concepts of supply and demand, so either the models as we have defined them do not fully capture this decision or the decision by this individual is unreasonable. Begin with the latter – does this individual's decision seem reasonable? Note that the indifference curves have the usual shape, so the decision is not some quirk of a special type of indifference curve. Beyond the basics of the model, it seems perfectly reasonable to me – working four hours per day generates $\$4,000$ of income under a $\$1,000$ per hour wage. The individual could work five 4 hour days per week and make $\$20,000$ per week. Extended to a year and the individual is making over $1 million per year. For sure they could make double that if they worked 8 hours per day, but there are tradeoffs and this particular individual might believe that earning $1 million per year while working 20 hours per week is better than earning $2 million per year and working 40 hours per week. Above some level of income individuals may shift away from work.

If the decision is reasonable, then our model of supply and demand must be incorrect. Considering the last statement of the previous paragraph – *above some level of income individuals may shift away from work* – suggests that there may not always be a positive relationship between the price of a good and its quantity supplied. Figure 18 shows an example of a backward bending labor supply curve. Labor supply is increasing up to a certain point and then begins to decrease. We have seen that both parts of that labor supply curve (the positive relationship and negative relationship) are possible using our indifference curve

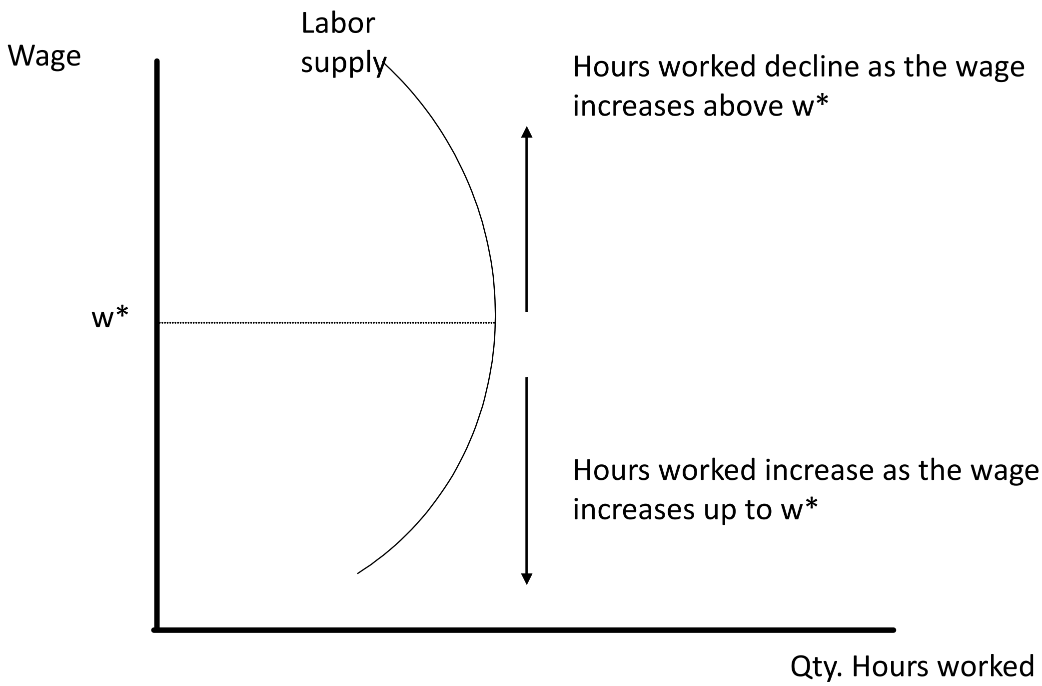

Figure 18: A backward bending labor supply curve.

analysis.[23] The backward bending labor supply curve is also related to the Laffer Curve used to motivate the initial discussion of income taxation. The Laffer Curve is based on the idea that tax rates influence individual labor-leisure decisions and, as we see in Figure 18, those labor-leisure decisions do not necessarily move in the same direction for all wage rates. As with the Laffer Curve, a key question to ask is what is the optimal w^* such that individuals begin to reduce their hours worked?[24]

Effect of Taxation Thus far we have developed a model of labor-leisure decisions but have not yet incorporated taxation. One important concept to keep in mind is that changes in policy that do not affect the underlying goods in the economy should not affect the indifference curves themselves. In our two-good labor-leisure model, the individual derives utility from income and from leisure – a policy that does not restrict what the individual purchases with the income earned or how leisure time is spent should not affect the shape of the indifference curves. If there is a policy that prohibits consumption of a good or changes preferences of a good then that would affect the indifference curves, but if the policy change merely affects the amount of income earned that should not affect the indifference curves themselves.

So where does a policy change affect our model of consumer choice? The budget constraint will be affected. It is important to note that different policies may affect the budget constraint in different ways. All the budget constraints we have seen thus far have been linear and that will still be the case if the income tax rate is a percentage rate that does not vary with income or wage. For example, if there is a 10% tax rate on all income, then the individual earning $35 per hour before taxes are collected will earn $31.50 per hour after taxes are collected, so only the slope of the budget constraint is affected. The same is true for the

[23] There is some debate on whether the effect of a larger income is important in determining working hours. For opposing views on the behavior of NYC cabdrivers see:
Camerer, et al. 1997 *Quarterly Journal of Economics*
https://www.jstor.org/stable/2951241
And Farber 2005 *Journal of Political Economy*
https://www.jstor.org/stable/10.1086/426040

[24] In reality the decision is more complicated than it is here because labor supply decisions affect not only income today, but also income for the future. This model is a static model that does not account for those intertemporal decisions.

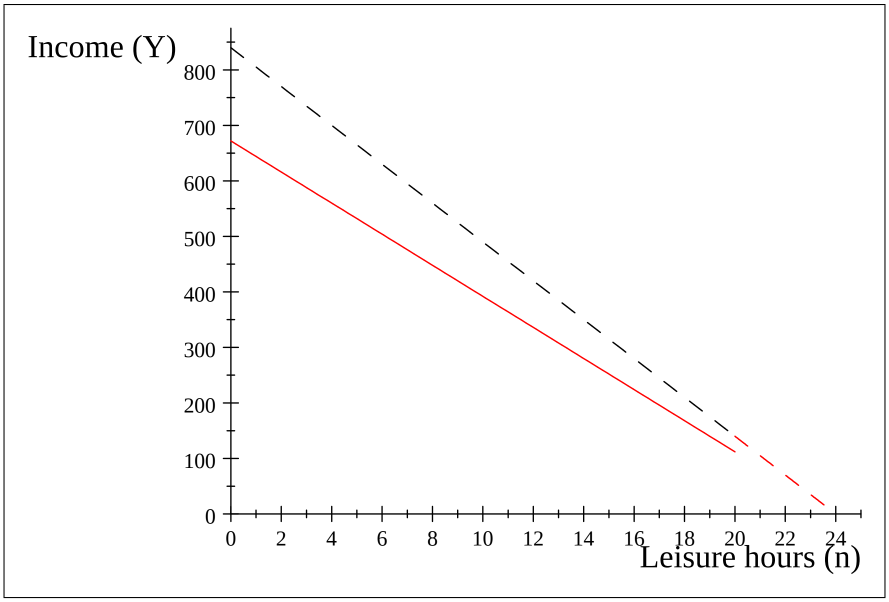

Figure 19: The budget constraint for a tax system where daily income greater than \$140 is taxed at a rate of 20%. The dashed line (both red and black portions) are the original budget constraint without a tax. The red line (both solid and dashed) are the budget constraint after the tax is imposed.

individual earning \$15 per hour before taxes are collected; after taxes are collected, that individual earns \$13.50 per hour, so again only the slope of the budget constraint is affected. While policies like those will alter behavior, they do not affect the budget constraint in any way that we have not seen.[25]

But suppose that individuals who earn more higher levels of income are taxed at higher rates than those with lower levels of income, or that individuals with lower levels of income are provided a subsidy. Suppose an individual works overtime hours, which changes the wage rate after a specified number of hours are worked. Those changes will affect the budget constraint but they will not affect the budget constraint in ways that we have seen in our standard two-good economy.

Return to our model of the \$35 per hour wage. Suppose there is a policy that states that an individual can earn up to \$140 per day and not pay any income tax; however, after earning \$140 before taxes in a day, total income is taxed at 20%. This tax policy means that if an individual earns \$141 in that day, then the individual must pay \$28.20 in taxes. Note that under this policy, the individual who earns \$175 before taxes earns \$140 after taxes, so earns the exact same amount as an individual who only earns \$140 that day and pays no taxes.[26] We could structure the policy so that only income above \$140 is taxed but that is a different system and the budget constraint would change in a different manner.

Figure 19 shows the budget constraint under this tax policy. The dashed line (both black and red) shows the original budget constraint without the tax; the red line (both solid and dashed) shows the budget constraint after the tax has been levied. Note that when leisure hours equals 20 (or work hours equals 4) the budget constraint is discontinuous. At 4 work hours income is \$140, but as soon as there is any work hour above 4 (even 4.01 or 4.001 or 4.0001, etc.) the income level drops to close to \$112 due to the tax.

What are the implications for our consumer? That depends on what the consumer's initial optimal labor-leisure decision is. Figure 20 shows one possible reallocation of labor and leisure hours.[27] Prior to the tax, the individual was working eight hours per day and receiving \$270 in income per day. After the

[25] Technically the y-intercept, which is $T * w$, is also affected. However, the x-intercept, which is just T, is not. So this shift of the budget constraint is like one that would occur with an increase in the price of the good on the y-axis (which we have been calling Good 1 in the general consumer choice problem).

[26] As with some of the earlier examples, that is a very sharp change in the tax rate, but a change of that magnitude makes it easier to illustrate the changes in the budget constraint.

[27] Note that I have restricted both axes to better focus on the discontinuity of the budget constraint as well as areas of interest in this individual's decision-making process.

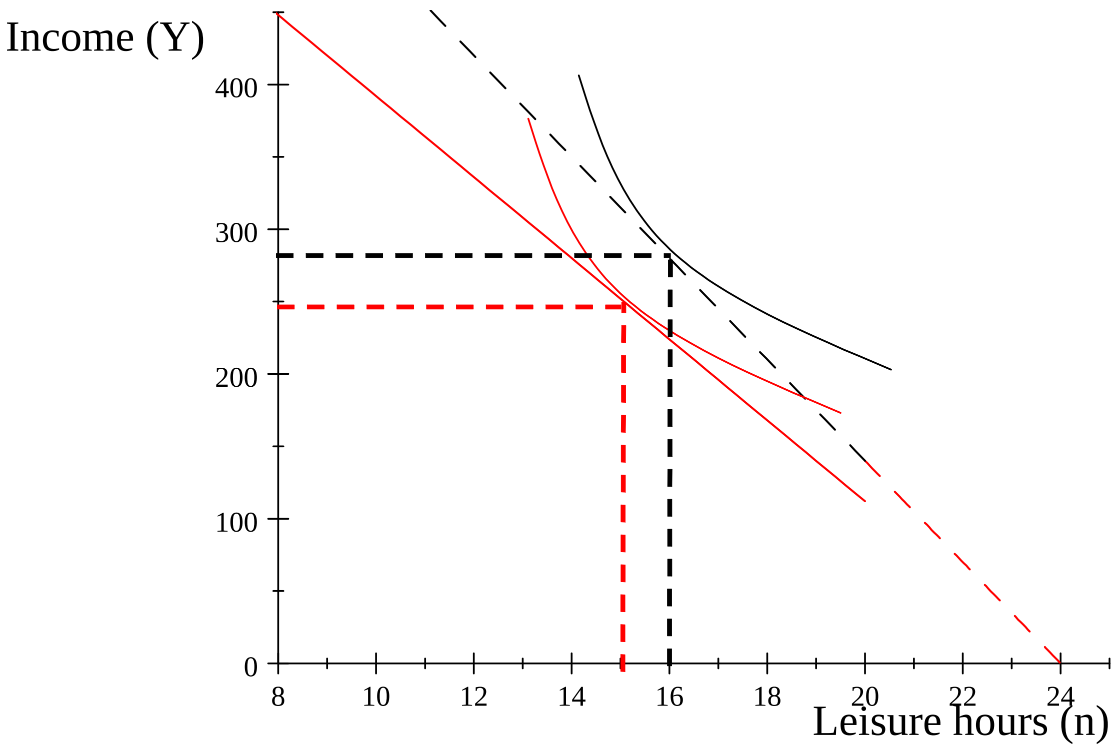

Figure 20: Reallocation of optimal choice for a consumer after the tax is levied.

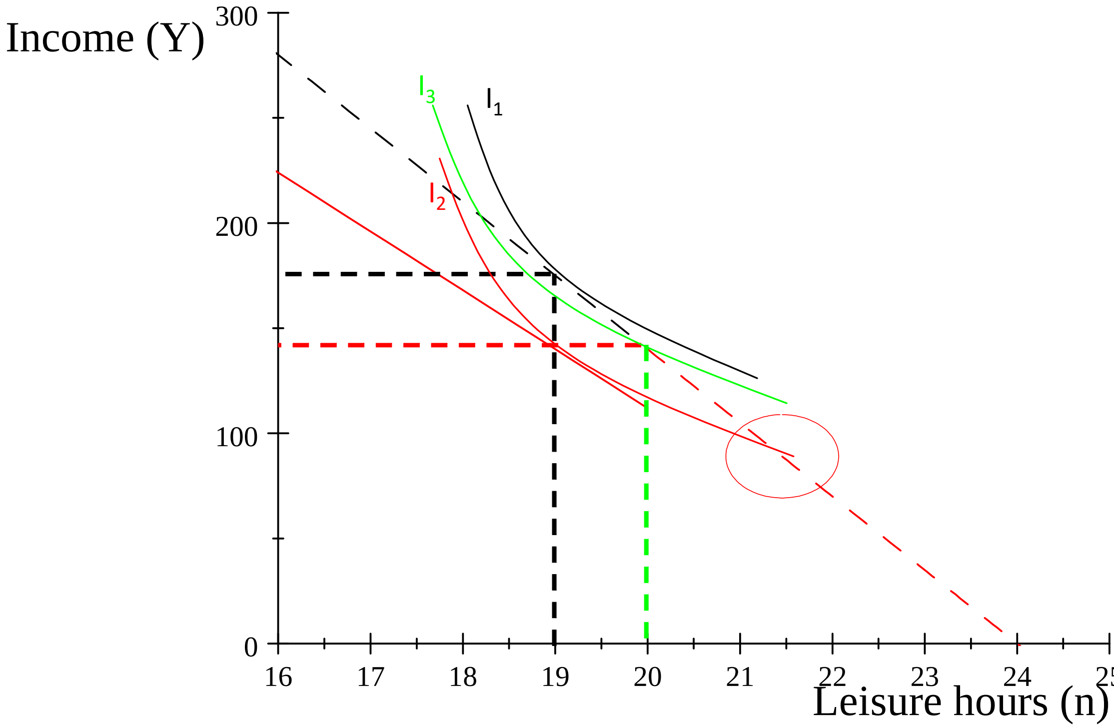

Figure 21: Indifference curve analysis for an individual's labor-leisure decision when the individual is choosing initial optimal hours near the discontinuity caused by the tax policy.

tax, the individual has shifted to nine hours per day and is now receiving \$252 in income. The indifference curve could have shifted such that the individual works ten hours per day and receives \$280 in income or even remained at eight hours of work. Given the initial labor-leisure choice and standard shaped indifference curves, it is very likely that this individual's optimal choice after the tax is on an indifference curve that is tangent to the solid portion of the after-tax budget constraint.

Now consider an individual who is only working five hours per day under the no-tax budget constraint. Figure 21 shows a potential outcome for this individual.[28] Under the pre-tax budget constraint the individual chose five hours of work and received \$175 in income. This decision is represented by the black indifference curve I_1. When the tax is imposed, if we find where the individual's indifference curve is tangent to the new budget constraint it remains at five hours of work but now the individual is only earning \$140 in income. This decision is represented by the black indifference curve I_2. However, is this choice optimal for the individual? Note the circled area – indifference curve I_2 intersects the red dashed portion of the budget constraint (which is still part of our budget constraint). Because indifference curve I_2 intersects the budget constraint, it seems possible that the individual could achieve a higher utility level based.

Consider indifference curve I_3 in green. Indifference curve I_3 is not the highest achievable indifference curve under the pre-tax budget constraint – clearly it provides less utility than I_1 which is the indifference curve at the optimal bundle in the pre-tax economy. However, I_3 does provide more utility than I_2. And I_3 is feasible because it just touches the corner of the red dashed portion of the after-tax budget constraint. Does it seem reasonable that the consumer would choose the feasible labor-leisure bundle given by I_3 over the one given by I_2? At I_2, the individual receives \$140 in income but works five hours; at I_3, the individual receives \$140 in income and works four hours. To me, it would seem unreasonable not to shift to the bundle given by I_3 – why work an extra hour only to receive the same income? While I have placed the indifference curves purposefully, the general idea that the consumer may be able to jump from an indifference curve that

[28] I have further restricted the axes to provide an even closer focus on the range of interest.

is tangent to the solid portion of the after-tax budget constraint to a higher indifference curve that just touches the corner of the dashed portion of the after-tax budget constraint will hold for some consumers.

Now, to be clear, I_3 is NOT tangent to any part of the dashed (pre-tax) budget constraint – it intersects that dashed budget constraint at 20 leisure hours and $140 of income.[29] But when there are corner solutions, which can happen when the budget constraint is not a straight line or the individual has unusual shaped indifference curves, there is no guarantee that the slope of the indifference curve is tangent to the slope of the budget constraint. As I have mentioned throughout class, we have theoretical models based on fairly restrictive assumptions, then we break those assumptions to make the model more realistic and reevaluate the predictions. In this particular case we have broken how the budget constraint is created. In other cases we could have indifference curves that lead the consumer to consume all of one good and none of the other good, which is the standard type of "corner solution" we describe in economics.

We can use indifference curve analysis to show these changes in individual behavior, which is interesting (at least to me) independently of any other goals. However, a more important goal is to highlight how changing the incentives an individual faces could change behavior. For the individual in Figure 20 who was working eight hours per day initially, that individual still makes a choice that is consistent with our general model of consumer choice – price of one good increases, the budget constraint shifts inward, the individual chooses a new optimal bundle tangent to the budget constraint. That result is fairly standard. But when the individual in Figure 21 shifts from five hours of work to four hours of work due to the tax policy, the individual ends up choosing a bundle that is not tangent to the budget constraint and the government actually collects no revenue from that individual because the individual's daily earnings are low enough to be exempt from the policy. When considering policy actions, it is especially important to consider how the policy would affect the incentives of those individuals who are choosing optimal bundles near the bounds of the policy actions,[30] as those individuals might have unexpected (to someone not trained to think about them) responses to the policy action.

3 Criticisms

You can likely find volumes to criticize tax policy itself, particularly specific taxes, but that is not the goal here. The goal is to consider the shortcomings of the models presented. As mentioned at the outset, the models are partial equilibrium models – they only consider the effect of the tax on the market being taxed or the effect of a tax on a single individual, but there may be effects in other markets that are being ignored. Most current analysis of tax policy uses much more complicated models, which rely on a good bit more math, to determine the effects of the tax. The NBER has a public economics program that is likely where you will find the cutting edge tax research in economics.

[29] It also intersects the dashed (pre-tax) budget constraint somewhere around 18 hours of leisure and $210 of income.

[30] In this particular example the policy action caused a discontinuity in the budget constraint. In other examples, the budget constraint may be kinked in that it is continuous but its slope changes because the tax rate changes based on income or hours worked.

If we were to impose a tax on all daily income earned over $140, and not a tax on all daily income once $140 is earned, then that budget constraint would be continuous but kinked at the point where leisure hours equals 20 and income equals $140. The slope would be flatter for leisure hours between $[0, 20]$ and steeper for leisure hours between $[20, 24]$.