

# PPOL 8640/ECON 6090 Assignment 4 Answers

Due: December 6<sup>th</sup> at 6pm

1. (30 points) In an eBay auction, bidders submit bids throughout the duration of the auction, which has a specified ending time. When a bidder places a bid, the auction price, which is visible to all bidders, becomes the second-highest existing bid plus some minimum increment. Assume the minimum increment is \$1. So if the current price is \$70 and Bidder A is currently winning the auction (with a bid of \$75, though other bidders do not know Bidder A's exact bid), if Bidder B bids \$120 then Bidder B will currently be winning the auction and the price will now be \$76 (the \$75 bid by Bidder A is now the second-highest bid and there is a \$1 minimum increment).

- a What type of auction format is most similar to the eBay auction? What bid should the bidders place, keeping in mind that bids are a function of players' values?

**Answer:**

The second-price sealed bid auction and ascending clock auctions are most similar to the eBay auctions, though neither are exact matches, as the eBay auction is essentially a combination of the two. Bidders should bid their value as they would in a second-price sealed bid auction or an ascending auction. If they bid more than their value then they run the risk of winning and paying a price greater than their value; if they bid less than their value then they run their risk of losing when they could have won and received a positive surplus.

Suppose now that Bidder C bids \$92 and that is the last bid in the auction.

- b Answer the following

- Who is the winner of the auction and what is the final price that is paid?

**Answer:**

Bidder B will win the auction and pay a price equal to \$93 because the second highest bid is \$92 and there is a \$1 minimum increment.

- Assuming that the winning bidder had a value of \$120, and this value was the highest among all bidders. Is the auction efficient and how much consumer surplus did the winner receive?

**Answer:**

The auction is efficient because the highest valued bidder won the item. The winner received \$27 in consumer surplus.

- Assuming the seller had a cost of \$50, how much profit did the seller receive?

**Answer:**

The seller received \$93, so with a cost of \$50 the seller received \$43 in profit.

Now assume that the seller employs a shill bidder, who is not interested in winning the item, but places bids in an effort to drive up the final sales price for the seller. Suppose the shill bidder bids \$110.

- c Answer the following

- Who is the winner of the auction and what is the final price that is paid?

**Answer:**

Bidder B bid \$120 so is still the winner of the auction because \$120 is the highest bid. The price paid is now \$111 because the shill bidder bid \$110 and there is the \$1 increment.

- Assuming that the winning bidder had a value of \$120, and this value was the highest among all bidders. Is the auction efficient and how much consumer surplus did the winner receive?

**Answer:**

The auction is still efficient because the bidder with the highest value won. However, consumer surplus is now only \$9 as Bidder B has a value of \$120 but pays \$111 for the item.

- Assuming the seller had a cost of \$50, how much profit did the seller receive?

**Answer:**

Employing the shill bidder is profitable – the seller’s profit is now  $\$111 - \$50 = \$61$  which is almost 50% more than what the seller earned when the shill bidder was not present.

- d** Do you believe shill bidding falls is per se illegal given antitrust regulations? Explain why or why not.

**Answer:**

While the practice seems like it should be per se illegal because one of the per se illegal activities is bid rigging, it is a gray area of the law. Shill bidding is illegal on eBay and many states have laws against the practice, depending upon the type of auction. There are some auctions that have a reserve price (a minimum price that must be met if the object is to be sold), and some auctions allow shill bidding up to the reserve price under the idea that the item is not going to sell if the price does not reach the reserve so there is really no harm done.

- e** Suppose that the potential buyers in the auction decide to discuss who will bid and which bid(s) they will make. Do you believe this practice by buyers is per se illegal given antitrust regulations? Explain why or why not.

**Answer:**

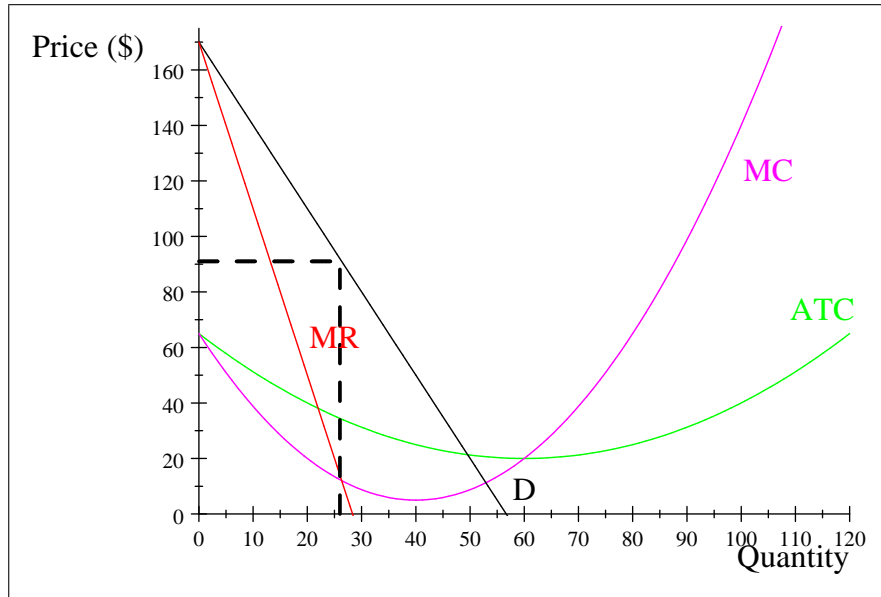
This practice is the definition of bid rigging and is per se illegal. It may make more sense to think about this behavior in an auction where the auctioneer is trying to procure a service (like repaving a road) from a contractor. If the contractors get together and decide who will bid and what bid to make, then they can keep prices high.

- f** Suppose that instead of conducting an auction, a seller posts a price for the good but is willing to accept offers from buyers. Suppose that the seller posts a price of \$130, but negotiates a final price of \$111 with the same person who won the auction in part **c**. Is this practice per se illegal or likely to violate any antitrust regulations? Compare with your answer to parts **c** and **d**, and explain why the process of completing a transaction is important in determining antitrust violations.

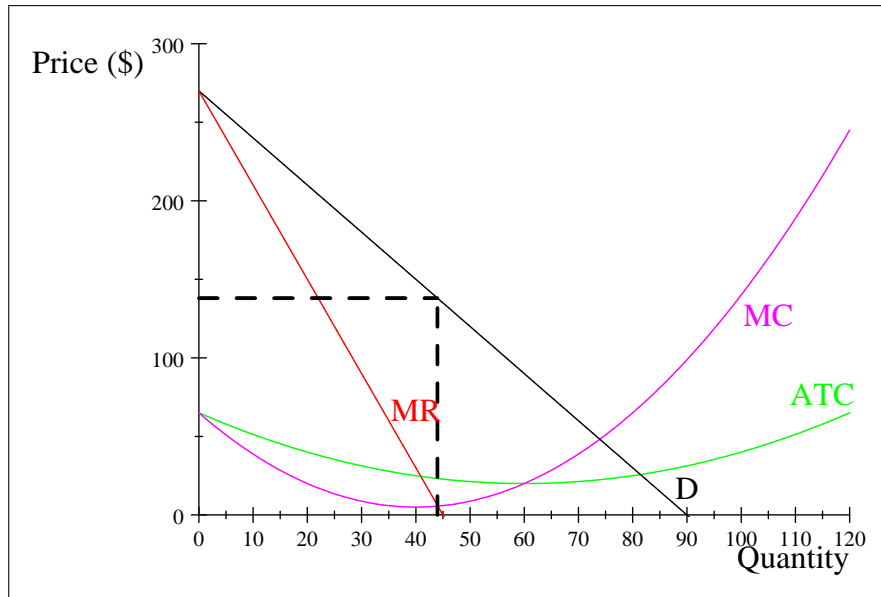
**Answer:**

There is nothing per se illegal about posting a high price and then negotiating a lower price with a buyer. The practice is also unlikely to violate any antitrust laws. Note that the outcome from this process is exactly the same as the outcome from employing the shill bidder in the auction – same bidder wins and pays the same price. However, the appearance of "fair" competition is important in determining antitrust violations. In an auction, the belief is that individual bidders are bidding against others who truly want the item, and not phantom bidders who are only trying to drive the price higher.

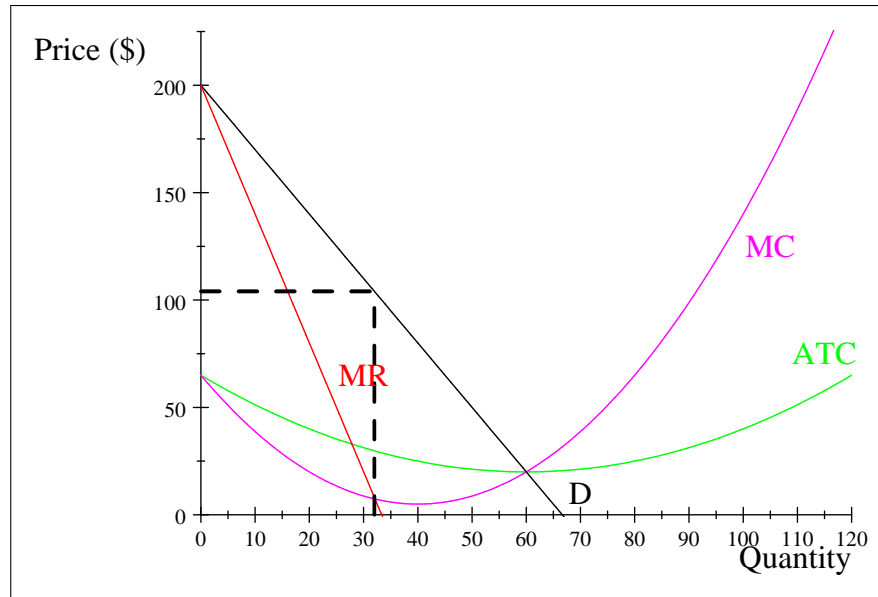
2. (15 points) We have discussed average cost pricing and marginal cost pricing. Suppose there are three monopolies with the demand and cost curves as given in the pictures:



Monopolist 1



Monopolist 2



Monopolist 3

Note that for Monopolist 3, demand,  $MC$ , and  $ATC$  all intersect at the same point (quantity of 60 and price of \$20)

a Identify each monopolist's profit-maximizing price and output on their respective graph.

**Answer:**

Identified on each graph. For Monopolist 1,  $Q^* \approx 26$  and  $P^* \approx \$91$ . For Monopolist 2,  $Q^* \approx 44$  and  $P^* \approx \$138$ . For Monopolist 3,  $Q^* \approx 32$  and  $P^* \approx \$104$ .

b Which of these monopolists, if any, meets the definition of natural monopoly? Explain.

**Answer:**

Monopolist 1 clearly meets the definition of natural monopoly because  $ATC$  is declining throughout the relevant range of demand, which is where  $ATC$  intersects demand. Monopolist 3 also meets the definition of natural monopoly, because we know that  $MC$  intersects  $ATC$  at its minimum, so its  $ATC$  is declining until it intersects the demand curve. However, Monopolist 2 is not a natural monopoly because demand intersects the  $ATC$  on the upward sloping portion of  $ATC$ . Note that the monopolist does not always have to have declining average total cost in order to be a natural monopolist, just that the average total costs need to be declining up until the point where  $ATC$  intersects demand.

c For each of these monopolists, would average cost pricing or marginal cost pricing lead to the most efficient, in terms of least deadweight loss, market? Explain.

**Answer:**

Marginal cost pricing will always lead to the least deadweight loss because by definition the gains from trade are maximized at the quantity where demand intersects marginal cost. However, in Monopolist 3, average cost pricing will also minimize deadweight loss because demand,  $MC$ , and  $ATC$  all intersect at the same point. Monopolist 3 is a unicorn situation for a regulator as pricing regulation in this case provides the same outcome as perfect competition (zero economic profit, no deadweight loss, price equal to the minimum of average total cost).

Monopolist 2 is interesting to discuss because this case illustrates a potential drawback of attempting pricing regulation with a monopoly that is not a natural monopolist. Marginal cost pricing still works as intended, where the price at which  $MC$  intersects demand is about \$48 and the quantity is about

74. There is an increase in production, the price is lower, and the deadweight loss from the monopolist is eliminated. However, the monopolist is still making a positive economic profit because  $P > ATC$  when  $Q = 74$ , though the profit is lower under marginal cost pricing. Recognize that the monopolist is adhering to the  $MR = MC$  rule because  $MR$  is a flat line at \$48 until the monopolist increases production beyond 74 (which is where  $MC$  intersects demand).

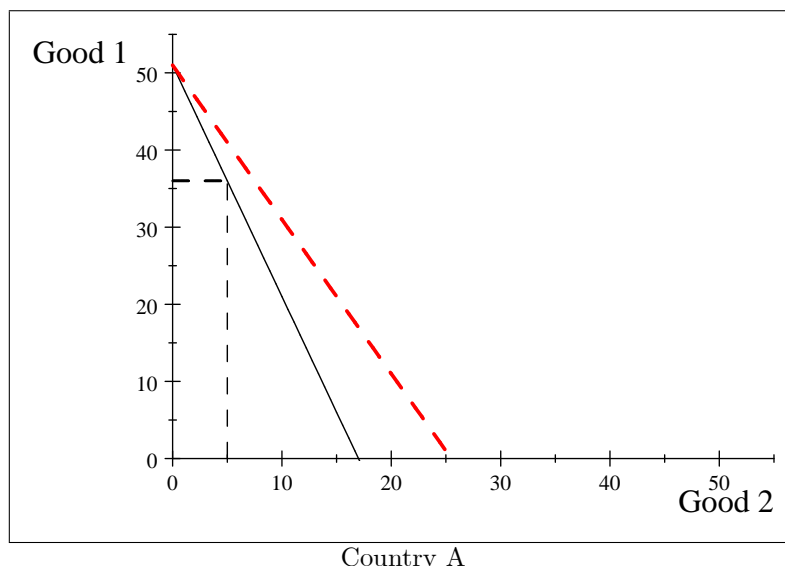
For average cost pricing, the price where  $ATC$  intersects demand for Monopolist 2 is about \$25.75 and the quantity is about 81. That seems even "better" than marginal cost pricing (lower price, larger quantity, and the monopolist is earning zero economic profit; clearly the regulator is ignoring that some of those last units have a marginal cost greater than the price which is inefficient, but the observables of price and quantity look better) but would Monopolist 2 actually produce a quantity of 81? The marginal revenue in this case is flat at \$25.75, and at a quantity of 81 Monopolist 2 has  $MC > MR$ . As Monopolist 2 is only regulated by price, not quantity, Monopolist 2 is free to choose its quantity, and it will not choose a quantity of 81 and earn zero economic profit when it could choose a lower quantity and earn a positive economic profit. So Monopolist 2 would actually cut back quantity to the point where  $MR = MC$ , which in this case would be a quantity level around 64. It still has to sell those units at \$25.75, but Monopolist 2's  $ATC$  at a quantity of 64 is about \$20.15, so it is earning a \$5.60 profit on each unit, which is better than earning \$0 profit on each unit if it were to sell 81 units. It may seem odd because there is a lower price AND lower production under average cost pricing for Monopolist 2 than under marginal cost pricing, but that is the outcome in this case when Monopolist 2 acts optimally given its constraints. There is also more deadweight loss under average cost pricing than marginal cost pricing for Monopolist 2 despite the lower price under average cost pricing. Note that Monopolist 2 could produce the efficient quantity of 74 (which we found under marginal cost pricing) and still have a positive economic profit, so deadweight loss could be minimized, but it is not in Monopolist 2's best interest to do so because  $MC > MR$  at a quantity of 74.

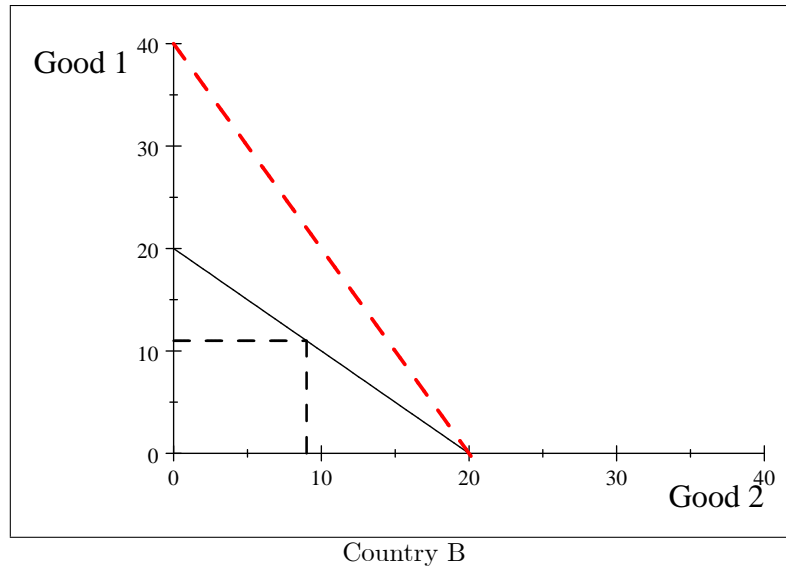
d Of the three monopolists, is there any monopolist for which marginal cost pricing is unsustainable? Explain.

**Answer:**

Marginal cost pricing is unsustainable for Monopolist 1 because  $ATC > P$  when using marginal cost pricing. Monopolist 3 is making a zero economic profit under marginal cost pricing while Monopolist 2 is making a positive economic profit under marginal cost pricing.

3. (25 points) Consider an international trade market between two countries producing Good 1 and Good 2. The production possibilities curves for the two countries are in the figures below.





Country A can produce 51 units of Good 1 if it produces zero Good 2 and 17 units of Good 2 if it produces zero Good 1. In autarky, it chooses its optimal bundle as 36 Good 1 and 5 Good 2. Country B can produce 20 units of Good 1 if it produces zero Good 2 and 20 units of Good 2 if it produces zero Good 1. In autarky, it chooses its optimal bundle as 11 Good 1 and 9 Good 2.

- a** How much of Good 1 does Country A have to give up in order to produce a unit of Good 2? How much of Good 1 does Country B have to give up in order to produce a unit of Good 2?

**Answer:**

Country A has to give up 3 units of Good 1 to produce a unit of Good 2. Country B has to give up 1 unit of Good 1 to produce a unit of Good 2.

- b** Which country is the lowest opportunity cost provider of each good? Should the countries specialize in different goods? Explain.

**Answer:**

Country A is the lowest opportunity cost provider of Good 1 (it gives up  $\frac{1}{3}$  of a unit of Good 2 to produce a unit of Good 1 as opposed to Country B which gives up a unit of Good 2). Country B is the lowest opportunity cost provider of Good 2 because it only gives up 1 unit of Good 1 to produce a unit of Good 2, whereas Country A gives up 3 units of Good 1.

Country A should produce Good 1 and Country B should produce Good 2.

- c** Suppose the terms of trade are 2 units of Good 1 for 1 unit of Good 2. Explain why both countries would agree to these terms and draw the trading possibilities curve for each country.

**Answer:**

If Country A wants to produce a unit of Good 2 domestically it must give up 3 units of Good 1; giving up 2 units of Good 1 is a better deal. Similarly, if Country B wants to produce a unit of Good 1 domestically it must give up 1 unit of Good 2; giving up a half a unit of Good 2 for 1 unit of Good 1 is a better deal. The trading possibilities curves are the dashed red lines in each country's graph. Country A's curve now intersects the Good 2 axis at 25.5, while Country B's curve now intersects the Good 1 axis at 40.

- d** Given the terms of trade in part **c**, can the countries both consume more of both goods (when compared to their autarky economy) through specialization and trade or would at least one country have to produce some amount of both goods? Explain.

**Answer:**

Country A produces 36 Good 1 and 5 Good 2 in autarky and 51 Good 1 by specializing. If they trade away 12 Good 1 they will receive 6 Good 2 and have 39 Good 1 and 6 Good 2.

Country B produces 11 Good 1 and 9 Good 2 in autarky and 20 Good 2 by specializing. If they trade away 6 Good 2 they will receive 12 Good 1 and have 12 Good 1 and 14 Good 2.

With that specific trade, 12 Good 1 from Country A for 6 Good 2 from Country B, both countries can be strictly better off by specializing and trading.

Another trade that could be made is 14 Good 1 for 7 Good 2. Country A would then have 37 Good 1 and 7 Good 2; Country B would have 14 Good 1 and 13 Good 2.

4. (30 points) Line City is a strip of land 30 miles long that runs east-west. As its name suggests, Line City is a perfect line. Residents live in houses which are distributed uniformly throughout the 30 miles – what this means is that they are evenly spaced along the line. There are two competing gas stations, Alpha Station and Bravo Station. They are attempting to determine where to locate their respective stations. They know that the residents of Line City will go to the gas station closest to their home, and if two gas stations are equidistant they will choose among the gas stations with equal probability (essentially the gas stations count these equidistant residents as half a customer). The objective of each station is to maximize the number (alternatively the proportion) of customers who visit their store and the firms only compete by choosing location.

a Define the concept of Nash equilibrium.

**Answer:**

A Nash equilibrium is a set of strategies such that no player can unilaterally deviate from the set of strategies and receive a strictly higher payoff. Alternatively, a Nash equilibrium is a set of strategies such that all players are playing best responses to each other.

- b Assume that residents of Line City are able to drive both east and west along Line City and that gas stations are free to locate anywhere along the 30 miles of the city. A Nash equilibrium to this game is a location choice for each gas station. There is one Nash equilibrium to this game. What is the Nash equilibrium? Explain why the set of locations you have identified is the Nash equilibrium.

**Answer:**

The only Nash equilibrium is for both to locate at the midpoint of the line. If both locate at the midpoint, both get half the customers. If one moves away from the midpoint, say to point 14, then that station will lose customers as it will lose all the customers who live between mile 14.5 and 15; thus they will get less than half ( $\frac{29}{60}$ ) of the customers.

There are other locations on the line which give equal customers to both stations. If the stations locate at different endpoints (Alpha at mile marker 0 and Bravo at mile marker 30) then they both get half the customers. However, that is not an equilibrium because both stations have the incentive to move right next to the other station; Alpha station should move to spot 29.9999 (as close as possible to 30) if Bravo is at 30. But then Bravo has the incentive to move just to the left of Alpha (say 29.9998). This leapfrogging continues until the stations get to the midpoint and both locate there.

The stations would also split the customers evenly if they located at the same point that is not the midpoint. Suppose both locate at mile marker 5. Then each station has the incentive to move to the side with the larger mass of customers, and the leapfrogging continues again.

This game structure is also used in examining how politicians position themselves along the political spectrum. There is a tendency for politicians to tend towards the center of the political spectrum, and it's the basis for discussions about the median voter theorem.

- c** A third gas station, Delta Station, has decided to compete. There is no Nash equilibrium to this game when there are three stations.<sup>1</sup> Explain why there are no Nash equilibria to this game.

**Answer:**

The third station introduces the problem that the leapfrogging behavior present at locations other than the midpoint when there are two stations now occurs at the midpoint. We can consider two potential types of equilibria – all locate at the same point, and all locate at different points. If all three locate at the midpoint, then each station receives  $\frac{1}{3}$  of the customers. However, each station has the incentive to move just left or right to capture almost half of the customers. If all three locate at some point other than the midpoint, then they all have the incentive to move to the side with the most mass of customers. If all three locate at different points, then the outer two stations have the incentive to move inward towards the station in the middle, and at some point the station in the middle would find it more profitable to become an outer station, moving to the side with the most mass. So there is no set of locations such that all stations want to remain where they are; whether they are all at the same point (regardless of what that point is) or all at different points, some station would like to change location.

- d** The citizens of Line City are distressed because they like equilibrium. They have proposed to alter their city in the following manner. Gas stations are only allowed to locate between miles 14 through 16, while residents are now uniformly distributed over miles 0 through 14 and 16 through 30 (half the people live between mile markers 0 through 14 and the other half from mile markers 16 through 30 – I know it involves a lot of moving houses around but that’s how seriously citizens of Line City take the concept of Nash equilibrium). Will this proposed structure solve the problem of no Nash equilibrium location choices? If so, find a Nash equilibrium. If not, explain why not.

**Answer:**

In this case, the structure does solve the problem of no Nash equilibrium locations. There is a Nash equilibrium (actually there are multiple Nash equilibria) where two stations locate at point 14 and the third station locates at point 16. The two stations at point 14 split the 50% of citizens who live between 0-14, so each receives 25% of the customers, while the station at point 16 receives 50% of the customers. The station by itself at point 16 has no incentive to move anywhere else because it cannot receive more than 50% of the customers. If it were to locate at point 14 then the station would drop to receiving only one-third of the customers. The two stations at 14 also do not have any incentive to move. If they move to the interior (somewhere between 14-16 but not at 14 or 16) then they would lose all their customers. If they moved to share point 16 with the third station, they would still receive 25% of the customers.

The same analysis holds if one station locates at 14 and the other two stations locate at 16.

- e** In essence the citizens of Line City rezoned some land for gas stations. Compared to part **b**, how does this rezoning affect the consumers? Do they travel more or less with the rezoning?

**Answer:**

In part **b** the citizens were traveling 7.5 miles on average to get gas; in part **d** that travel distance is reduced to 7 miles. So the citizens travel less, at least on average, which may or may not offset any utility loss that the citizens received from locating their houses between mile markers 14-16.

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<sup>1</sup>Technically there are Nash equilibria but they involve the stations choosing their locations by using a probability distribution, such as Alpha Station chooses to locate at mile marker 1 with 50% probability and mile marker 14 with 50% probability. Our concern is with the stations choosing a specific location (mile marker 9) with certainty.